Name: SOLUTIONS

Unit 2- Curve Sketching

WORKBOOK

MCV4U

graph feature		f(x)		f'(x)		f''(x)	Notes
rising (L to R)		slope > 0		+			
falling (L to R)		slope < 0		-			
extrema	maximum	slope = 0		= 0	+ on L - on R	− at x_{max}	derivative may not exist at a max or min, e.g. or
extrema	minimum	slope = 0		= 0	- on L + on R	+ at x_{min}	
inflection pt.		Curvature changes: ↔				= 0 potential inflection point	Check f''(x) on either side of a potential inflection point.
concave up				-	+	+	
concave down				+	-	-	

W1 – Increasing / Decreasing Unit 2
MCV4U
Jensen

SOLUTIONS

1) Use critical numbers and the first derivative test to determine when the function is increasing or decreasing.

a) $f(x) = x^3 + 3x^2 + 1$

$f'(x) = 3x^2 + 6x$

$0 = 3x(x+2)$

$x_1 = 0 \quad x_2 = -2$

	$-\infty$ to -2	-2 to 0	0 to ∞
Test value	-3	-1	1
$f'(x)$	+	−	+
$f(x)$	increasing	decreasing	increasing

increasing: $x < -2$, $x > 0$

decreasing: $-2 < x < 0$

b) $f(x) = x^5 - 5x^4 + 100$

$f'(x) = 5x^4 - 20x^3$

$0 = 5x^3(x-4)$

$x_1 = 0 \quad x_2 = 4$

	$-\infty$ to 0	0 to 4	4 to ∞
Test value	-1	1	5
$f'(x)$	+	−	+
$f(x)$	inc.	dec.	inc.

increasing: $x < 0$, $x > 4$

decreasing: $0 < x < 4$

c) $f(x) = 3x^4 + 4x^3 - 12x^2$

$f'(x) = 12x^3 + 12x^2 - 24x$

$0 = 12x(x^2 + x - 2)$

$0 = 12x(x+2)(x-1)$

$x_1 = 0 \quad x_2 = -2 \quad x_3 = 1$

	$-\infty$ to -2	-2 to 0	0 to 1	1 to ∞
Test	-3	-1	0.5	2
$f'(x)$	−	+	−	+
$f(x)$	dec.	inc.	dec.	inc.

increasing: $-2 < x < 0$, $x > 1$

decreasing: $x < -2$, $0 < x < 1$

d) $f(x) = (2x-1)^2(x^2-9)$

$f'(x) = 2(2x-1)(2)(x^2-9) + 2x(2x-1)^2$

$0 = 2(2x-1)[2(x^2-9) + x(2x-1)]$

$0 = 2(2x-1)(4x^2 - x - 18)$

$0 = 2(2x-1)[4x^2 - 9x + 8x - 18]$

$0 = 2(2x-1)[x(4x-9) + 2(4x-9)]$

$0 = 2(2x-1)(4x-9)(x+2)$

$x_1 = \frac{1}{2} \quad x_2 = \frac{9}{4} = 2.25 \quad x_3 = -2$

$-\infty \quad -2 \quad 0.5 \quad 2.25 \quad \infty$

Test	-3	0	1	3
$f'(x)$	−	+	−	+
$f(x)$	dec.	inc.	dec	inc.

increasing: $-2 < x < 0.5$, $x > 2.25$

decreasing: $x < -2$, $0.5 < x < 2.25$

2) Suppose that $f(x)$ is a differentiable function with the given derivative. Determine the values of x for which $f(x)$ is increasing and decreasing.

a) $f'(x) = (x-1)(x+2)(x+3)$

$0 = (x-1)(x+2)(x+3)$

$x_1 = 1 \quad x_2 = -2 \quad x_3 = -3$

$-\infty \quad -3 \quad -2 \quad 1 \quad \infty$

test value	-4	-2.5	0	2
$f'(x)$	−	+	−	+
$f(x)$	dec.	inc.	dec.	inc.

Increasing: $-3 < x < -2$, $x > 1$

decreasing: $x < -3$, $-2 < x < 1$

b) $f'(x) = x^2 + 2x - 4$

$0 = x^2 + 2x - 4$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{20}}{2}$

$x = \dfrac{-2 \pm 2\sqrt{5}}{2}$

$x = -1 \pm \sqrt{5}$

$x_1 \simeq 1.24 \quad x_2 \simeq -3.24$

$-\infty \quad -1-\sqrt{5} \quad -1+\sqrt{5} \quad \infty$

Test Value	-4	0	2
$f'(x)$	+	−	+
$f(x)$	inc.	dec.	inc.

increasing: $x < -1-\sqrt{5}$, $x > -1+\sqrt{5}$

decreasing: $-1-\sqrt{5} < x < -1+\sqrt{5}$

c) $f'(x) = x^3 + 3x^2 - 4x - 12$

$0 = x^2(x+3) - 4(x+3)$

$0 = (x+3)(x^2-4)$

$0 = (x+3)(x-2)(x+2)$

$x_1 = -3 \quad x_2 = 2 \quad x_3 = -2$

$-\infty \quad -3 \quad -2 \quad 2 \quad \infty$

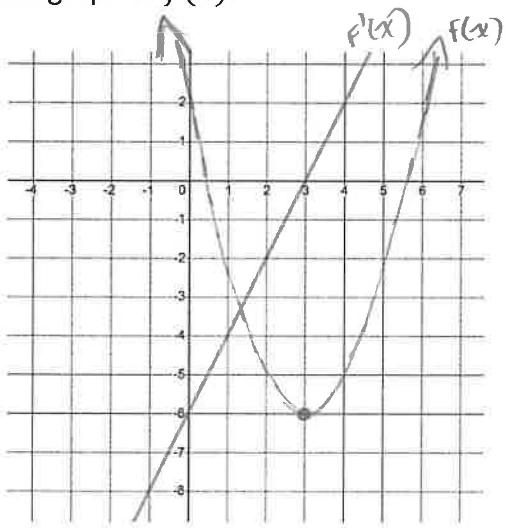
Test value	-4	-2.5	0	3
$f'(x)$	−	+	−	+
$F(x)$	dec.	inc	dec.	inc.

Increasing: $-3 < x < -2,\ x > 2$

decreasing: $x < -3,\ -2 < x < 2$

3) Given each graph of $f'(x)$, state the intervals of increase and decrease for the function $f(x)$. Then sketch a possible graph of $f(x)$.

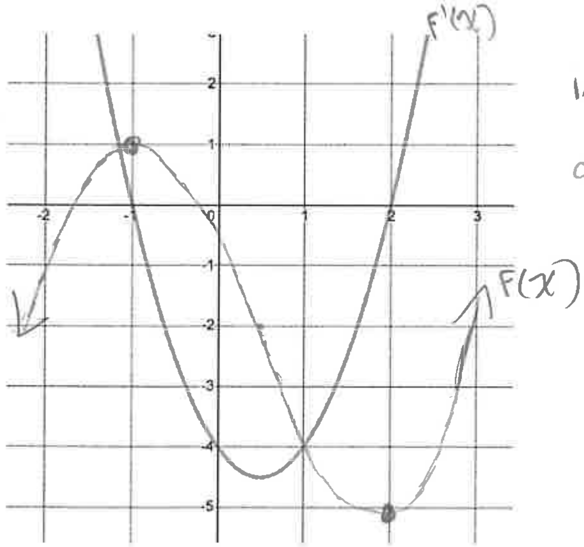
a)



increasing: $x > 3$

decreasing: $x < 3$

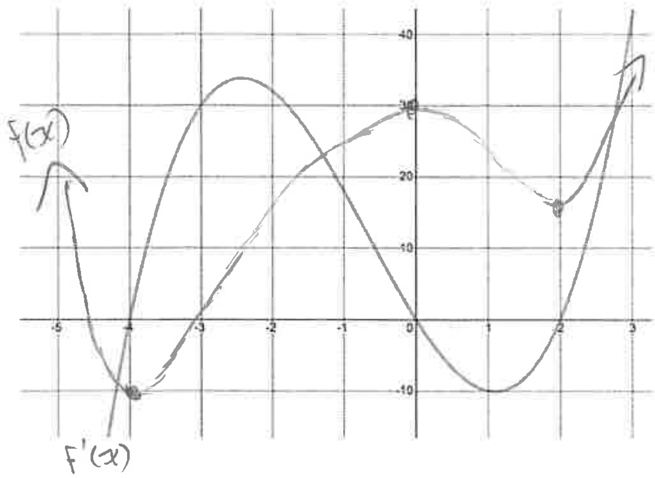
b)



Increasing: $x < -1,\ x > 2$

decreasing: $-1 < x < 2$

c)

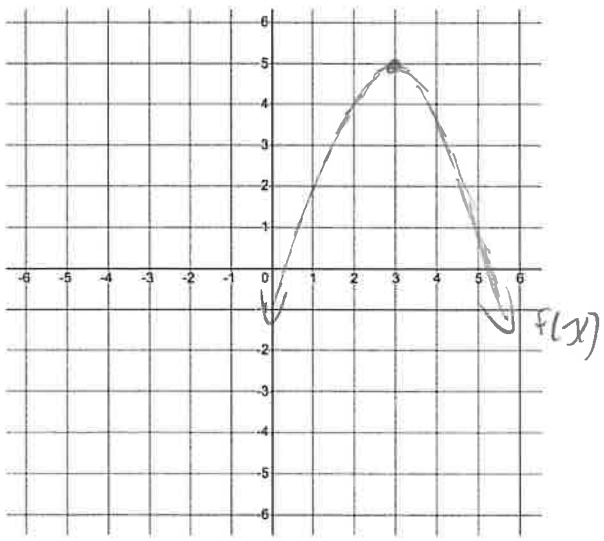


increasing: $-4 < x < 0$, $x > 2$

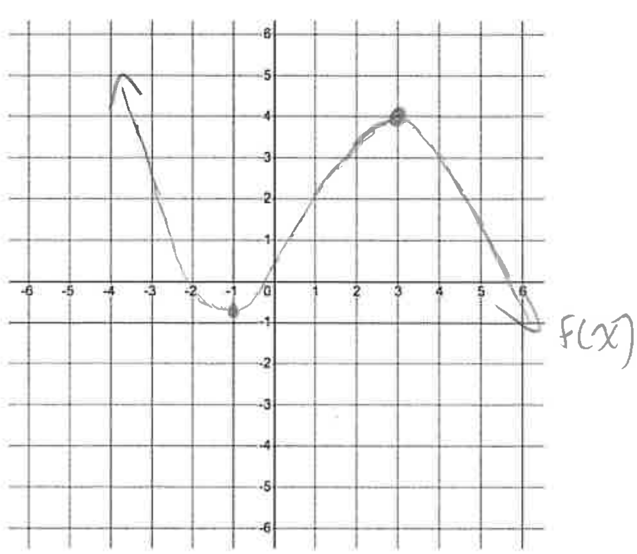
decreasing: $x < -4$, $0 < x < 2$

4) Sketch a continuous graph of $f(x)$ given each set of conditions.

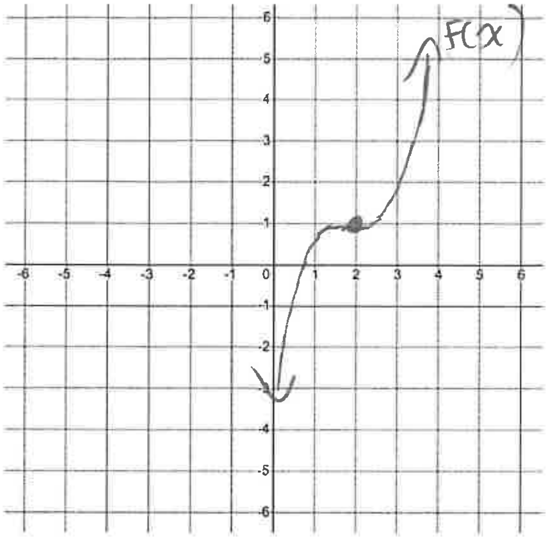
a) $f'(x) > 0$ (increasing) when $x < 3$, $f'(x) < 0$ (decreasing) when $x > 3$, $f(3) = 5$



b) $f'(x) > 0$ (increasing) when $-1 < x < 3$, $f'(x) < 0$ (decreasing) when $x < -1$ and when $x > 3$, $f(-1) = -\frac{20}{27}$, $f(3) = 4$



c) $f'(x) > 0$ when $x \neq 2$, $f(2) = 1$



Answers:

1)a) increasing: $x < -2, x > 0$
decreasing: $-2 < x < 0$

b) increasing: $x < 0, x > 4$
decreasing: $0 < x < 4$

c) increasing: $-2 < x < 0, x > 1$
decreasing: $x < -2, 0 < x < 1$

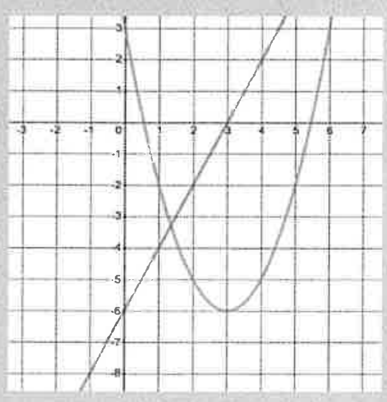
d) increasing: $-2 < x < 0.5, x > 2.25$
decreasing: $x < -2, 0.5 < x < 2.25$

2)a) increasing: $-3 < x < -2, x > 1$
decreasing: $x < -3, -2 < x < 1$

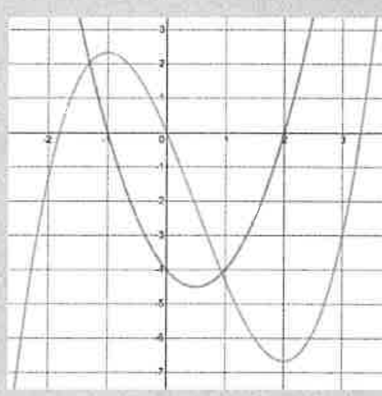
b) increasing: $x < -1 - \sqrt{5}, x > -1 + \sqrt{5}$
decreasing: $-1 - \sqrt{5} < x < -1 + \sqrt{5}$

c) increasing: $-3 < x < -2, x > 2$
decreasing: $x < -3, -2 < x < 2$

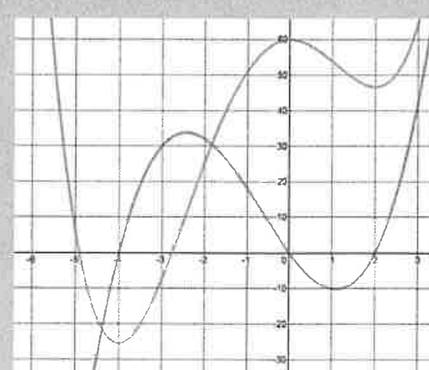
3a) increasing: $x > 3$
decreasing: $x < 3$



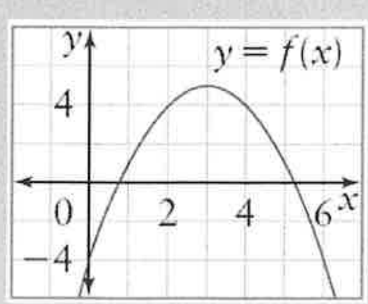
b) increasing: $x < -1, x > 2$
decreasing: $-1 < x < 2$



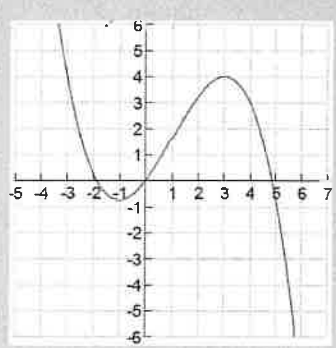
c) increasing: $-4 < x < 0, x > 2$
decreasing: $x < -4, 0 < x < 2$



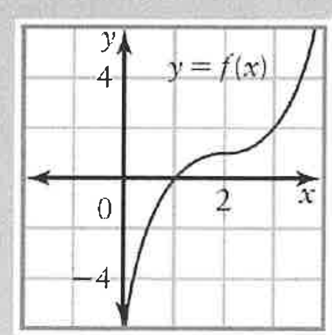
4)a)



b)



c)



W2 – Maxima and Minima Unit 2

MCV4U

Jensen

SOLUTIONS

1) Find the critical numbers for each function

a) $f(x) = -x^2 + 6x + 2$

$f'(x) = -2x + 6$

$0 = -2x + 6$

$x = 3$

b) $f(x) = x^3 - 2x^2 + 3x$

$f'(x) = 3x^2 - 4x + 3$

$0 = 3x^2 - 4x + 3$

$b^2 - 4ac = (-4)^2 - 4(3)(3)$

$b^2 - 4ac = -20$

$b^2 - 4ac < 0 \therefore$ no critical #'s

c) $g(x) = 2x^3 - 3x^2 - 12x + 5$

$g'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x_1 = -1 \quad x_2 = 2$

d) $y = x - \sqrt{x}$

$\frac{dy}{dx} = 1 - \frac{1}{2}x^{-1/2}$

$\frac{dy}{dx} = 1 - \frac{1}{2\sqrt{x}}$

$0 = 1 - \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1$

$1 = 2\sqrt{x}$

$\left(\frac{1}{2}\right)^2 = x$

$x = \frac{1}{4}$

2) Determine the absolute extreme values of each function on the given interval.

a) $y = 3x^2 - 12x + 7, 0 \le x \le 4$

$\frac{dy}{dx} = 6x - 12$

$0 = 6x - 12$

$x = 2$ is a critical #

$y(0) = 3(0)^2 - 12(0) + 7$
$= 7$

$y(2) = 3(2)^2 - 12(2) + 7$
$= -5$

$y(4) = 3(4)^2 - 12(4) + 7$
$= 7$

Absolute min: $(2, -5)$

Absolute max: $(0, 7)$ and $(4, 7)$

b) $g(x) = 2x^3 - 3x^2 - 12x + 2, -3 \le x \le 3$

$g'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

critical #'s : $x_1 = 2$ $x_2 = -1$

$g(-3) = 2(-3)^3 - 3(-3)^2 - 12(-3) + 2$
$= -43$

$g(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 2$
$= 9$

$g(2) = 2(2)^3 - 3(2)^2 - 12(2) + 2$
$= -18$

$g(3) = 2(3)^3 - 3(3)^2 - 12(3) + 2$
$= -7$

absolute min: $(-3, -43)$

absolute max: $(-1, 9)$

c) $f(x) = x^3 + x, 0 \le x \le 10$

$f'(x) = 3x^2 + 1$

$0 = 3x^2 + 1$

$x = \pm\sqrt{\frac{-1}{3}}$

so no critical #'s

$f(0) = 0$

$f(10) = 10^3 + 10$
$= 1010$

absolute min: $(0, 0)$

absolute max: $(10, 1010)$

3) Find and classify the critical points of each function as a local max, local min, or neither.

a) $y = 4x - x^2$

$y' = 4 - 2x$

$0 = 4 - 2x$

$x = 2$

$-\infty$		2	∞
Test value	1	3	
$f'(x)$	+	−	
$f(x)$	inc. ↗	dec. ↘	

local max

$y(2) = 4(2) - (2)^2$

$y(2) = 4$

(2,4) is a critical point

$\boxed{(2,4) \text{ is a local MAX}}$

b) $f(x) = (x-1)^4$

$f'(x) = 4(x-1)^3(1)$

$0 = 4(x-1)^3$

$0 = (x-1)^3$

$0 = x-1$

$x = 1$

$f(1) = (1-1)^4$

$f(1) = 0$

$(1,0)$ is a critical point

	$-\infty$ to 1	1 to ∞
Test Value	0	2
$f'(x)$	$-$	$+$
$f(x)$	dec. ↘	inc. ↗

local min $(1,0)$

$\boxed{(1,0) \text{ is a local MIN}}$

c) $g(x) = 2x^3 - 24x + 5$

$g'(x) = 6x^2 - 24$

$0 = 6(x^2-4)$

$0 = 6(x-2)(x+2)$

$x_1 = 2 \quad x_2 = -2$

$f(2) = -27 \quad f(-2) = 37$

critical points: $(2,-27)$ and $(-2,37)$

	$-\infty$ to -2	-2 to 2	2 to ∞
Test	-3	0	3
$f'(x)$	$+$	$-$	$+$
$f(x)$	inc. ↗	dec. ↘	inc. ↗

local max (at -2), local min (at 2)

$\boxed{\begin{array}{l}(-2,37) \text{ is a local MAX} \\ (2,-27) \text{ is a local MIN}\end{array}}$

d) $y = \frac{1}{4}x^4 - \frac{2}{3}x^3$

$\frac{dy}{dx} = x^3 - 2x^2$

$0 = x^2(x-2)$

$x_1 = 0 \quad x_2 = 2$

$y(0) = 0 \quad y(2) = \frac{-4}{3}$

critical points: $(0,0)$ and $(2,-\frac{4}{3})$

	$-\infty$ to 0	0 to 2	2 to ∞
Test Value	-1	1	3
$f'(x)$	$-$	$-$	$+$
$f(x)$	dec ↘	dec. ↘	inc. ↗

local min (at 2)

$(2,-\frac{4}{3})$ is a local MIN

$(0,0)$ is neither

4)a) Find the critical numbers of $f(x) = 2x^3 - 3x^2 - 12x + 5$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$\boxed{\text{Critical \#'s: } x_1 = 2 \quad x_2 = -1}$

$f(2) = -15 \quad f(-1) = 12$

critical points: $(2,-15)$ and $(-1,12)$

$f'(x) = 6(x-2)(x+1)$

b) Find any local extrema of $f(x)$.

Test	-2	0	3
$f'(x)$	$+$	$-$	$+$
$f(x)$	inc. ↗	dec. ↘	inc. ↗

$-\infty$, -1, 2, ∞

-1: local max

2: local MIN

$\boxed{\begin{array}{l}(-1,12) \text{ is a local MAX} \\ (2,-15) \text{ is a local MIN}\end{array}}$

c) Find the absolute extrema of $f(x)$ in the interval $[-2,4]$.

$f(-2) = 1$

$f(-1) = 12$

$f(2) = -15$

$F(4) = 37$

$\boxed{\begin{array}{l}\text{Absolute MIN: } (2,-15) \\ \text{Absolute MAX: } (4,37)\end{array}}$

5) A section of rollercoaster is in the shape of $f(x) = -x^3 - 2x^2 + x + 15$, where x is between -2 and 2.

a) Find all local extrema and explain what portions of the rollercoaster they represent.

$f'(x) = -3x^2 - 4x + 1$

$0 = -3x^2 - 4x + 1$

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(-3)(1)}}{2(-3)}$

$x = \dfrac{4 \pm 2\sqrt{7}}{-6} = \dfrac{2 \pm \sqrt{7}}{-3}$

$x_1 \simeq -1.55$ $\quad x_2 \simeq 0.22$

$f(-1.55) \simeq 12.37$ $\quad F(0.22) \simeq 15.11$

critical points: $(-1.55, 12.37)$ and $(0.22, 15.11)$

Test	-2	0	1
$f'(x)$	$-$	$+$	$-$
$F(x)$	dec ↘	inc ↗	dec. ↘

$-\infty$, -1.55, 0.22, ∞

The coaster starts going down a hill at $x=-2$, reaches a min at $(-1.55, 12.37)$, goes up to a max at $(0.22, 15.11)$, then continues down until $x=2$.

b) Is the highest point of this section of the ride at the beginning, the end, or neither?

$F(-2) = 13$

$F(2) = 1$

∴ The absolute max is at $(0.22, 15.11)$; NOT at the beginning or end.

Answers:

1)a) $x = 3$ **b)** no critical numbers **c)** $x = -1{,}2$ **d)** $x = \frac{1}{4}$

2)a) absolute max at $(0{,}7)$ and $(4{,}7)$
absolute min at $(2, -5)$

b) absolute max at $(-1{,}9)$
absolute min at $(-3, -43)$

c) absolute max at $(10{,}1010)$
absolute min at $(0{,}0)$

3)a) $(2{,}4)$ is a local max **b)** $(1{,}0)$ is a local min **c)** $(-2{,}37)$ is a local max; $(2, -27)$ is a local min
d) $(0{,}0)$ is neither; $\left(2, -\frac{4}{3}\right)$ is a local min

4)a) $x = -1{,}2$ **b)** $(-1{,}12)$ is a local max; $(2, -15)$ is a local min **c)** $(2, -15)$ is the absolute min, $(4{,}37)$ is the absolute max

5)a) The coaster starts down a hill from $x = -2$, reaching a local min at the bottom of a hill at $(-1.55{,}12.37)$. It then increases height until it reaches a local max at the top of a hill at $(0.22{,}15.11)$. It then continues downward until $x = 2$.
b) The highest point is at $(0.22{,}15.11)$, not either of the endpoints.

W3 – Concavity and the Second Derivative Unit 2

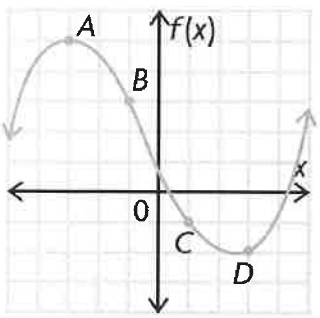
MCV4U

Jensen

SOLUTIONS

1) For each function, state whether the value of the of the second derivative is positive or negative at each of points A, B, C, and D.

a)



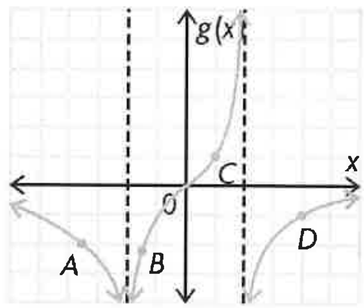
A: $f''(x) < 0$

B: $f''(x) < 0$

C: $f''(x) > 0$

D: $f''(x) > 0$

b)



A: $f''(x) < 0$

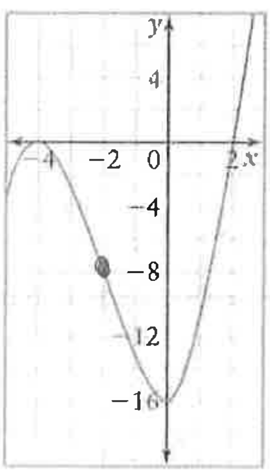
B: $f''(x) < 0$

C: $f''(x) > 0$

D: $f''(x) < 0$

2) For each graph, identify the intervals over which the graph is concave up and the intervals over which it is concave down.

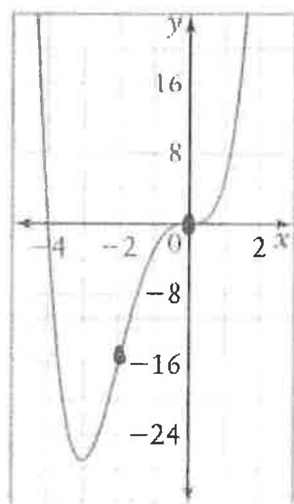
a)



concave up: $x > -2$

concave down: $x < -2$

b)

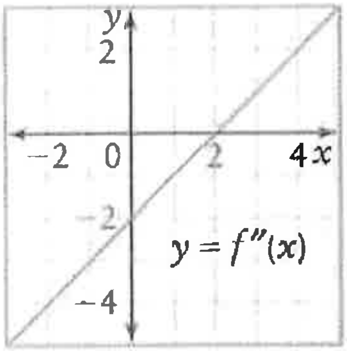


concave up: $x < -2$, $x > 0$

concave down: $-2 < x < 0$

3) Given each graph of $f''(x)$, state the intervals of concavity for the function $f(x)$. Also indicate where any points of inflection occur for $f(x)$.

a)

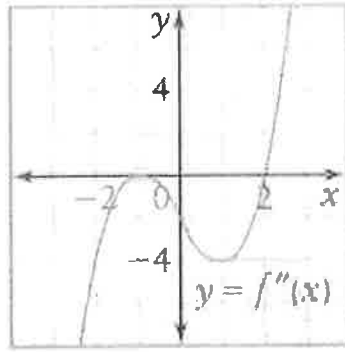


concave up: $x > 2$

concave down: $x < 2$

PoI when $x = 2$

b)



Concave up: $x > 2$

Concave down: $x < -1$, $-1 < x < 2$

PoI when $x = 2$

4) For each function, find the intervals of concavity and the coordinates of any points of inflection.

a) $y = 6x^2 - 7x + 5$

$y' = 12x - 7$

$y'' = 12$

$\therefore$ y is always concave up; No PoI's.

b) $g(x) = -2x^3 + 12x^2 - 9$

$g'(x) = -6x^2 + 24x$

$g''(x) = -12x + 24$

$0 = -12x + 24$

$x = 2$

$g(2) = 23$

(2, 23) is a possible PoI

	$-\infty$ to 2	2 to ∞
Test	1	3
$f''(x)$	+	−
$F(x)$	concave up ∪	concave down ∩

concave up: $x < 2$

concave down: $x > 2$

PoI: (2, 23)

5) For each function, find and classify all the critical points using the second derivative test.

a) $y = x^2 + 10x - 11$

$y' = 2x+10$

$0 = 2x+10$

$x = -5$

$y(-5) = -36$

$(-5,-36)$ is a critical point

2^{nd} Derivative Test:

$y'' = 2$

$y''(-5) = 2$

$\therefore$ y is concave up when $x = -5$

$(-5,-36)$ is a local min.

b) $f(x) = x^4 - 6x^2 + 10$

$f'(x) = 4x^3 - 12x$

$0 = 4x(x-3)$

$x_1 = 0 \quad x_2 = 3$

$f(0) = 10 \quad f(3) = 37$

$(0,10)$ and $(3,37)$ are critical points

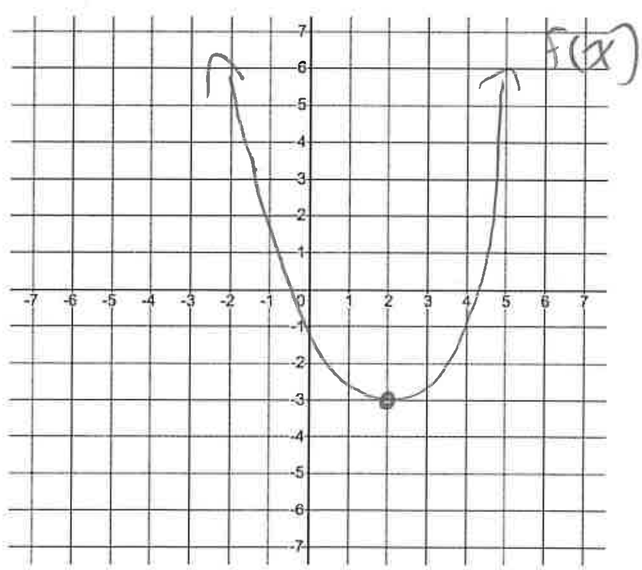
2^{nd} derivative test: $f''(x) = 8x - 12$

$f''(0) = -12$ concave down; $(0,10)$ is a local max

$f''(3) = 12$ concave up; $(3,37)$ is a local min

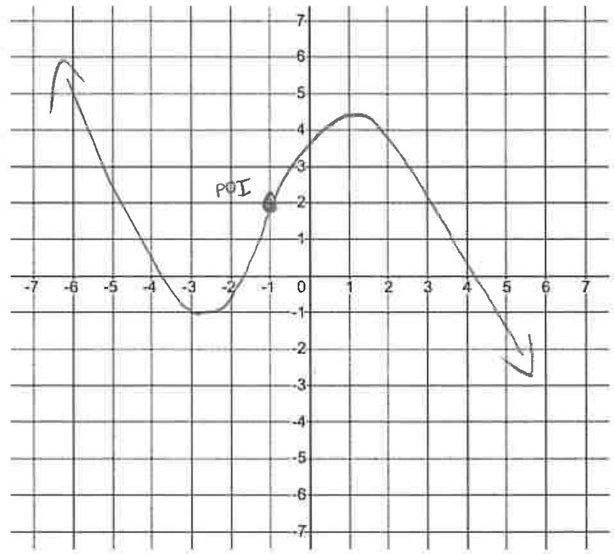
6) Sketch a graph of a function that satisfies each set of conditions

a) $f''(x) = 2$ for all x, $f'(2) = 0$, $f(2) = -3$



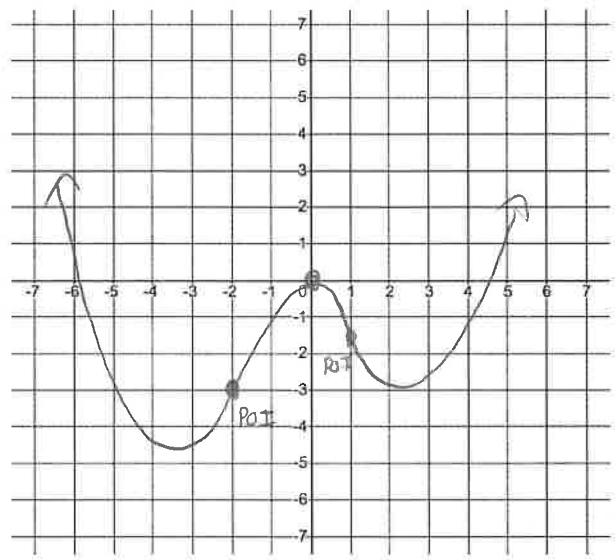
con. up   con. down

b) $f''(x) > 0$ when $x < -1$, $f''(x) < 0$ when $x > -1$, $f'(-1) = 1$, $f(-1) = 2$



con. down   con. up

c) $f''(x) < 0$ when $-2 < x < 1$, $f''(x) > 0$ when $x < -2$ and $x > 1$, $f(-2) = -3$, $f(0) = 0$



Answers:

1)a) A-neg, B-neg, C-pos, D-pos **b)** A-neg, B-neg, C-pos, D-neg

2)a) concave up: $x > -2$
concave down: $x < -2$

b) concave up: $x < -2$, $x > 0$
concave down: $-2 < x < 0$

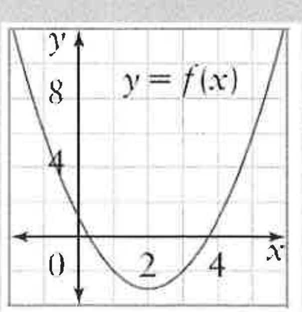
3)a) concave up: $x > 2$; concave down: $x < 2$; POI when $x = 2$

b) concave up: $x > 2$; concave down: $x < -1$ and $-1 < x < 2$; POI when $x = 2$

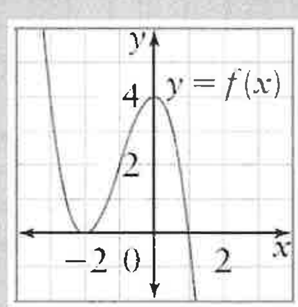
4)a) always concave up **b)** concave up: $x < 2$; concave down: $x > 2$; POI at (2,23).

5)a) $(-5, -36)$ is a local min point **b)** $(-\sqrt{3}, 1)$ and $(\sqrt{3}, 1)$ are local mins, (0,10) is a local max

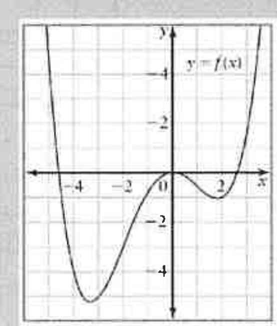
6)a)



b)



c)



W4 – Rational Functions Unit 2
MCV4U
Jensen

SOLUTIONS

1) Find the equation of any asymptotes for the following functions. Then, find the one-sided limits approaching the vertical asymptotes.

a) $f(x) = \frac{x+3}{x^2-4} = \frac{x+3}{(x-2)(x+2)}$

HA: $y=0$
VA: $x=2$, $x=-2$

Tests: $f(1.99) \simeq -125$
$f(2.01) \simeq 125$
$f(-1.99) \simeq -25$
$f(-2.01) \simeq 25$

Limits: $\lim_{x\to 2^-} f(x) = -\infty$
$\lim_{x\to 2^+} f(x) = \infty$
$\lim_{x\to -2^-} f(x) = \infty$
$\lim_{x\to -2^+} f(x) = -\infty$

b) $y = \frac{x^2}{x^2-3x+2} = \frac{x^2}{(x-2)(x-1)}$

HA: $y=1$
VA: $x=2$, $x=1$

Tests: $y(1.99) \simeq -400$
$y(2.01) \simeq 400$
$y(0.99) \simeq 97$
$y(1.01) \simeq -103$

Limits: $\lim_{x\to 2^+} y = \infty$
$\lim_{x\to 2^-} y = -\infty$
$\lim_{x\to 1^+} y = -\infty$
$\lim_{x\to 1^-} y = \infty$

c) $y = 2x + \frac{1}{x} = \frac{2x^2+1}{x}$

S.A: $y=2x$
VA: $x=0$

Tests: $y(-0.01) \simeq -100$
$y(0.01) \simeq 100$

Limits:
$\lim_{x\to 0^+} y = \infty$
$\lim_{x\to 0^-} y = -\infty$

d) $g(x) = \frac{2x-3}{x^2-6x+9} = \frac{2x-3}{(x-3)^2}$

HA: $y=0$
VA: $x=3$

Tests: $g(2.99) = 29800$
$g(3.01) = 30200$

Limits:
$\lim_{x\to 3^+} g(x) = \infty$
$\lim_{x\to 3^-} g(x) = \infty$

2) Find the derivative of each function. Then, determine whether the function has any local extrema.

a) $f(x) = \frac{2}{x+3}$

$f'(x) = \frac{0(x+3) - 1(2)}{(x+3)^2}$

$f'(x) = \frac{-2}{(x+3)^2}$

No critical points since $f'(x) \neq 0$ and $x = -3$ is not in the domain of $f(x)$

$\therefore$ no local extrema.

b) $h(x) = \frac{-3}{(x-2)^2}$

$h'(x) = \frac{0(x-2)^2 - 2(x-2)(1)(-3)}{[(x-2)^2]^2}$

$h'(x) = \frac{6(x-2)}{(x-2)^4}$

$h'(x) = \frac{6}{(x-2)^3}$

$h'(x) \neq 0$ and $x = 2$ is not in the domain of $h(x)$.

$\therefore$ no critical points and no local extrema.

3) Consider the function $f(x) = \frac{-2}{(x+1)^2}$ VA: $x = -1$

a) Find the intervals of increase and decrease for $f(x)$.

$f'(x) = \frac{0(x+1)^2 - 2(x+1)(1)(-2)}{(x+1)^4}$

$f'(x) = \frac{4(x+1)}{(x+1)^4}$

$f'(x) = \frac{4}{(x+1)^3}$

No critical points.
Only use VA as a dividing point when testing.

$-\infty$ to -1		-1 to ∞
Test	-2	0
$f'(x)$	$-$	$+$
$f(x)$	dec. ↘	inc. ↗

increasing: $x > -1$
decreasing: $x < -1$

b) Find the intervals of concavity for $f(x)$.

$f''(x) = \frac{0(x+1)^3 - 3(x+1)^2(1)(4)}{(x+1)^6}$

$f''(x) = \frac{-12(x+1)^2}{(x+1)^6}$

$f''(x) = \frac{-12}{(x+1)^4}$

$f''(x) \neq 0$

only possible change in concavity is at the VA

$-\infty$ to -1		-1 to ∞
Test	-2	0
$f''(x)$	$-$	$-$
$f(x)$	con. down	con. down

concave down: $x < -1$, $x > -1$
concave up: never

4) Consider the function $h(x) = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)}$

a) Write the equations of the asymptotes

HA: $y=0$

VA: $x=2$, $x=-2$

b) Make a table showing the increasing and decreasing intervals for the function

$h'(x) = \frac{0(x^2-4) - 2x(1)}{(x-2)^2(x+2)^2}$

$h'(x) = \frac{-2x}{(x-2)^2(x+2)^2}$

$0 = -2x$

$x = 0$

$h(0) = -0.25$

critical point $(0, -0.25)$

Test:

$-\infty$ | -2 | 0 | 2 | ∞

Test	-3	-1	1	3
$f'(x)$	+	+	−	−
$F(x)$	inc. ↗	inc. ↗	dec. ↘	dec. ↘

increasing: $x<-2$, $-2<x<0$

decreasing: $0<x<2$, $x>2$

c) How can you use the table from part b) to determine the behavior of $f(x)$ near the vertical asymptotes?

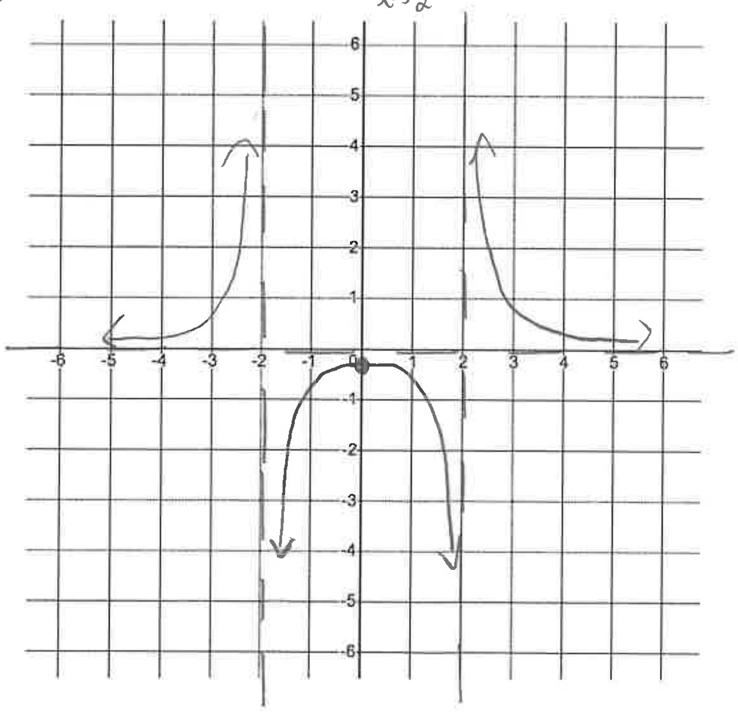
$h(x)$ is increasing to the left of $x=-2$; $\lim_{x\to -2^-} h(x) = \infty$

$h(x)$ is increasing to the right of $x=-2$; $\lim_{x\to 2^+} h(x) = -\infty$

$h(x)$ is decreasing to the left of $x=2$; $\lim_{x\to 2^-} h(x) = -\infty$

$h(x)$ is decreasing to the right of $x=2$; $\lim_{x\to 2^+} h(x) = \infty$

d) Sketch a graph of the function.



Answers:

1)a) VA: $x = 2$ and $x = -2$; HA: $y = 0$; $\lim\limits_{x\to 2^+} = \infty$, $\lim\limits_{x\to 2^-} = -\infty$, $\lim\limits_{x\to -2^+} = -\infty$, $\lim\limits_{x\to -2^-} = \infty$

b) VA: $x = 1$ and $x = 2$; HA: $y = 1$; $\lim\limits_{x\to 2^+} = \infty$, $\lim\limits_{x\to 2^-} = -\infty$, $\lim\limits_{x\to 1^+} = -\infty$, $\lim\limits_{x\to 1^-} = \infty$

c) VA: $x = 0$; SA: $y = 2x$; $\lim\limits_{x\to 0^+} = \infty$, $\lim\limits_{x\to 0^-} = -\infty$

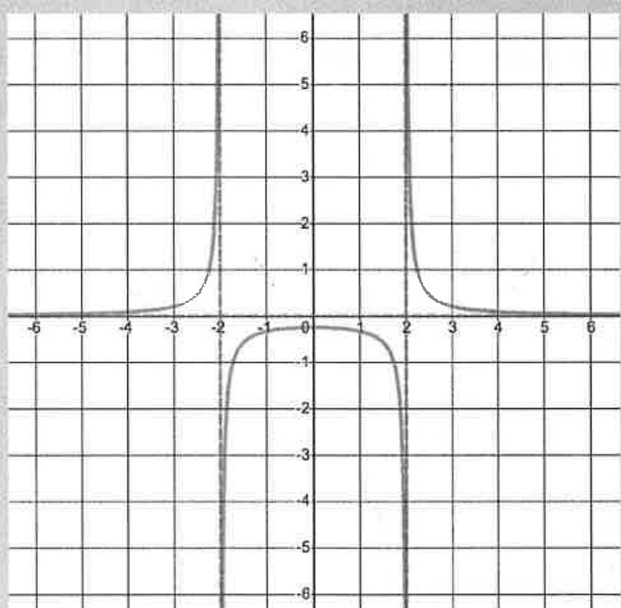
d) VA: $x = 3$; HA: $y = 0$; $\lim\limits_{x\to 3^+} = \infty$, $\lim\limits_{x\to 3^-} = \infty$

2)a) $f'(x) = \frac{-2}{(x+3)^2}$; no local extrema **b)** $h'(x) = \frac{6}{(x-2)^3}$; no local extrema

3)a) decreasing when $x < -1$, increasing when $x > -1$ **b)** concave down when $x < -1$ or $x > -1$

4)a) VA: $x = 2$ and $x = -2$; HA: $y = 0$
b) increasing when $x < -2$ or $-2 < x < 0$; decreasing when $0 < x < 2$ or $x > 2$
c) Since the curve is increasing to the left of $x = -2$, $\lim\limits_{x\to -2^-} = \infty$
Since the curve is increasing to the right of $x = -2$, $\lim\limits_{x\to -2^+} = -\infty$
Since the curve is decreasing to the left of $x = 2$, $\lim\limits_{x\to 2^-} = -\infty$
Since the curve is decreasing to the right of $x = 2$, $\lim\limits_{x\to 2^+} = \infty$

d)



SOLUTIONS

1) For each function, determine the coordinates of the local extrema. Classify each point as a local max or min.

a) $y = 2x - 3x^2$

$y' = 2 - 6x$

$0 = 2 - 6x$

$x = \frac{1}{3}$

$y\left(\frac{1}{3}\right) = \frac{1}{3}$

critical point: $\left(\frac{1}{3}, \frac{1}{3}\right)$

2nd derivative test:

$y'' = -6$

$y''\left(\frac{1}{3}\right) = -6$; concave down

$\therefore \left(\frac{1}{3}, \frac{1}{3}\right)$ is a local MAX

b) $y = 2t^3 + 6t^2 + 6t + 4$

$y' = 6t^2 + 12t + 6$

$0 = 6(t^2 + 2t + 1)$

$0 = (t+1)^2$

$t = -1$

$y(-1) = 2$

2nd derivative test:

$y'' = 12t + 12$

$y''(-1) = 0$; $(-1, 2)$ is a point of inflection not a local min or max

2) For each function, determine the coordinates of any points of inflection.

a) $f(x) = 2x^3 - 4x^2$

$f'(x) = 6x^2 - 8x$

$f''(x) = 12x - 8$

$0 = 12x - 8$

$x = \frac{2}{3}$

$f\left(\frac{2}{3}\right) = \frac{-32}{27}$

Possible PoI is $\left(\frac{2}{3}, \frac{-32}{27}\right)$

Test:

$-\infty$ ... $2/3$ ... ∞	0	1
$f''(x)$	$-$	$+$
$f(x)$	con. down $\frown$	con. up $\smile$

$\boxed{\text{PoI: } \left(\frac{2}{3}, \frac{-32}{27}\right)}$

b) $f(x) = 3x^5 - 5x^4 - 40x^3 + 120x^2$

$f'(x) = 15x^4 - 20x^3 - 120x^2 + 240x$

$f''(x) = 60x^3 - 60x^2 - 240x + 240$

$0 = 60(x^3 - x^2 - 4x + 4)$

$0 = x^2(x-1) - 4(x-1)$

$0 = (x-1)(x^2-4)$

$0 = (x-1)(x-2)(x+2)$

$x_1 = 1$ $\quad x_2 = 2$ $\quad x_3 = -2$

$f(1) = 78$ $\quad f(2) = 176$ $\quad f(-2) = 624$

Test:

$-\infty$ ... -2 ... 1 ... 2 ... ∞	-3	0	1.5	3
$f''(x)$	$-$	$+$	$-$	$+$
$f(x)$	down $\frown$	up $\smile$	down $\frown$	up $\smile$

$\boxed{\text{PoI's: } (-2, 624), (1, 78), \text{ and } (2, 176)}$

3) Use the algorithm for curve sketching to analyze the key features of each of the following functions and to sketch a graph of them.

a) $f(x) = x^4 - 8x^3$

1. No domain restrictions; no asymptotes

2. $0 = x^3(x-8)$ x-int: $(0,0)$, $(8,0)$ $f(0) = 0$ y-int: $(0,0)$

$x_1 = 0$ $x_2 = 0$

3. $f'(x) = 4x^3 - 24x^2$

$0 = 4x^2(x-6)$

$x_1 = 0$ $x_2 = 6$

$f(0) = 0$ $f(6) = -432$

critical points: $(0,0)$, $(6,-432)$

4. $f''(x) = 12x^2 - 48x$

$0 = 12x(x-4)$

$x_1 = 0$ $x_2 = 4$

$f(0) = 0$ $f(4) = -256$

possible points of inflection: $(0,0)$, $(4,-256)$

5/6.

	$-\infty$ to 0	0 to 4	4 to 6	6 to ∞
Test	-1	1	5	7
$f'(x)$	$-$	$-$	$-$	$+$
$f''(x)$	$+$	$-$	$+$	$+$
$F(x)$	Con. UP Decreasing	Con. Down Decreasing	Con. UP Decreasing	Con. UP Increasing

At 0: POI; at 4: POI; at 6: local min

7. Local min: $(6,-432)$

Local max: NONE

Points of inflection: $(0,0)$ and $(4,-256)$

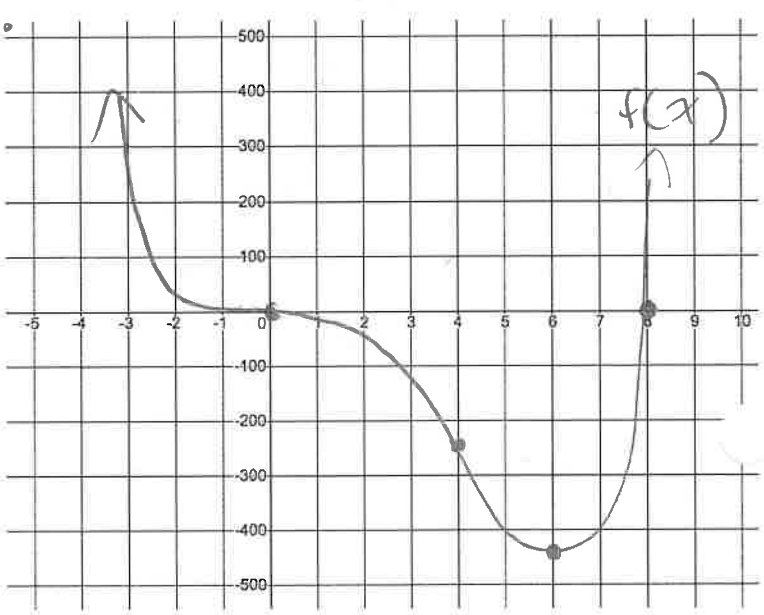
increasing: $x > 6$

decreasing: $x < 6$

C.U.: $x < 0$, $x > 4$

C.D.: $0 < x < 4$

8.



b) $g(x) = 3x^3 + 7x^2 + 3x - 1$

① No restrictions on the domain ; no asymptotes

② x-int

Test: $g(-1) = 0$ $\therefore x+1$ is a Factor

$0 = 3x^3 + 7x^2 + 3x - 1$

$0 = (x+1)(3x^2 + 4x - 1)$

$x_1 = -1$ $\quad x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-4 \pm \sqrt{28}}{6}$

$x_2 \simeq 0.215 \quad x_3 \simeq -1.549$

$$\begin{array}{r|rrrr} -1 & 3 & 7 & 3 & -1 \\ & \downarrow & -3 & -4 & 1 \\ \hline \times & 3 & 4 & -1 & 0 \\ & x^2 & x & * & R \end{array}$$

x-int: $(-1, 0)$, $(0.215, 0)$, and $(-1.549, 0)$

y-int:

$g(0) = -1$

$(0, -1)$

③ $g'(x) = 9x^2 + 14x + 3$

$0 = 9x^2 + 14x + 3$

$x = \frac{-14 \pm \sqrt{(14)^2 - 4(9)(3)}}{2(9)}$

$x = \frac{-14 \pm \sqrt{88}}{18}$

$x_1 \simeq -0.26 \quad x_2 \simeq -1.30$

$g(-0.26) = -1.36 \quad g(-1.3) = 0.34$

Critical Points: $(-0.26, -1.36)$, $(-1.3, 0.34)$

④ $g''(x) = 18x + 14$

$0 = 18x + 14$

$x = -\frac{7}{9} \simeq -0.78$

$g\left(-\frac{7}{9}\right) = \frac{-124}{243} \simeq -0.51$

Possible PoI: $(-0.78, -0.51)$

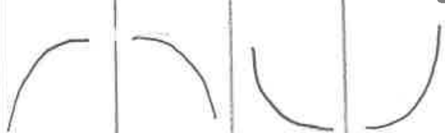
⑦ Local min: $(-0.26, -1.36)$

Local max: $(-1.3, 0.34)$

Point of Inflection: $(-0.78, -0.51)$

5/6

	$-\infty$ to -1.3	-1.3 to -0.78	-0.78 to -0.26	-0.26 to ∞
Test	-2	-1	-0.5	0
$g'(x)$	+	−	−	+
$g''(x)$	−	−	+	+
	Con. DOWN Increasing	Con. DOWN Decreasing	Con. UP decreasing	Con. UP increasing
$g(x)$				



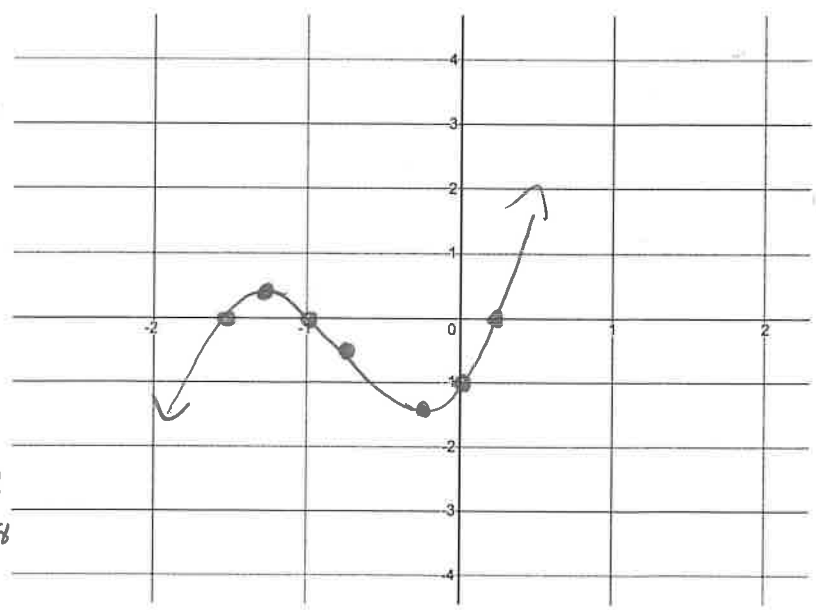
local max (at -1.3) · PoI (at -0.78) · local min (at -0.26)

increasing: $x < -1.3$, $x > -0.26$

decreasing: $-1.3 < x < -0.26$

C.U.: $x > -0.78$

C.D.: $x < -0.78$



c) $h(x) = 2x^4 - 26x^2 + 72$

① no restrictions ; no asymptotes

② x-int

$0 = 2x^4 - 26x^2 + 72$

$0 = x^4 - 13x^2 + 36$

$0 = (x^2)^2 - 13(x^2) + 36$

$0 = (x^2-9)(x^2-4)$

$0 = (x-3)(x+3)(x-2)(x+2)$

$x_1 = -3 \quad x_2 = -2 \quad x_3 = 2 \quad x_4 = 3$

x-int: $(-3,0), (-2,0), (2,0), (3,0)$

y-int

$h(0) = 72$

$(0, 72)$

③ $h'(x) = 8x^3 - 52x$

$0 = 4x(2x^2 - 13)$

$x_1 = 0 \qquad 2x^2 - 13 = 0$

$h(0) = 72 \qquad x = \pm\sqrt{13/2}$

$x_2 \simeq 2.55 \quad x_3 \simeq -2.55$

$h(2.55) \simeq -12.5 \quad h(-2.55) \simeq -12.5$

critical points: $(0,72), (2.55,-12.5), (-2.55,-12.5)$

④ $h''(x) = 24x^2 - 52$

$0 = 24x^2 - 52$

$x = \pm\sqrt{13/6}$

$x_1 \simeq 1.47 \quad x_2 \simeq -1.47$

$h(1.47) \simeq 25.16 \quad h(-1.47) \simeq 25.16$

Possible POI's: $(1.47, 25.16)$, $(-1.47, 25.16)$

5/6

	$-\infty$ to -2.55	-2.55 to -1.47	-1.47 to 0	0 to 1.47	1.47 to 2.55	2.55 to ∞
	-3	-2	-1	1	2	3
$h'(x)$	$-$	$+$	$+$	$-$	$-$	$+$
$h''(x)$	$+$	$+$	$-$	$-$	$+$	$+$
$h(x)$	CU Decreasing	CU Increasing	CD incr.	CD decr.	CU decr.	CU incr.
	min	POI	max	POI	min	

Increasing: $-2.55 < x < 0$, $x > 2.55$

decreasing: $x < -2.55$, $0 < x < 2.55$

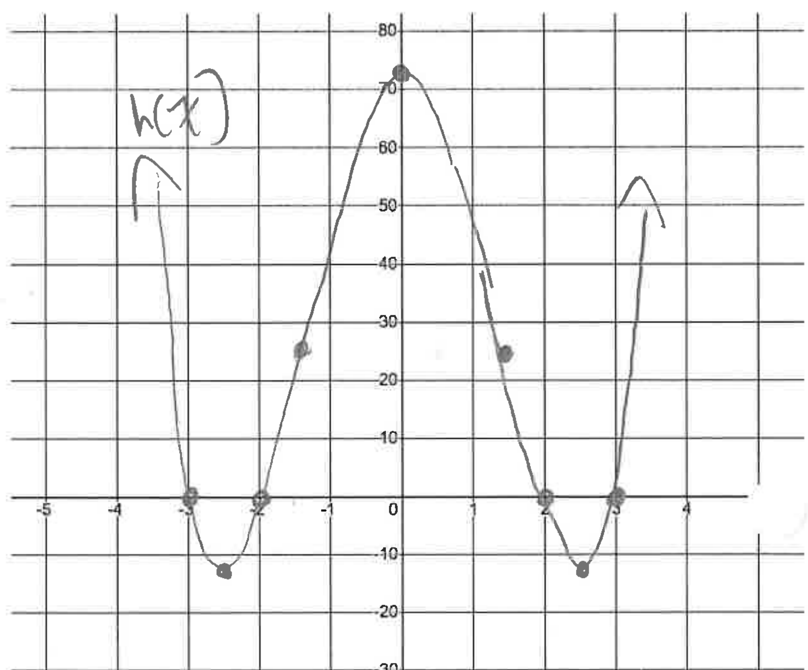
CU: $x < -1.47$, $x > 1.47$

CD: $-1.47 < x < 1.47$

⑦ Local min: $(-2.55, -12.5)$ and $(2.55, -12.5)$

Local max: $(0, 72)$

POI's: $(-1.47, 25.16)$ and $(1.47, 25.16)$



d) $j(x) = \frac{x^2+2x-4}{x^2}$

① $x \neq 0$; VA at $x=0$
HA at $y=1$

② x-int

$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(-4)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{20}}{2}$

$x = \frac{-2 \pm 2\sqrt{5}}{2}$

$x = -1 \pm \sqrt{5}$

$(1.24, 0)$
$(-3.24, 0)$

y-int:

$j(0) = \frac{-4}{0}$ = undefined

∴ no y-intercept.

③ $j'(x) = \frac{(2x+2)(x^2) - 2x(x^2+2x-4)}{x^4}$

$j'(x) = \frac{\cancel{x}[(2x+2)(x) - 2(x^2+2x-4)]}{x^{\cancel{4}3}}$

$j'(x) = \frac{2x^2+2x-2x^2-4x+8}{x^3}$

$j'(x) = \frac{-2x+8}{x^3}$

$0 = -2x+8$
$x = 4$
$j(4) = 1.25$

$x=0$ is not a critical # because it is NOT in the domain of $j'(x)$

Critical #: $(4, 1.25)$

④ $j''(x) = \frac{-2(x^3) - 3x^2(-2x+8)}{x^6}$

$j''(x) = \frac{-2x^3 + 6x^3 - 24x^2}{x^6}$

$j''(x) = \frac{4x^3 - 24x^2}{x^6}$

$j''(x) = \frac{4x^2(x-6)}{x^6}$

$j''(x) = \frac{4(x-6)}{x^4}$

$0 = 4(x-6)$
$x = 6$
$j(6) = 1.22$

possible POI is $(6, 1.22)$

5/6

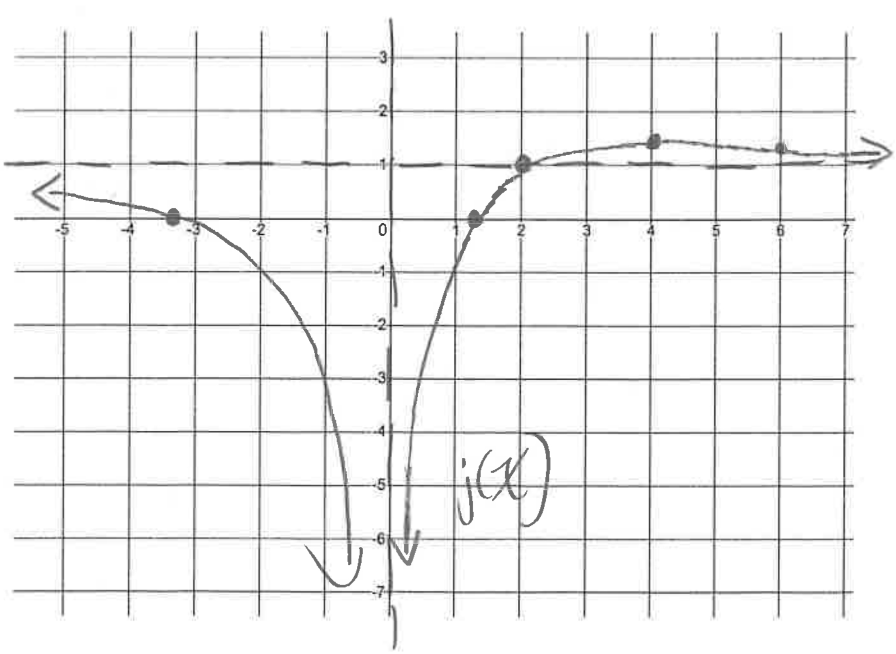
	$-\infty$ to 0: -1	0 to 4: 1	4 to 6: 5	6 to ∞: 7
$j'(x)$	−	+	−	−
$j''(x)$	−	−	−	+
$j(x)$	CD decrease	CD increase	CD decrease	CU decrease
	VA	Max	POI	

increasing: $0 < x < 4$

decreasing: $x < 0$, $x > 4$

CU: $x > 6$

CD: $x < 0$, $0 < x < 6$

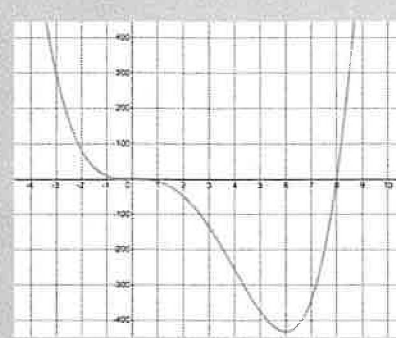


Answers:

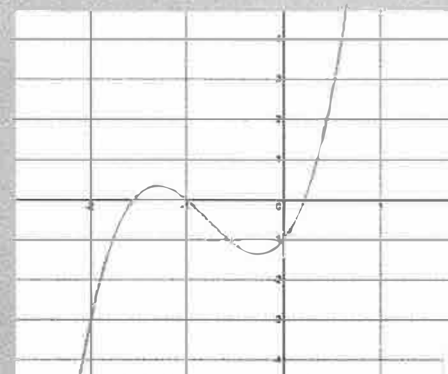
1)a) max: $\left(\frac{1}{3}, \frac{1}{3}\right)$ **b)** no local extrema; $(-1,2)$ is an inflection point NOT a max or min

2)a) $\left(\frac{2}{3}, -\frac{32}{27}\right)$ **b)** $(-2, 624)$, $(2,176)$, and $(1,78)$

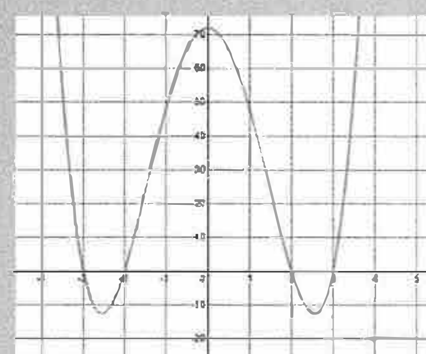
3)a) x-int: $(0,0)$ and $(8,0)$; y-int: $(0,0)$; local max: none; local min: $(6, -432)$; POI: $(0,0)$ and $(4, -256)$; increasing: $x > 6$; decreasing: $x < 0$ and $0 < x < 6$; concave up: $x < 0$ and $x > 4$; concave down: $0 < x < 4$



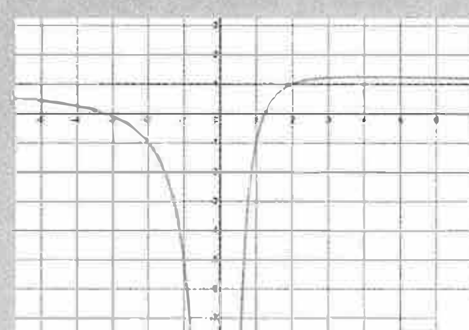
b) x-int: $(-1,0)$, $(0.215,0)$, and $(-1.549,0)$; y-int: $(0, -1)$; local max: $(-1.3, 0.34)$; local min: $(-0.26, -1.36)$; POI: $(-0.78, -0.51)$; increasing: $x < -1.3$ and $x > -0.26$; decreasing: $-1.3 < x < -0.26$; concave up: $x > -0.78$; concave down: $x < -0.78$



c) x-int: $(-3,0)$, $(-2,0)$, $(2,0)$ and $(3,0)$; y-int: $(0,72)$; local max: $(0,72)$; local min: $(-2.55, -12.5)$ and $(2.55, -12.5)$; POI: $(-1.47, 25.16)$ and $(1.47, 25.16)$; increasing: $-2.55 < x < 0$ and $x > 2.55$; decreasing: $x < -2.55$, and $0 < x < 2.55$; concave up: $x < -1.47$ and $x > 1.47$; concave down: $-1.47 < x < 1.47$



d) VA: $x = 0$; HA: $y = 1$; x-int: $(-3.24,0)$, and $(1.24,0)$; y-int: none; local max: $(4,1.25)$; local min: none; POI: $(6,1.22)$; increasing: $0 < x < 4$ and; decreasing: $x < 0$, and $x > 4$; concave up: $x > 6$; concave down: $x < 0$ and $0 < x < 6$



W6 – Optimization Problems Unit 2

MCV4U

Jensen

SOLUTIONS

1) A rectangular pen is to be built with 1200 m of fencing. The pen is to be divided into three parts using two parallel partitions. Find the max possible area of the pen.

width $= x$

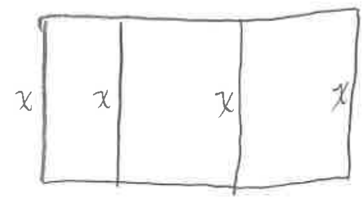
$1200 = 4x + 2l$

$\frac{1200-4x}{2} = l$

length $= 600 - 2x$

$A(x) = x(600-2x)$

$A(x) = 600x - 2x^2$



Critical Points:

$A'(x) = 600 - 4x$

$0 = 600 - 4x$

$x = 150$

$A(150) = 45\,000 \text{ m}^2$

2nd Derivative Test:

$A''(x) = -4$

$A''(150) = -4$; concave down

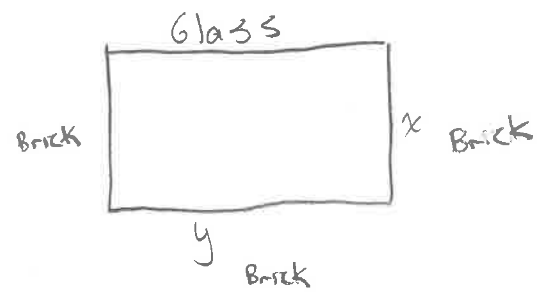
$\therefore$ $(150, 45000)$ is a max point.

$\boxed{\text{The max area is } 45000 \text{ m}^2}$

2) A showroom for a car dealership is to be built in the shape of a rectangle with brick on the back and sides, and glass on the front. The floor of the showroom is to have an area of 500 m^2. If a brick wall costs \$1200/m while a glass wall costs \$600/m, what dimensions would minimize the cost of the showroom? What is the min cost?

$xy = 500$

$y = \frac{500}{x}$



$C(x) = 600\left(\frac{500}{x}\right) + 1200\left(\frac{500}{x}\right) + 1200(2x)$

$C(x) = \frac{300\,000}{x} + \frac{600\,000}{x} + 2400x$

$C(x) = \frac{900\,000}{x} + 2400x$

Critical Points:

$C'(x) = -900000x^{-2} + 2400$

$0 = \frac{-900000}{x^2} + 2400$

$-2400x^2 = -900\,000$

$x^2 = 375$

$x \approx 19.36 \text{ m}$

$C(19.36) \approx \$92\,951.6$

2nd Derivative Test:

$C''(x) = \frac{1800\,000}{x^3}$

$C''(19.36) \approx 248$

$\therefore$ concave up and $(19.36, 92951.6)$ is a min point. The dimensions that obtain a min cost of \$92 951.6 are 19.36m by 25.83m.

3) A soup can is to have a capacity of 250 cm^3 and the diameter of the can must be no less than 4 cm and no greater than 8 cm. Wat are the dimensions of the can that can be constructed using the LEAST amount of material?

Domain: $2 \le r \le 4$

$SA = 2\pi r^2 + 2\pi rh$

$SA(r) = 2\pi r^2 + 2\pi r\left(\frac{250}{\pi r^2}\right)$

$SA(r) = 2\pi r^2 + \frac{500}{r}$

$SA'(r) = 4\pi r - \frac{500}{r^2}$

$0 = 4\pi r - \frac{500}{r^2}$

$r^3 = \frac{500}{4\pi}$

$r = 3.41$

From Volume Equation:

$V = \pi r^2 h$

$250 = \pi r^2 h$

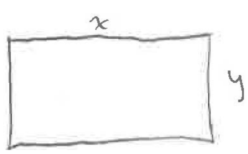
$h = \frac{250}{\pi r^2}$

Test endpoints of domain and critical number:

$SA(2) = 275.1$ cm³
$SA(3.41) = 219.7$ cm³
$SA(2) = 225.5$ cm³

Therefore a $radius = 3.41$ cm and a $height = \frac{250}{\pi(3.41)^2} = 6.84$ cm will minimize the amount of material needed to make the can.

4) A rectangular piece of paper with perimeter 100 cm is to be rolled to form a cylindrical tube. Find the dimensions of the paper that will produce a tube with maximum volume. What is the max volume?



$x = 2\pi r$

$r = \frac{x}{2\pi}$

$100 = 2x + 2y$

$\frac{100 - 2x}{2} = y$

$V(x) = \pi\left(\frac{x}{2\pi}\right)^2(50 - x)$

$V(x) = \pi\left(\frac{x^2}{4\pi^2}\right)(50 - x)$

$V(x) = \frac{x^2}{4\pi}(50 - x)$

$V(x) = \frac{25x^2}{2\pi} \quad \frac{x^3}{4\pi}$

$V(x) - \frac{25}{2\pi}x^2 - \frac{1}{4\pi}x^3$

$V'(x) = \frac{25}{\pi}x - \frac{3}{4\pi}x^2$

$0 = x\left(\frac{25}{\pi} - \frac{3}{4\pi}x\right)$

$x_1 = 0$

$0 = \frac{25}{\pi} - \frac{3x}{4\pi}$

$\frac{3x}{4\pi} = \frac{25}{\pi}$

$3x = 100$

$x = \frac{100}{3}$

$V''(x) = \frac{25}{\pi} - \frac{3}{2\pi}x$

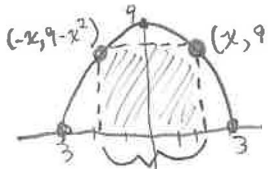
$V''(0) = \frac{25}{\pi}$ ∴ concave up (min)

$V''\left(\frac{100}{3}\right) \approx -8$ ∴ concave down (max)

$V\left(\frac{100}{3}\right) \approx 1473.66\ cm^3$

A max volume of $1473.66\ cm^3$ can be obtained with a length of $\frac{100}{3}$ cm and width of $\frac{50}{3}$ cm

5) Find the area of the largest rectangle that can be inscribed between the x-axis and the graph defined by $y = 9 - x^2$. $= (3-x)(3+x)$



$x-(-x)$
$=2x$

$A(x) = 2x(9-x^2)$

$A(x) = 18x - 2x^3$

$A'(x) = 18 - 6x^2$

$0 = 18 - 6x^2$

$x^2 = 3$

$x = \pm\sqrt{3}$

$A(\sqrt{3}) \simeq 20.78$

~~$A(-\sqrt{3}) \simeq 20.78$~~

Can't have $x<0$

2nd derivative test

$A''(x) = -12x$

$A''(\sqrt{3}) \simeq -20.8$ $\therefore$ concave down (max)

$\therefore$ $(\sqrt{3}, 20.78)$ is a max point.

The max area is about 20.78 units^2.

6) For an outdoor concert, a ticket price of \$30 typically attracts 5000 people. For each \$1 increase in the ticket price, 100 fewer people will attend. The revenue, R, is the product of the number of people attending and the price per ticket. Let x equal the number of \$1 increases in price. Find the ticket price that maximizes the revenue. What is the max revenue?

$R(x) = (30+x)(5000-100x)$

$R'(x) = 1(5000-100x) + (-100)(30+x)$

$R'(x) = 2000 - 200x$

$0 = 2000 - 200x$

$x = 10$

2nd derivative test:

$R''(x) = -200$

$R''(10) = -200$ $\therefore$ concave down

$\therefore$ 10 price increases will maximize revenue.

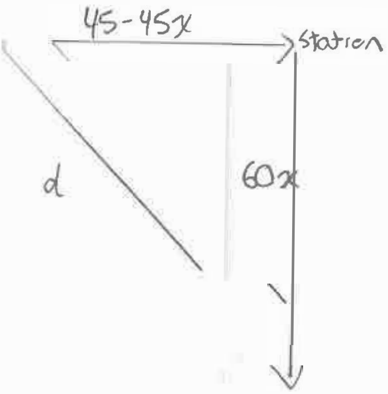
$R(10) = (30+10)(5000-100(10))$

$R(10) = (40)(4000)$

$R(10) = 160\,000$

A \$40 ticket will maximize the revenue to \$160000.

7) A train leaves the station at 10:00 am and travels SOUTH at a speed of 60 km/h. Another train heading WEST at 45 km/h reaches the same station at 11:00 am. At what time were the two trains closest together?



$$d(x) = \sqrt{(45-45x)^2 + (60x)^2}$$

$$d(x) = (2025 - 4050x + 2025x^2 + 3600x^2)^{1/2}$$

$$d(x) = (5625x^2 - 4050x + 2025)^{1/2}$$

$$d'(x) = \frac{1}{2}(5625x^2 - 4050x + 2025)^{-1/2}(11250x - 4050)$$

$$d'(x) = \frac{5625x - 2025}{\sqrt{5625x^2 - 4030x + 2025}}$$

$$0 = 5625x - 2025$$

$$x = 0.36$$

1st. derivative test:

	$-\infty$	0	0.36	∞
$d'(x)$		$-$	$+$	
$d(x)$		decreasing ↘	increasing ↗	

local min

∴ they are closest together 0.36 hours after 10:00 a.m.

(10:22 am)

Answers:

1) 45000 m^2
2) 19.4 m by 25.8 m; min cost is \$92952
3) $r = 3.41$ cm and $h = 6.83$ cm
4) $\frac{50}{3}$ cm by $\frac{100}{3}$ cm; volume is 1473.7 cm^3
5) $12\sqrt{3}$ units^2
6) \$40; max revenue is \$160 000
7) 0.36 hours after the first train left the station (10:22 am)