Calculus Midterm Exam Review

① a) $y=4$
$y'=0$

b) $y=3x$
$y'=3$

c) $y=5x^3+3x-7$
$y'=15x^2+3$

d) $y=\frac{3}{2}x^2$
$y'=3x$

e) $y=\frac{3}{x^4}$
$y=3x^{-4}$
$y'=-12x^{-5}$
$y'=\frac{-12}{x^5}$

F) $y=4x^{1/3}-8$
$y'=\frac{4}{3}x^{-2/3}$
$y'=\frac{4}{3x^{2/3}}$

g) $y=(2x+3)(5x-1)$
$y'=2(5x-1)+5(2x+3)$
$y'=10x-2+10x+15$
$y'=20x+13$

h) $y=(4x^2-x+2)(x-3)$
$y'=(8x-1)(x-3)+1(4x^2-x+2)$
$y'=8x^2-24x-x+3+4x^2-x+2$
$y'=12x^2-26x+5$

i) $y=(2x+1)(x^2-3)(3x-2)$
$y'=(6x^2+2x-6)(3x-2)+3(2x+1)(x^2-3)$
$y'=18x^3-12x^2+6x^2-4x-18x+12+6x^3-18x+3x^2-9$
$y'=24x^3-3x^2-40x+3$

Note: $\frac{d}{dx}(2x+1)(x^2-3)$
$=2(x^2-3)+2x(2x+1)$
$=2x^2-6+4x^2+2x$
$=6x^2+2x-6$

j) $y = (3x-2)^2$

$y' = 2(3x-2)(3)$

$y' = 6(3x-2)$

$y' = 18x-12$

k) $y = (2x-5)^{1/2}$

$y' = \frac{1}{2}(2x-5)^{-1/2}(2)$

$y' = \frac{1}{\sqrt{2x-5}}$

l) $y = (4x^2-3x)^{-3}$

$y' = -3(4x^2-3x)^{-4}(8x-3)$

$y' = \frac{-3(8x-3)}{(4x^2-3x)^4}$

m) $y = (x^2+2x)^3(4x+1)^2$

$y' = 3(x^2+2x)^2(2x+2)(4x+1)^2 + 2(4x+1)(4)(x^2+2x)^3$

$y' = (x^2+2x)^2(4x+1)\left[3(2x+2)(4x+1) + 8(x^2+2x)\right]$

$y' = (x^2+2x)^2(4x+1)\left[24x^2+6x+24x+6+8x^2+16x\right]$

$y' = (x^2+2x)^2(4x+1)(32x^2+46x+6)$

$y' = 2(x^2+2x)^2(4x+1)(16x^2+23x+3)$

n) $y = \dfrac{2x}{x-5}$

$y' = \dfrac{2(x-5) - 1(2x)}{(x-5)^2}$

$y' = \dfrac{2x-10-2x}{(x-5)^2}$

$y' = \dfrac{-10}{(x-5)^2}$

o) $y = \dfrac{3x}{x^2+2x-4}$

$y' = \dfrac{3(x^2+2x-4) - (2x+2)(3x)}{(x^2+2x-4)^2}$

$y' = \dfrac{3x^2+6x-12-6x^2-6x}{(x^2+2x-4)^2}$

$y' = \dfrac{-3x^2-12}{(x^2+2x-4)^2}$

$y' = \dfrac{-3(x^2+4)}{(x^2+2x-4)^2}$

p) $y = \dfrac{3x}{(x-2)^2}$

$y' = \dfrac{3(x-2)^2 - 2(x-2)(1)(3x)}{[(x-2)^2]^2}$

$y' = \dfrac{(x-2)[3(x-2)-6x]}{(x-2)^4}$

$y' = \dfrac{3x-6-6x}{(x-2)^3}$

$y' = \dfrac{-3x-6}{(x-2)^3}$

$y' = \dfrac{-3(x+2)}{(x-2)^3}$

q) $y = [(4x+x^2)^{-3}]^3$

$y' = (4x+x^2)^{-9}$

$y' = -9(4x+x^2)^{-10}(4+2x)$

$y' = \dfrac{-9(4+2x)}{(4x+x^2)^{10}}$

$y' = \dfrac{-18(2+x)}{(4x+x^2)^{10}}$

r) $y = 2x^{-1/2} + \frac{1}{\sqrt{3}}x + 6x^{1/3}$

$y' = -1x^{-3/2} + \frac{1}{\sqrt{3}} + 2x^{-2/3}$

$y' = \frac{-1}{x^{3/2}} + \frac{2}{x^{2/3}} + \frac{1}{\sqrt{3}}$

(2) $f(x) = \frac{3}{(2x-3)^2} = 3(2x-3)^{-2}$

$f'(x) = -6(2x-3)^{-3}(2)$

$f'(x) = -12(2x-3)^{-3}$

$f''(x) = 36(2x-3)^{-4}(2)$

$f''(x) = \frac{72}{(2x-3)^4}$

$f''(-1) = \frac{72}{[2(-1)-3]^4}$

$f''(-1) = \frac{72}{625}$

(3) $y = (2x^2-3)(x+1)^2$

Point:

$y(1) = [2(1)^2-3](1+1)^2$

$y(1) = -4$

$(1, -4)$

Slope:

$y' = 4x(x+1)^2 + 2(x+1)(2x^2-3)$

$y'(1) = 4(1)(1+1)^2 + 2(1+1)[2(1)^2-3]$

$y'(1) = 16 + 4(-1)$

$y'(1) = 12$

Eq^n: $y = mx+b$

$-4 = 12(1) + b$

$b = -16$

$\boxed{y = 12x - 16}$

④ $y = -x^3 + 6x^2$

$y' = -3x^2 + 12x$

$12 = -3x^2 + 12x$

$0 = -3x^2 + 12x - 12$

$0 = -3(x^2 - 4x + 4)$

$0 = -3(x-2)^2$

$x = 2$

⑤ Slope

$x + 5y - 10 = 0$

$5y = -x + 10$

$y = \frac{-1}{5}x + 2$

$\perp m = 5$

Point:

$y = 3x^2 - 7x + 5$

$y' = 6x - 7$

$5 = 6x - 7$

$x = 2$

$y(2) = 3(2)^2 - 7(2) + 5$

$y(2) = 3$

$(2, 3)$

Eqn

$y = mx + b$

$3 = 5(2) + b$

$b = -7$

$\boxed{y = 5x - 7}$

⑥ a) $V(5) = 3000\left(1 - \frac{5}{10}\right)^2$

$V(5) = 750$ L

b) $V'(t) = 6000\left(1 - \frac{t}{10}\right)\left(\frac{-1}{10}\right)$

$V'(t) = -600\left(1 - \frac{t}{10}\right)$

i) $V'(3) = -420$ L/min

ii) $V'(8) = -120$ L/min

⑦ $f(x) = x^3 - x^2 - x + 1$

$f'(x) = 3x^2 - 2x - 1$

$0 = 3x^2 - 2x - 1$

$0 = 3x^2 - 3x + 1x - 1$

$0 = 3x(x-1) + 1(x-1)$

$0 = (x-1)(3x+1)$

$x = 1 \quad x = -\frac{1}{3}$

$f(1) = 0 \quad (1, 0)$

$f\left(\frac{-1}{3}\right) = \frac{32}{27} \quad \left(\frac{-1}{3}, \frac{32}{27}\right)$

(8) $P(t) = (t^{1/4}+3)^3$

$P'(t) = 3(t^{1/4}+3)^2\left(\frac{1}{4}t^{-3/4}\right)$

$P'(16) = 3(16^{1/4}+3)^2\left[\frac{1}{4}(16)^{-3/4}\right]$

$P'(16) = \frac{75}{32} \simeq 2.34$ ppm / year

(9) $s(t) = 8 - 7t - t^2$

$v(t) = s'(t) = -7-2t$

$v(5) = -7-2(5)$

$v(5) = -17$ m/s

$a(t) = v'(t) = -2$

$a(5) = -2$ m/s^2

(10) x: # sold
p = price
n = # of price decreases

a) $x = 250 + 20n \longrightarrow n = \frac{x-250}{20}$

$p = 100 - 5n$

$p(x) = 100 - 5\left(\frac{x-250}{20}\right)$

$p(x) = 100 - \left(\frac{x-250}{4}\right)$

$p(x) = 100 - 0.25x + 62.5$

$\boxed{p(x) = -0.25x + 162.5}$

b) $R(x) = x \cdot p(x)$

$R(x) = x(-0.25x + 162.5)$

$R(x) = -0.25x^2 + 162.5x$

c) $R(250) = \$25000$

$R(290) = \$26100$

d) $R'(x) = -0.5x + 162.5$

$R'(290) = \$17.50$ / pair sold.

(12) a) $R(500) = 30(500) - 0.025(500)^2$

$= \$8750$

$R'(x) = 30 - 0.05x$

$R'(500) = 30 - 0.05(500)$

$R'(500) = \$5$ / item sold.

b) $C(500) = 2(500) + 5$

$= \$1005$

$C'(x) = 2$

$C'(500) = \$2$ / item sold.

c) $P(x) = R(x) - C(x)$

$P(x) = 30x - 0.025x^2 - (2x+5)$

$P(x) = 28x - 0.025x^2 - 5$

$P(500) = \$7745$

$P'(x) = 28 - 0.05x$

$P'(500) = \$3$ / item sold.

(13) $y = 2x - \frac{8}{x} = 2x - 8x^{-1}$

$y' = 2 + 8x^{-2}$

$y' = 2 + \frac{8}{x^2}$

$4 = 2 + \frac{8}{x^2}$

$2 = \frac{8}{x^2}$

$x^2 = 4$

$x = \pm 2$

Points:

$y(2) = 0$

$y(-2) = 0$

Eqⁿ:

$0 = 4(2) + b$

$b = -8$ $\boxed{y = 4x - 8}$

AND

$0 = 4(-2) + b$

$b = 8$ $\boxed{y = 4x + 8}$

(14) see answer key

(15) increase: $x > 1$

decrease: $x < -2$, $-2 < x < 1$

see answer key for graph.

(16) concave up: $-3 < x < 0$, $x > 3$

concave down: $x < -3$, $0 < x < 3$

(17) $f(x) = x^3 + 3x^2 + 1$

$f'(x) = 3x^2 + 6x$

$0 = 3x^2 + 6x$

$0 = 3x(x+2)$

$x = 0 \quad x = -2$

Test critical #'s AND endpoints:

$f(-2) = 5$

$f(0) = 1$

$F(2) = 21$

Absolute Min: $(0,1)$

Absolute Max: $(2,21)$

(18) $f(x) = -4x^3 + 6x^2 + 2$

a) $f'(x) = -12x^2 + 12x$

$0 = -12x(x-1)$

$x = 0 \quad x = 1$

$f(0) = 2 \quad F(1) = 4$

critical Points: $(0,2)$ and $(1,4)$

2nd Derivative test:

$f''(x) = -24x + 12$

$f''(0) = 12$; $f(0)$ is concave up
$(0,2)$ is a min.

$f''(1) = -12$; $f(1)$ is concave down
$(1,4)$ is a max.

b) $0 = -24x + 12$

$x = \frac{1}{2}$

$F(\frac{1}{2}) = (3)$

Possible POI $(0.5, 3)$

$-\infty$	0.5	∞
	0	1
$f''(x)$	+	−
$F(x)$	CU ∪	CD ∩

POI

concave up: $x < \frac{1}{2}$

concave down: $x > \frac{1}{2}$

POI at $(0.5, 3)$

(19) a) $y = 81 - x^4$

$y' = -4x^3$

$0 = -4x^3$

$x = 0$

$y(0) = 81$

critical point: $(0, 81)$

1st derivative test

	$-\infty$ to 0	0 to ∞
	-1	1
y'	$+$	$-$
y	inc. ↗	dec. ↘
	Max (at 0)	

Local Max at $(0, 81)$

b) $F(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 9$

$f'(x) = 12x^3 + 24x^2 - 12x - 24$

$0 = 12(x^3 + 2x^2 - x - 2)$

$0 = x^2(x+2) - 1(x+2)$

$0 = (x^2 - 1)(x+2)$

$0 = (x-1)(x+1)(x+2)$

$x = 1$ $\quad x = -1$ $\quad x = -2$

$f(1) = -10$ $\quad f(-1) =$ $\quad F(-2) =$

critical points: $(1, -10)$, $(-1, 22)$, $(-2, 17)$

1st Derivative test:

	$-\infty$ to -2	-2 to -1	-1 to 1	1 to ∞
	-3	-1.5	0	2
$f'(x)$	$-$	$+$	$-$	$+$
$F(x)$	dec ↘	inc. ↗	dec. ↘	inc. ↗
		min (at -2)	max (at -1)	min (at 1)

Local min: $(-2, 17)$ and $(1, -10)$

Local max: $(-1, 22)$

c) $f(x) = -3x^3 + 9x^2 - 8$

$f'(x) = -9x^2 + 18x$

$0 = -9x(x-2)$

$x = 0$ $\quad x = 2$

$f(0) = -8$ $\quad f(2) = 4$

critical points: $(0, -8)$ and $(2, 4)$

2nd derivative test:

$f''(x) = -18x + 18$

$f''(0) = 18$; $f(0)$ is concave up

Local min at $(0, -8)$

$f''(2) = -18$; $f(2)$ is concave down

Local max at $(2, 4)$

(20) a) $y = x^3 - 12x + 8$

$y' = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$0 = 3(x-2)(x+2)$

critical #'s: $x = 2$ $\quad x = -2$

$-\infty \quad -2 \quad 2 \quad \infty$

	-3	0	3
y'	$+$	$-$	$+$
y	inc. ↗	dec. ↘	inc. ↗
		max	min

increasing: $x < -2$, $x > 2$

decreasing: $-2 < x < 2$

b) $f(x) = 3x^4 - 2x^3 - 9x^2$

$f'(x) = 12x^3 - 6x^2 - 18x$

$0 = 6x(2x^2 - x - 3)$

$0 = 6x(2x^3 - 3x + 2x - 3)$

$0 = 6x[x(2x-3) + 1(2x-3)]$

$0 = 6x(2x-3)(x+1)$

critical #'s: $x = 0$ $\quad x = 1.5$ $\quad x = -1$

$-\infty \quad -1 \quad 0 \quad 1.5 \quad \infty$

	-2	-0.5	1	2
$f'(x)$	$-$	$+$	$-$	$+$
$f(x)$	dec. ↘	inc. ↗	dec. ↘	inc. ↗
	min	max	min	

increasing: $-1 < x < 0$, $x > 1.5$

decreasing: $x < -1$, $0 < x < 1.5$

(21) a) $y = \frac{1}{3}x^4 - 8x^2$

$y' = \frac{4}{3}x^3 - 16x$

$y'' = 4x^2 - 16$

$0 = 4(x^2 - 4)$

$0 = 4(x-2)(x+2)$

$x = 2$ $\quad x = -2$

$y(2) = -\frac{80}{3}$ $\quad y(-2) = -\frac{80}{3}$

Possible PoI's: $(2, -\frac{80}{3})$ and $(-2, -\frac{80}{3})$

$-\infty \quad -2 \quad 2 \quad \infty$

	-3	0	3
y''	$+$	$-$	$+$
y	CU $\cup$	CD $\cap$	CU $\cup$
	PoI	PoI	

concave up: $x < -2$, $x > 2$

concave down: $-2 < x < 2$

points of inflection: $(2, -\frac{80}{3})$ and $(-2, -\frac{80}{3})$

b) $f(x) = x^4 - 8x^3 + 3x - 5$

$f'(x) = 4x^3 - 24x^2 + 3$

$f''(x) = 12x^2 - 48x$

$0 = 12x(x-4)$

$x = 0 \qquad x = 4$

$f(0) = -5 \qquad f(4) = -249$

Possible POI's: $(0,-5)$ and $(4,-249)$

$-\infty \quad 0 \quad 4 \quad \infty$

	-1	1	5
$f''(x)$	+	−	+
$f(x)$	CU $\cup$	CD $\cap$	CU $\cup$

POI   POI

Concave up: $x<0$, $x>4$

concave down: $0<x<4$

POI's: $(0,-5)$ and $(4,-249)$

22 a)

1) No restriction, no asymptotes

2) x-int:

$0 = x^3 + 6x^2 + 9x + 4$

$0 = (x+1)(x^2+5x+4)$

$0 = (x+1)(x+4)(x+1)$

$(-1,0)$ and $(-4,0)$

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 9 & 4 \\ & \downarrow & -1 & -5 & -4 \; + \\ \hline \times & 1 & 5 & 4 & 0 \\ & x^2 & x & \# & R \end{array}$$

y-int: $(0,4)$

3) $h'(x) = 3x^2 + 12x + 9$

$0 = 3(x^2 + 4x + 3)$

$0 = 3(x+3)(x+1)$

$x = -3 \qquad x = -1$

$h(-3) = 4 \qquad h(-1) = 0$

critical points: $(-3,4)$ and $(-1,0)$

4) $h''(x) = 6x + 12$

$0 = 6(x+2)$

$x = -2$

$h(-2) = 2$

Possible POI: $(-2,2)$

5/6/7)

$-\infty \quad -3 \quad -2 \quad -1 \quad \infty$

	-4	-2.5	-1.5	0
$h'(x)$	+	−	−	+
$h''(x)$	−	−	+	+
$h(x)$	Inc. CD	Dec. CD	Dec. CU	Inc. CU

MAX   POI   MIN

8) See answer key for graph.

b) $f(x) = -3x^4 - 2x^3 + 15x^2 - 12x + 2$

1) no restrictions ; no asymptotes

2) x-int:

$0 = -3x^4 - 2x^3 + 15x^2 - 12x + 2$

1	-3	-2	15	-12	2
	↓	-3	-5	10	-2 +
x	-3	-5	10	-2	0

$0 = (x-1)(-3x^3 - 5x^2 + 10x - 2)$

1	-3	-5	10	-2
	↓	-3	-8	2 +
x	-3	-8	2	0

$0 = (x-1)^2(-3x^2 - 8x + 2)$

$x = 1$ $\qquad x = \frac{8 \pm \sqrt{(-8)^2 - 4(-3)(2)}}{2(-3)}$

$x \simeq -2.9 \qquad x \simeq 0.23$

$(1,0)$, $(-2.9, 0)$, and $(0.23, 0)$

y-int: $(0, 2)$

3)

$f'(x) = -12x^3 - 6x^2 + 30x - 12$

$0 = -6(2x^3 + x^2 - 5x + 2)$

1	2	1	-5	2
	↓	2	3	-2
	2	3	-2	0

$0 = -6(x-1)(2x^2 + 3x - 2)$

$0 = -6(x-1)(2x^2 + 4x - x - 2)$

$0 = -6(x-1)[2x(x+2) - 1(x+2)]$

$0 = -6(x-1)(x+2)(2x-1)$

$x = 1 \qquad x = -2 \qquad x = 0.5$

$f(1) = 0 \qquad f(-2) = 54 \qquad f(0.5) = -\frac{11}{16} \simeq -0.7$

critical points: $(1,0)$, $(0.5, -0.7)$, and $(-2, 54)$

4) $f''(x) = -36x^2 - 12x + 30$

$0 = -6(6x^2 + 2x - 5)$

$x = \frac{-2 \pm \sqrt{124}}{12}$

$x \simeq 0.76 \qquad x \simeq -1.1$

$f(0.76) \simeq -0.3 \qquad f(-1.1) \simeq 31.6$

Possible POI's: $(0.76, -0.3)$ and $(-1.1, 31.6)$

5/6/7)

	$-\infty$ to -2	-2 to -1.1	-1.1 to 0.5	0.5 to 0.76	0.76 to 1	1 to ∞
	-3	-1.5	-1	0.7	0.8	2
$f'(x)$	+	−	−	+	+	−
$f''(x)$	−	−	+	+	−	−
$f(x)$	inc CD	dec CD	dec CU	inc CU	inc CD	dec CD

MAX (at -2) POI (at -1.1) MIN (at 0.5) POI (at 0.76) MAX (at 1)

8) see answer key

c) $f(x) = -x^4 - x^3 + 2x^2$

1) No restrictions ; no asymptotes

2) x-int:

$$0 = -x^4 - x^3 + 2x^2$$

$$0 = -x^2(x^2 + x - 2)$$

$$0 = -x^2(x+2)(x-1)$$

$x = 0 \quad x = -2 \quad x = 1$

$(0,0), (-2,0), (1,0)$

y-int: $(0,0)$

3) $f'(x) = -4x^3 - 3x^2 + 4x$

$$0 = -x(4x^2 + 3x - 4)$$

$x = 0$ $\qquad x = \frac{-3 \pm \sqrt{73}}{8}$

$f(0) = 0$

$x \simeq 0.7 \qquad x \simeq -1.44$

$f(0.7) \simeq 0.4 \qquad f(-1.44) \simeq 2.8$

critical points: $(0,0)$, $(0.7, 0.4)$, $(-1.44, 2.8)$

4) $f''(x) = -12x^2 - 6x + 4$

$$0 = -2(6x^2 + 3x - 2)$$

$$x = \frac{-3 \pm \sqrt{57}}{12}$$

$x \simeq 0.38 \qquad x \simeq -0.88$

$f(0.38) \simeq 0.21 \qquad f(-0.88) \simeq 1.63$

Possible POI: $(0.38, 0.21)$ and $(-0.88, 1.63)$

5/6/7)

	$-\infty$ to -1.44	-1.44 to -0.88	-0.88 to 0	0 to 0.38	0.38 to 0.7	0.7 to ∞
	-2	-1	-0.5	0.1	0.5	0.8
$f'(x)$	+	−	−	+	+	−
$f''(x)$	−	−	+	+	−	−
$f(x)$	inc CD	dec CD	dec CU	inc CU	inc CD	dec CD

MAX (-1.44) POI (-0.88) MIN (0) POI (0.38) MAX (0.7)

8) see answer key

d) $y = \frac{5x}{x^2-1} = \frac{5x}{(x-1)(x+1)}$

1) VA: $x=1$ and $x=-1$

HA: $y=0$

2) x-int: $0=5x$

$x=0$

$(0,0)$

y-int: $(0,0)$

3)

$$y' = \frac{5(x^2-1) - 2x(5x)}{(x^2-1)^2}$$

$$y' = \frac{5x^2-5-10x^2}{(x^2-1)^2}$$

$$y' = \frac{-5x^2-5}{(x^2-1)^2}$$

$$y' = \frac{-5(x^2+1)}{(x^2-1)^2}$$

$x^2+1 \neq 0$; no critical points

4)

$$y'' = \frac{-10x(x^2-1)^2 - 2(x^2-1)(2x)(-5x^2-5)}{(x^2-1)^4}$$

$$y'' = \frac{(x^2-1)\left[-10x(x^2-1) - 4x(-5x^2-5)\right]}{(x^2-1)^4}$$

$$y'' = \frac{10x^3+30x}{(x^2-1)^3}$$

$$y'' = \frac{10x(x^2+3)}{(x^2-1)^3}$$

$x=0$

$y(0)=0$

Possible POI: $(0,0)$

5/6/7)

$-\infty$ -1 0 ∞

	-2	-0.5	0.5	2
y'	$-$	$-$	$-$	$-$
y''	$-$	$+$	$-$	$+$
y	dec CD	dec CU	dec CD	dec CU

8) see answer key

e) $y = \frac{1-x^2}{x^2+1} = \frac{(1-x)(1+x)}{x^2+1}$

1) VA: NONE ; $x^2+1 \neq 0$

HA: $y = -1$

2) x-int: $0 = (1-x)(1+x)$

$x = 1 \quad x = -1$

$(1,0)$ and $(-1,0)$

y-int: $(0,1)$

3) $y' = \frac{-2x(x^2+1) - 2x(1-x^2)}{(x^2+1)^2}$

$y' = \frac{-2x^3 - 2x - 2x + 2x^3}{(x^2+1)^2}$

$y' = \frac{-4x}{(x^2+1)^2}$

$x = 0$

$F(0) = 1$

critical point: $(0,1)$

4) $y'' = \frac{-4(x^2+1)^2 - 2(x^2+1)(2x)(-4x)}{(x^2+1)^4}$

$y'' = \frac{(x^2+1)\left[-4(x^2+1) + 16x^2\right]}{(x^2+1)^4}$

$y'' = \frac{12x^2 - 4}{(x^2+1)^3}$

$0 = 4(3x^2 - 1)$

$x = \pm\frac{1}{\sqrt{3}} \approx \pm 0.58$

$f(-0.58) \approx 0.5 \qquad F(0.58) \approx 0.5$

Possible POI: $(-0.58, 0.5)$ and $(0.58, 0.5)$

5/6/7)

$-\infty \qquad -0.58 \qquad 0 \qquad 0.58 \qquad \infty$

	-1	-0.5	0.5	1
y'	+	+	−	−
y''	+	−	−	+
y	inc CU	inc CD	dec. CD	dec CU

POI $\qquad$ MAX $\qquad$ POI

8) see answer key

f) $F(x) = 3x^4 - 16x^3 + 18x^2$

1) No restrictions
No asymptotes

2) x-int: $0 = x^2(3x^2 - 16x + 18)$

$x = 0$ $\qquad$ $x = \dfrac{16 \pm \sqrt{40}}{6}$

$x \simeq 3.72 \qquad x \simeq 1.61$

$(0,0) \qquad (3.72, 0) \qquad (1.61, 0)$

y-int: $(0,0)$

3) $F'(x) = 12x^3 - 48x^2 + 36x$

$0 = 12x(-x^2 - 4x + 3)$

$0 = 12x(x-3)(x-1)$

$x = 0 \qquad x = 3 \qquad x = 1$

$f(0) = 0 \qquad F(3) = -27 \qquad F(1) = 5$

critical points: $(0,0)$, $(3,-27)$, and $(1,5)$

4) $f''(x) = 36x^2 - 96x + 36$

$0 = 12(3x^2 - 8x + 3)$

$x = \dfrac{8 \pm \sqrt{28}}{6}$

$x \simeq 2.2 \qquad x \simeq 0.45$

$F(2.2) \simeq -13 \qquad F(0.45) \simeq 2.3$

Possible POI's: $(2.2, -13)$ and $(0.45, 2.3)$

5/6/7)

Boundaries: $-\infty$, 0, 0.45, 1, 2.2, 3, ∞

	-1	0.1	0.5	2	2.5	4
$F'(x)$	$-$	$+$	$+$	$-$	$-$	$+$
$F''(x)$	$+$	$+$	$-$	$-$	$+$	$+$
$F(x)$	dec CU	inc CU	inc CD	Dec. CD	Dec CU	Inc. CU

MIN (at 0) $\quad$ POI (at 0.45) $\quad$ MAX (at 1) $\quad$ POI (at 2.2) $\quad$ MIN (at 3)

8) see answer key.

23

$V = x^2 y$

$4800 = x^2 y$

$y = \frac{4800}{x^2}$

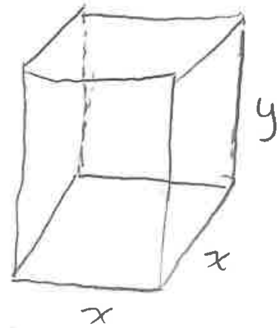
$SA = 2x^2 + 4xy$

$SA(x) = 2x^2 + 4x\left(\frac{4800}{x^2}\right)$

$SA(x) = 2x^2 + \frac{19200}{x}$

$C(x) = 0.002(2x^2) + 0.001\left(\frac{19200}{x}\right)$

$C(x) = 0.004x^2 + \frac{19.2}{x}$



$C'(x) = 0.008x - 19.2x^{-2}$

$0 = 0.008x - \frac{19.2}{x^2}$

$\frac{19.2}{x^2} = 0.008x$

$\frac{19.2}{0.008} = x^3$

$x^3 = 2400$

$x \simeq 13.4\text{cm}$

$y = \frac{4800}{13.4^2} \simeq 26.7\text{cm}$

2nd derivative test:

$C''(x) = 0.008 + 38.4x^{-3}$

$C''(13.4) \simeq 0.024 \therefore C(13.4)$ is concave up.

$C(13.4)$ is a min.

The dimensions that minimize the cost are 13.4 cm by 13.4 cm by 26.7 cm.

(24) $V = \pi r^2 h$

a) $500 = \pi r^2 h$

$h = \frac{500}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r h$

$SA(r) = 2\pi r^2 + 2\pi r\left(\frac{500}{\pi r^2}\right)$

$SA(r) = 2\pi r^2 + \frac{1000}{r}$

$SA'(r) = 4\pi r - \frac{1000}{r^2}$

$0 = 4\pi r - \frac{1000}{r^2}$

$\frac{1000}{r^2} = 4\pi r$

$\frac{1000}{4\pi} = r^3$

$r \simeq 4.3$ cm

$h = \frac{500}{\pi(4.3)^2} \simeq 8.6$ cm

b) $C(r) = 2(2\pi r^2) + \frac{1000}{r}$

$C(r) = 4\pi r^2 + \frac{1000}{r}$

$C'(r) = 8\pi r - \frac{1000}{r^2}$

$0 = 8\pi r - \frac{1000}{r^2}$

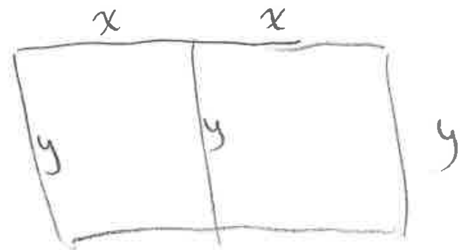
$\frac{1000}{r^2} = 8\pi r$

$\frac{1000}{8\pi} = r^3$

$r \simeq 3.4$ cm

$h = \frac{500}{\pi(3.4)^2} \simeq 13.8$ cm

(25)



$P = 4x + 3y$

$800 = 4x + 3y$

$\frac{800 - 4x}{3} = y$

$A = 2xy$

$A(x) = 2x\left(\frac{800-4x}{3}\right)$

$A(x) = \frac{1600x}{3} - \frac{8x^2}{3}$

$A'(x) = \frac{1600}{3} - \frac{16x}{3}$

$0 = \frac{1600 - 16x}{3}$

$16x = 1600$

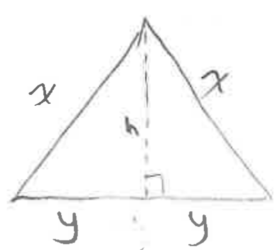
$x = 100$ m

$y = \frac{800 - 4(100)}{3}$

$y \simeq 133.3$ m

The rectangle formed by the two pens is 200m by 133.3m.

(26)



$P = 2x + 2y$

$200 = 2x + 2y$

$x = \frac{200 - 2y}{2}$

$x = 100 - y$

$h^2 = x^2 - y^2$

$h = \sqrt{x^2 - y^2}$

$h = \sqrt{(100-y)^2 - y^2}$

$h = \sqrt{10000 - 200y + y^2 - y^2}$

$h = \sqrt{10000 - 200y}$

$h = \sqrt{100(100-2y)}$

$h = 10\sqrt{100-2y}$

$A(y) = \frac{1}{2}(2y)(10\sqrt{100-2y})$

$A(y) = 10y\sqrt{100-2y}$

$A'(y) = 10\sqrt{100-2y} + \frac{1}{2}(100-2y)^{-1/2}(-2)(10y)$

$A'(y) = 10\sqrt{100-2y}\left[1 - y(100-2y)^{-1}\right]$

$0 = 10\sqrt{100-2y}$

$0 = 100 - 2y$

$y = 50$

$0 = 1 - \frac{y}{100-2y}$

$\frac{y}{100-2y} = 1$

$y = 100 - 2y$

$3y = 100$

$y = \frac{100}{3}$

Test on next page...

1^{st} derivative test:

limit of domain.

	$-\infty$	33.3	50	∞
	10	40	60	
$A'(y)$	+	−	undefined	
$A(y)$	↗	↘		
		Max		

local max at $y = \frac{100}{3}$ cm $\quad x = 100 - \frac{100}{3} = \frac{200}{3}$

Dimensions of triangle are $2y$ by x by $x = \frac{200}{3}$ by $\frac{200}{3}$ by $\frac{200}{3}$ cm.

27) a) $y = 2x + 3e^x$

$y' = 2 + 3e^x$

b) $y = 10^x$

$y' = 10^x \ln(10)$

c) $y = 4^{3x^2}$

$y' = 4^{3x^2}[\ln(4)](6x)$

$y' = 6x(4)^{3x^2}\ln(4)$

d) $y = (x^4)2^x$

$y' = 4x^3(2^x) + 2^x[\ln(2)](x^4)$

$y' = x^3 2^x[4 + x\ln(2)]$

e) $y = 3\sin x + \cos(2x)$

$y' = 3\cos x - [\sin(2x)](2)$

$y' = 3\cos x - 2\sin(2x)$

f) $y = \tan(3x)$

$y' = [\sec^2(3x)](3)$

$y' = 3\sec^2(3x)$

g) $y = [\sin(2x)]e^{3x}$

$y' = [2\cos(2x)]e^{3x} + 3e^{3x}[\sin(2x)]$

$y' = e^{3x}[2\cos(2x) + 3\sin(2x)]$

h) $y = x^2 \sin x$

$y' = 2x \sin x + (-\cos x) x^2$

$y' = x\,(2\sin x - x\cos x)$

i) $y = 5(2)^x$

$y' = 5(2)^x \ln(2)$

j) $y = -2(10)^{3x}$

$y' = -2(10)^{3x}\left[\ln(10)\right](3)$

$y' = -6(10)^{3x} \ln(10)$

k) $y = \sin^3(x^2)$

$y = \left[\sin(x^2)\right]^3$

$y' = 3\left[\sin(x^2)\right]^2\left[\cos(x^2)\right](2x)$

$y' = 6x \sin^2(x^2)\cos(x^2)$

l) $y = \tan\left[(1-x)^{1/2}\right]$

$y' = \left[\sec^2\left[(1-x)^{1/2}\right]\right]\left[\frac{1}{2}(1-x)^{-1/2}(-1)\right]$

$y' = \dfrac{-\sec^2\sqrt{1-x}}{2\sqrt{1-x}}$

m) $y = \left[\sin(2x)+1\right]^4$

$y' = 4\left[\sin(2x)+1\right]^3\left[\cos(2x)\right](2)$

$y' = 8\left[\sin(2x)+1\right]^3\cos(2x)$

n) $y = \sin\left[\cos(x^2)\right]$

$y' = \cos\left(\cos(x^2)\right)\left[-\sin(x^2)\right](2x)$

$y' = -2x\cos\left[\cos(x^2)\right]\sin(x^2)$

o) $y = 2\sin x - 3\cos(5x)$

$y' = 2\cos x - 3[-\sin(5x)](5)$

$y' = 2\cos x + 15\sin(5x)$

p) $x^2y + y^2x = 1$

$2xy + y'(x^2) + 2yy'x + 1y^2 = 0$

$y'x^2 + 2yy'x = -2xy - y^2$

$y'(x^2 + 2xy) = -2xy - y^2$

$$y' = \frac{-2xy - y^2}{x^2 + 2xy}$$

q) $x = \sin(y)$

$1 = [\cos(y)]\,y'$

$$y' = \frac{1}{\cos y}$$

r) $y^5 + xy = 3$

$5y^4y' + 1y + y'x = 0$

$5y^4y' + y'x = -y$

$y'(5y^4 + x) = -y$

$$y' = \frac{-y}{5y^4 + x}$$

s) $\sin y + \sin x = 1$

$(\cos y)(y') + \cos x = 0$

$$y' = -\frac{\cos x}{\cos y}$$

t) $yx^2 + e^y = x$

$y'x^2 + 2xy + e^y y' = 1$

$y'x^2 + e^4y' = 1 - 2xy$

$y'(x^2 + e^y) = 1 - 2xy$

$$y' = \frac{1 - 2xy}{x^2 + e^y}$$

u) $y = \ln(x^3+3x)$

$y' = \frac{1}{(x^3+3x)\ln e}(3x^2+3)$

$y' = \frac{3x^2+3}{x^3+3x}$

v) $y = 4\log_5 x$

$y' = 4\left(\frac{1}{x\ln 5}\right)$

$y' = \frac{4}{x\ln(5)}$

w) $y = \log_2(\cos x)$

$y' = \frac{1}{\cos x\ln(2)}(-\sin x)$

$y' = \frac{-\sin x}{\cos x\ln(2)}$

x) $y = \log(4x^2)$

$y' = \frac{1}{4x^2\ln(10)}(8x)$

$y' = \frac{2}{x\ln(10)}$

y) $y = x^2 e^{\sin x}$

$y' = 2x e^{\sin x} + e^{\sin x}(\cos x)x^2$

$y' = xe^{\sin x}(2 + x\cos x)$

28

$y = \sin(\cos x)$

$y' = [\cos(\cos x)](-\sin x)$

$y'(\frac{\pi}{2}) = [\cos(\cos\frac{\pi}{2})](-\sin\frac{\pi}{2})$

$y'(\frac{\pi}{2}) = \cos(0)(-1)$

$y'(\frac{\pi}{2}) = (1)(-1)$

$y'(\frac{\pi}{2}) = -1$

$m = -1$

29

$y = 2x^2 + \sin(4x)$

Point:

$y(\frac{\pi}{3}) = 2(\frac{\pi}{3})^2 + \sin(\frac{4\pi}{3})$

$y(\frac{\pi}{3}) = \frac{2\pi^2}{9} - \frac{\sqrt{3}}{2}$

Slope

$y' = 4x + 4\cos(4x)$

$y'(\frac{\pi}{3}) = 4(\frac{\pi}{3}) + 4\cos(\frac{4\pi}{3})$

$y'(\frac{\pi}{3}) = \frac{4\pi}{3} + 4(\frac{-1}{2})$

$y'(\frac{\pi}{3}) = \frac{4\pi}{3} - 2$

$m = \frac{4\pi}{3} - 2$

$m = \frac{4\pi - 6}{3}$

Eq^n $y = mx + b$

$\frac{2\pi^2}{9} - \frac{\sqrt{3}}{2} = (\frac{4\pi}{3} - 2)(\frac{\pi}{3}) + b$

$\frac{2\pi^2}{9} - \frac{\sqrt{3}}{2} = \frac{4\pi^2}{9} - \frac{2\pi}{3} + b$

$\frac{-2\pi^2}{9} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3} = b$

$\frac{-4\pi^2 - 9\sqrt{3} + 12\pi}{18} = b$

$$\boxed{y = \frac{4\pi - 6}{3}x + \frac{-4\pi^2 - 9\sqrt{3} + 12\pi}{18}}$$

(30)

a) $I(6) = 9\cos[120\pi(6)] = 9\text{ A}$

b) $E(t) = I'(t) = 9[-\sin(120\pi t)](120\pi)$

$I'(t) = -1080\pi\sin(120\pi t)$

c) $E(3) = -1080\pi\sin[120\pi(3)]$

$= 0\text{ V}$

d) Max $= 1080\pi \simeq 3392.9\text{ V}$

$1080\pi = -1080\pi\sin(120\pi t)$

$-1 = \sin(120\pi t)$

$120\pi t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$

$t = \frac{3}{240}, \frac{7}{240}, \frac{11}{240}, \ldots$

Max voltage when $t = \frac{3+4k}{240}, k \in \mathbb{Z}, k \geq 0$

Min $= -1080\pi \simeq -3392.9\text{ V}$

$-1080\pi = -1080\pi\sin(120\pi t)$

$1 = \sin(120\pi t)$

$120\pi t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

$t = \frac{1}{240}, \frac{5}{240}, \frac{9}{240}, \ldots$

Min voltage when

$t = \frac{1+4k}{240}, k \in \mathbb{Z}, k \geq 0$

(31) $h(t) = 4\sin t$

a) $v(t) = h'(t)$

$v(t) = 4\cos t$

$v(5) = 4\cos(5)$

$= 1.13$ cm/s

b) $a(t) = v'(t)$

$a(t) = 4(-\sin t)$

$a(t) = -4\sin t$

$a(5) = -4\sin(5)$

$a(5) \simeq 3.84$ cm/s^2

(32) $P(t) = 125(2)^{t/4}$

a) $p(0) = 125$ people

b) $p(28) = 125(2)^7$

$p(28) = 16000$ people.

c) $p'(t) = 125(2)^{t/4}\ln(2)\left(\frac{1}{4}\right)$

$p'(28) = 125(2)^7\ln(2)\left(\frac{1}{4}\right)$

$\simeq 2773$ people/day

d) $22\,500 = 125(2)^{t/4}$

$180 = 2^{t/4}$

$\log_2(180) = \frac{t}{4}$

$4\log_2(180) = t$

$t \simeq 30$ days.

33) $y = \left(\frac{1}{2}\right)^x$

Slope:

$$y' = \left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right)$$

$$y'(-3) = \left(\frac{1}{2}\right)^{-3} \ln\left(\frac{1}{2}\right)$$

$$y'(-3) = 8 \ln\left(\frac{1}{2}\right)$$

Eq^n

$$y = mx + b$$

$$8 = 8\ln\left(\tfrac{1}{2}\right)(-3) + b$$

$$8 + 24\ln\left(\tfrac{1}{2}\right) = b$$

$$y = 8\ln\left(\tfrac{1}{2}\right)x + 8 + 24\ln\left(\tfrac{1}{2}\right)$$

34) $y = x \sin x$

Point:

$$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2}(1)$$

$$= \frac{\pi}{2}$$

$$\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Slope:

$$y' = 1\sin x + \cos x\,(x)$$

$$y'\left(\frac{\pi}{2}\right) = 1\sin\left(\frac{\pi}{2}\right) + \left(\cos\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right)$$

$$y'\left(\frac{\pi}{2}\right) = 1 + 0\left(\frac{\pi}{2}\right)$$

$$y'\left(\frac{\pi}{2}\right) = 1$$

$$m = 1$$

Eq^n

$$y = mx + b$$

$$\frac{\pi}{2} = 1\left(\frac{\pi}{2}\right) + b$$

$$b = 0$$

$$\boxed{y = x}$$

35

$y = xe^x + 3e^x$

$y' = 1e^x + e^x(x) + 3e^x$

$y' = e^x(1 + x + 3)$

$y' = e^x(x+4)$

$0 = e^x(x+4)$

$x = -4$

$y(-4) = (-4)e^{-4} + 3e^{-4}$

$y(-4) = -e^{-4}$

1st derivative test:

	$-\infty$	-5	-4	-3	∞
y'		$-$		$+$	
y		↘		↗	
			MIN		

Local min at $(-4, -e^{-4})$

36 $y = 2e^{3x}$

Slope:

$-6x + y = 2$

$y = 6x + 2$

$// m = 6$

Point:

$y' = 2e^{3x}(3)$

$y' = 6e^{3x}$

$6 = 6e^{3x}$

$1 = e^{3x}$

$3x = \ln(1)$

$x = \frac{\ln(1)}{3}$

$x = \frac{0}{3}$

$x = 0$

$y(0) = 2e^{3(0)}$

$y(0) = 2$

$(0, 2)$

Eq^n: $y = mx + b$

$2 = 0(0) + b$

$b = 2$

$\boxed{y = 6x + 2}$