UNIT 1

Chapter 1- Polynomial Functions

Lesson Package

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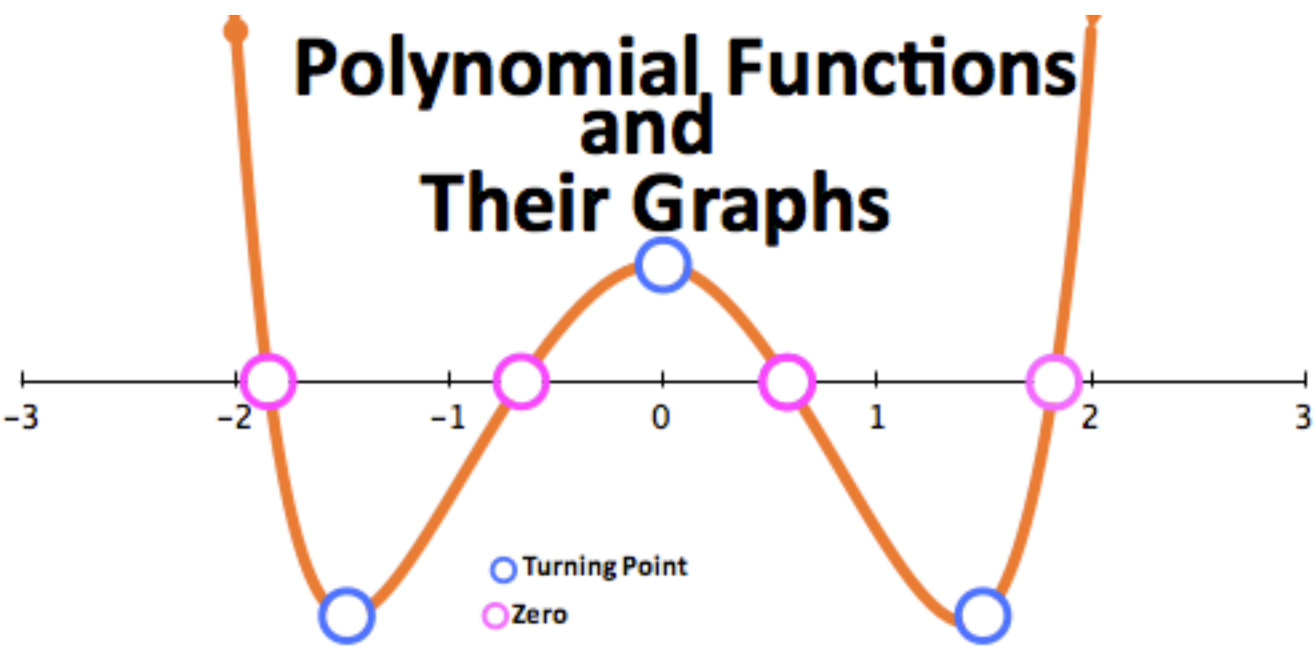

Chapter 1 Outline

Unit Goal: By the end of this unit, you will be able to identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions.

Section	Subject	Learning Goals	Curriculum Expectations
L1	Power Functions	- describe key features of graphs of power functions - learn interval notation - be able to describe end behaviour	C1.1, 1.2, 1.3
L2	Characteristics of Polynomial Functions	- describe characteristics of equations and graphs of polynomial functions - learn how degree related to turning points and x-intercepts	C1.1, 1.2, 1.3, 1.4
L3	Factored Form Polynomial Functions	- connect how factored form equation related to x-intercepts of graph of polynomial function - given graph, determine equation in factored form	C1.5, 1.7, 1.8
L4	Transformations of Polynomial Functions	- understand how the parameters a, k, d, and c transform power functions	C1.6
L5	Symmetry in Polynomial Functions	- understand the properties of even and odd polynomial functions	C1.9

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Properties of Polynomial Functions	F		P	
PreTest Review	F/A		P	
Test - Functions	O	C1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9	P	K(21%), T(34%), A(10%), C(34%)

L1 – 1.1 – Power Functions Lesson
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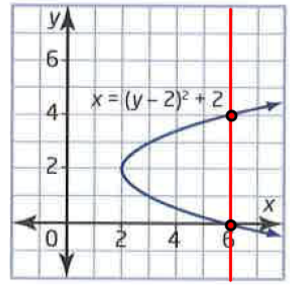
Things to Remember About Functions

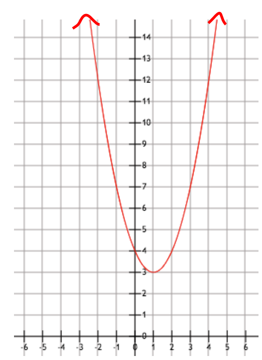


- A relation is a function if for every x-value there is only 1 corresponding y-value. The graph of a relation represents a function if it passes the ______________________________, that is, if a vertical line drawn anywhere along the graph intersects that graph at no more than one point.

- The ____________of a function is the complete set of all possible values of the independent variable (x)
  - Set of all possible x-vales that will output real y-values

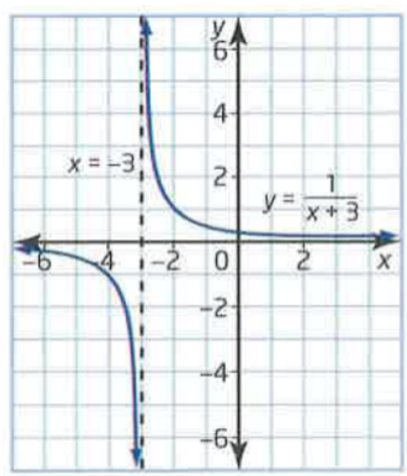
- The ____________of a function is the complete set of all possible resulting values of the dependent variable (y)
  - Set of all possible y-values we get after substituting all possible x-values

- For the function $f(x) = (x - 1)^2 + 3$



- The degree of a function is the highest exponent in the expression
  - $f(x) = 6x^3 - 3x^2 + 4x - 9$ has a degree of ____.

- An ____________________ is a line that a curve approaches more and more closely but never touches.



The function $y = \frac{1}{x+3}$ has two asymptotes:

Vertical Asymptote: Division by zero is undefined. Therefore the expression in the denominator of the function can not be zero. Therefore $x \neq -3$. This is why the vertical line $x = -3$ is an asymptote for this function.

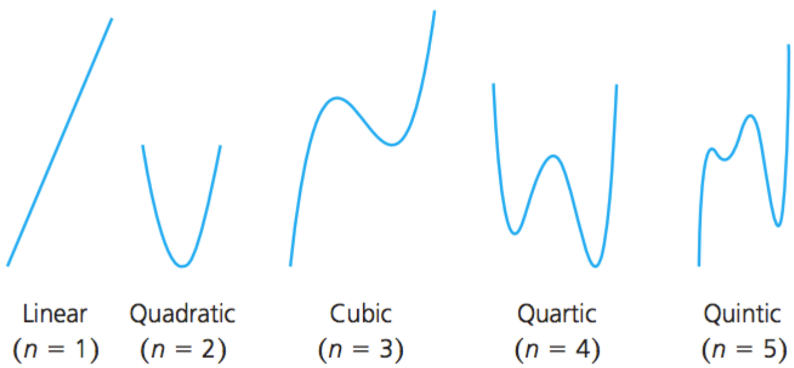
Horizontal Asymptote: For the range, there can never be a situation where the result of the division is zero. Therefore the line $y = 0$ is a horizontal asymptote. For all functions where the denominator is a higher degree than the numerator, there will by a horizontal asymptote at $y = 0$.

Polynomial Functions

A ***polynomial function*** has the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x^1 + a_0$$

- n Is a whole number
- x Is a variable
- the ______________________ $a_0, a_1, \ldots, a_n$ are real numbers
- the _________________ of the function is n, the exponent of the greatest power of x
- a_n, the coefficient of the greatest power of x, is the _____________________________
- a_0, the term without a variable, is the ________________________
- The domain of a polynomial function is the set of real numbers ________________
- The range of a polynomial function may be all real numbers, or it may have a lower bound or an upper bound (but not both)
- The graph of polynomial functions do not have horizontal or vertical asymptotes
- The graphs of polynomial functions of degree 0 are _______________________. The shapes of other graphs depends on the degree of the function. Five typical shapes are shown for various degrees:



A ________________________is the simplest type of polynomial function and has the form:

$$f(x) = ax^n$$

- a is a real number
- x is a variable
- n is a whole number

Example 1: Determine which functions are polynomials. State the degree and the leading coefficient of each polynomial function.

a) $g(x) = \sin x$

b) $f(x) = 2x^4$

c) $y = x^3 - 5x^2 + 6x - 8$

d) $g(x) = 3^x$

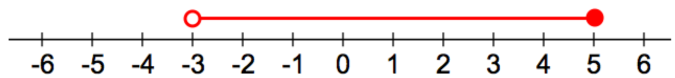
<u>**Interval Notation**</u>

In this course, you will often describe the features of the graphs of a variety of types of functions in relation to real-number values. Sets of real numbers may be described in a variety of ways:

1) as an inequality $-3 < x \leq 5$

2) interval (or bracket) notation $(-3, 5]$

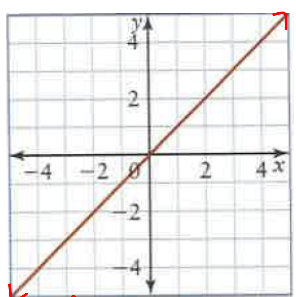
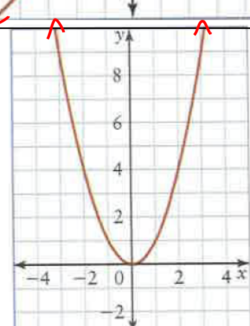
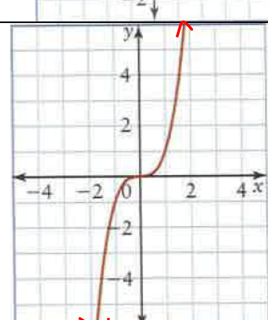
3) graphically on a number line



Note:

- Intervals that are infinite are expressed using ________________ or ______________________
- __________________indicate that the end value is included in the interval
- __________________indicate that the end value is NOT included in the interval
- A ____________ bracket is always used at infinity and negative infinity

Example 2: Below are the graphs of common power functions. Use the graph to complete the table.

Power Function	Special Name	Graph	Domain	Range	End Behaviour as $x \to -\infty$	End Behaviour as $x \to \infty$
$y = x$	Linear					
$y = x^2$	Quadratic					
$y = x^3$	Cubic					

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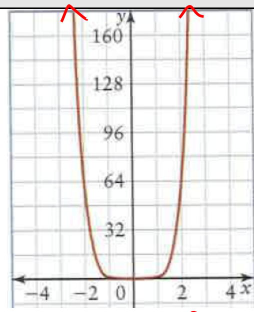
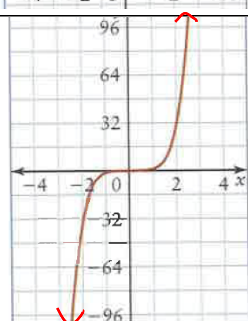
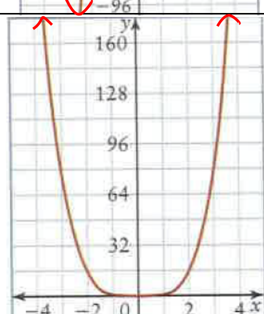
Power Function	Special Name	Graph	Domain	Range	End Behaviour as $x \to -\infty$	End Behaviour as $x \to \infty$
$y = x^4$	Quartic					
$y = x^5$	Quintic					
$y = x^6$	Sextic					
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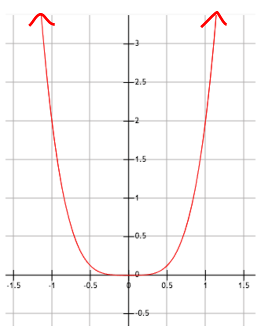
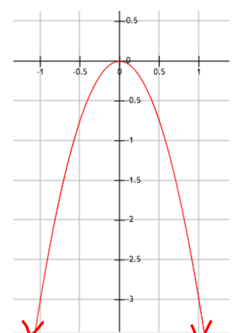
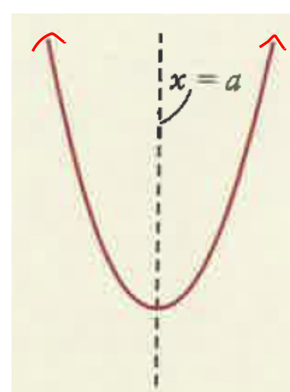
Key Features of EVEN Degree Power Functions

When the leading coefficient (a) is positive		When the leading coefficient (a) is negative	
End behaviour		**End behaviour**	
Domain		**Domain**	
Range		**Range**	
Example: $f(x) = 2x^4$		**Example:** $f(x) = -3x^2$	

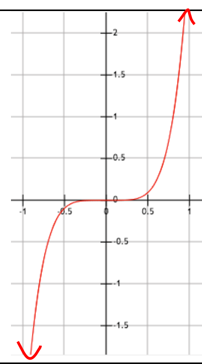
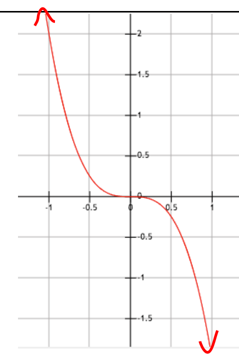
Line Symmetry

A graph has line symmetry if there is a vertical line $x = a$ that divides the graph into two parts such that each part is a reflection of the other.

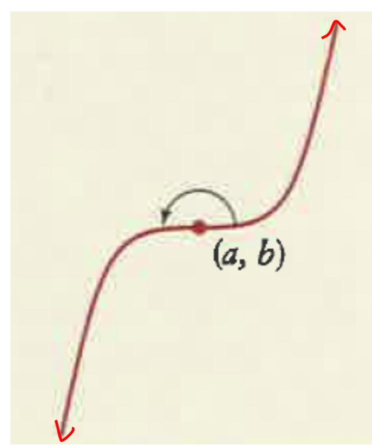


Note:

Key Features of ODD Degree Power Functions

When the leading coefficient (a) is positive		When the leading coefficient (a) is negative	
End behaviour		**End behaviour**	
Domain		**Domain**	
Range		**Range**	
Example: $f(x) = 3x^5$		**Example:** $f(x) = -2x^3$	

Point Symmetry



A graph has point point symmetry about a point (a, b) if each part of the graph on one side of (a, b) can be rotated 180° to coincide with part of the graph on the other side of (a, b).

Note:

Example 3: Write each function in the appropriate row of the second column of the table. Give reasons for your choices.

$y = 2x$ $\quad$ $y = 5x^6$ $\quad$ $y = -3x^2$ $\quad$ $y = x^7$

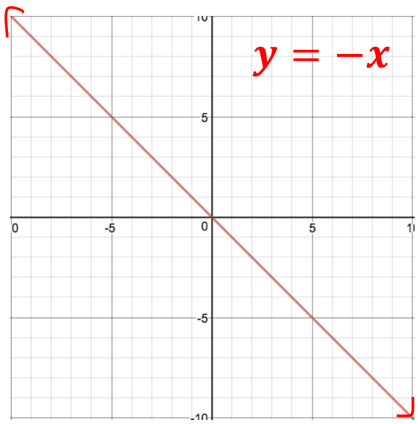
$y = -\frac{2}{5}x^9$ $\quad$ $y = -4x^5$ $\quad$ $y = x^{10}$ $\quad$ $y = -0.5x^8$

End Behaviour	Functions	Reasons
Q3 to Q1		
Q2 to Q4		
Q2 to Q1		
Q3 to Q4		

Example 4: For each of the following functions

i) State the domain and range
ii) Describe the end behavior
iii) Identify any symmetry

a)

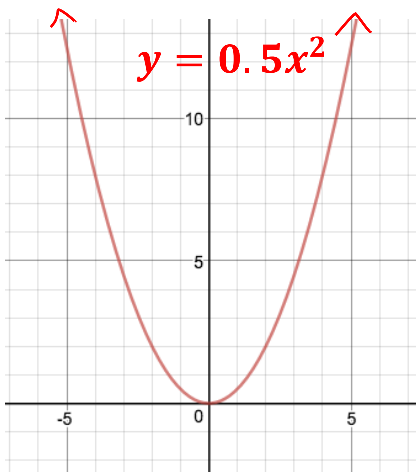


i) Domain: Range:

ii) As ________________ and as ________________
The graph extends from quadrant ____ to ____

iii)

b)

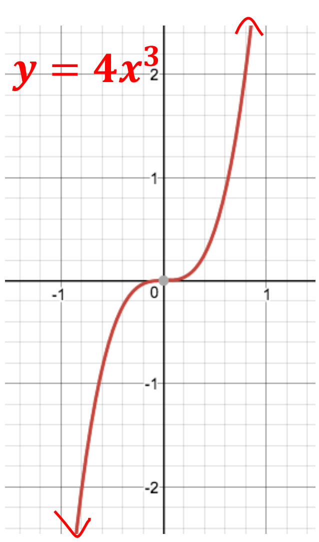


i) Domain: Range:

ii) As ________________ and as ________________
The graph extends from quadrant ____ to ____

iii)

c)



i) Domain: Range:

ii) As ________________ and as ________________
The graph extends from quadrant ____ to ____

iii)

L2 – 1.2 – Characteristics of Polynomial Functions Lesson
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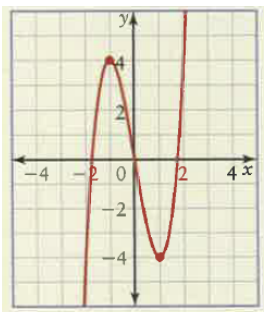
In section 1.1 we looked at power functions, which are single-term polynomial functions. Many polynomial functions are made up of two or more terms. In this section we will look at the characteristics of the graphs and equations of polynomial functions.

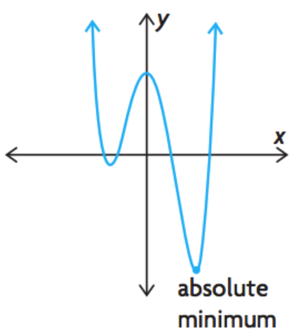
New Terminology – Local Min/Max vs. Absolute Min/Max

Local Min or Max Point – Points that are minimum or maximum points on some interval around that point.

Absolute Max or Min – The greatest/least value attained by a function for ALL values in its domain.



In this graph, (-1, 4) is a ________________ and (1, -4) is a ________________. These are not absolute min and max points because there are other points on the graph of the function that are smaller and greater. Sometimes local min and max points are called _________________________.



On the graph of this function…

There are ___ local min/max points. ___ are local min and ___ is a local max.

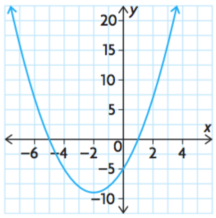
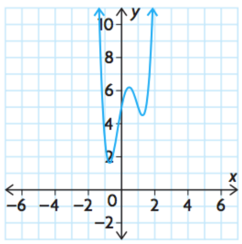
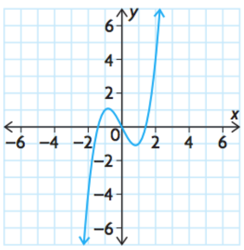
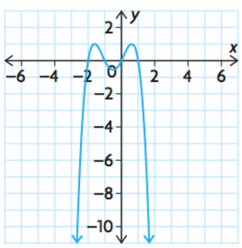
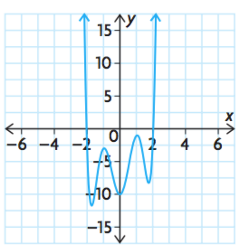
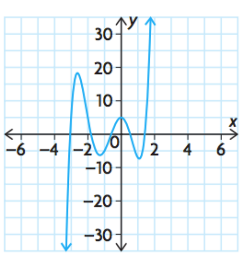
One of the local min points is also an absolute min (it is labeled).

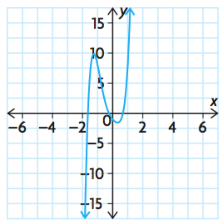
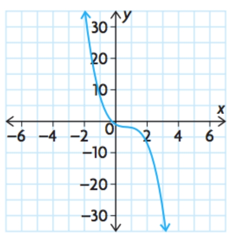
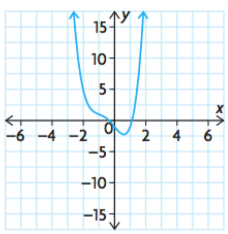
Investigation: Graphs of Polynomial Functions

The degree and the leading coefficient in the equation of a polynomial function indicate the end behaviours of the graph.

The degree of a polynomial function provides information about the shape, turning points (local min/max), and zeros (x-intercepts) of the graph.

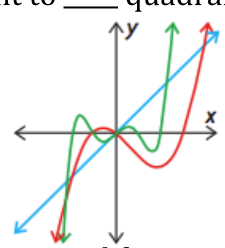
Complete the following table using the equation and graphs given:

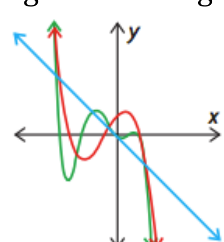
Equation and Graph	Degree	Even or Odd Degree?	Leading Coefficient	End Behaviour	Number of turning points	Number of x-intercepts
$f(x) = x^2 + 4x - 5$ 						
$f(x) = 3x^4 - 4x^3 - 4x^2 + 5x + 5$ 						
$f(x) = x^3 - 2x$ 						
$f(x) = -x^4 - 2x^3 + x^2 + 2x$ 						
$f(x) = 2x^6 - 12x^4 + 18x^2 + x - 10$ 						
$f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$ 						

Equation and Graph	Degree	Even or Odd Degree?	Leading Coefficient	End Behaviour	Number of turning points	Number of x-intercepts
$f(x) = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$ 						
$f(x) = -2x^3 + 4x^2 - 3x - 1$ 						
$f(x) = x^4 + 2x^3 - 3x - 1$ 						

Summary of Findings:

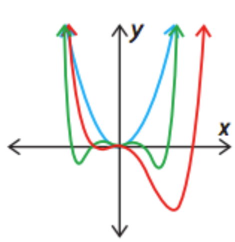
- A polynomial function of degree n has at most ____________ local max/min points (turning points)
- A polynomial function of degree n may have up to ____ distinct zeros (x-intercepts)
- If a polynomial function is ________ degree, it must have at least one x-intercept, and an even number of turning points
- If a polynomial function is _______ degree, it may have no x-intercepts, and an odd number of turning points
- An odd degree polynomial function extends from…
  - ____ quadrant to ____ quadrant if it has a positive leading coefficient
  - ____ quadrant to ____ quadrant if it has a negative leading coefficient

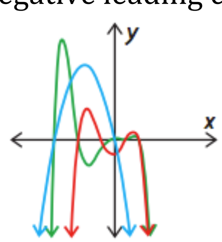




> ***Note:*** *Odd degree polynomials have OPPOSITE end behaviours*

- An even degree polynomial function extends from…
  - ____ quadrant to ____ quadrant if it has a positive leading coefficient
  - ____ quadrant to ____ quadrant if it has a negative leading coefficient





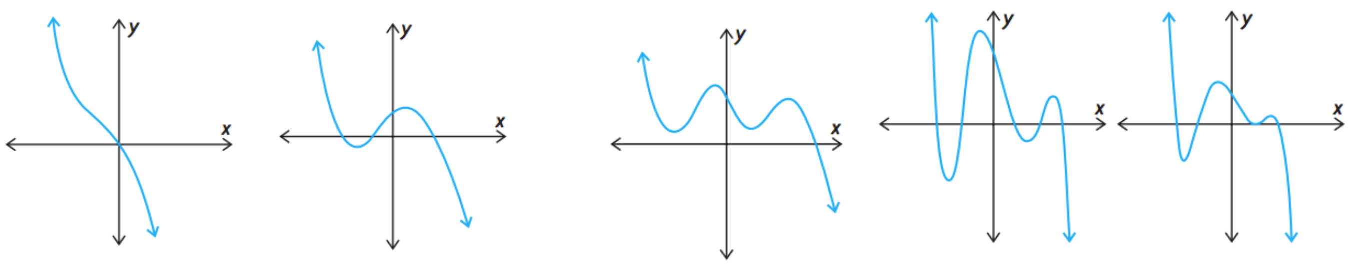
> ***Note:*** *Even degree polynomials have THE SAME end behaviour.*

Example 1: Describe the end behaviours of each function, the possible number of turning points, and the possible number of zeros. Use these characteristics to sketch possible graphs of the function

a) $f(x) = -3x^5 + 4x^3 - 8x^2 + 7x - 5$

> ***Note:*** *Odd degree functions must have an even number of turning points.*

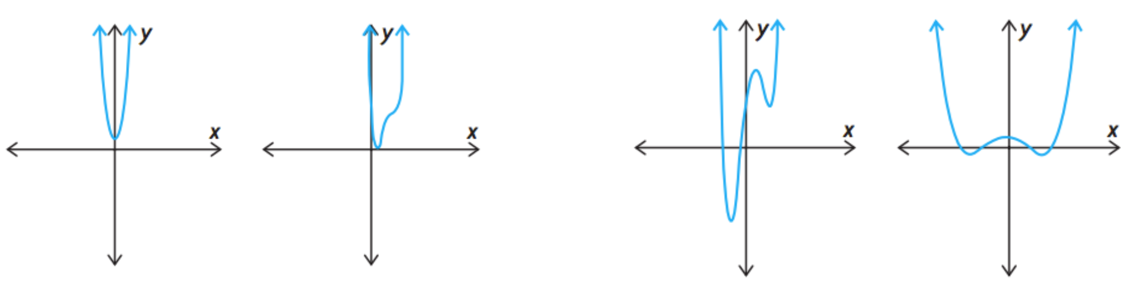
Possible graphs of 5th degree polynomial functions with a negative leading coefficient:



b) $g(x) = 2x^4 + x^2 + 2$

> ***Note:*** *Even degree functions must have an odd number of turning points.*

Possible graphs of 4th degree polynomial functions with a positive leading coefficient:

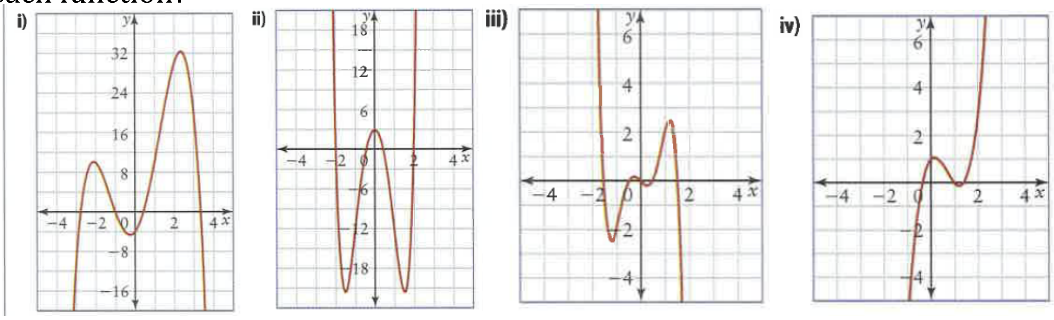


Example 2: Fill out the following chart

Degree	Possible # of x-intercepts	Possible # of turning points
1		
2		
3		
4		
5		

Example 3: Determine the key features of the graph of each polynomial function. Use these features to match each function with its graph. State the number of x-intercepts, the number of local max/min points, and the number of absolute max/min points for the graph of each function. How are these features related to the degree of each function?

a) $f(x) = 2x^3 - 4x^2 + x + 1$
b) $g(x) = -x^4 + 10x^2 + 5x - 4$
c) $h(x) = -2x^5 + 5x^3 - x$
d) $p(x) = x^6 - 16x^2 + 3$



a)

b)

c)

d)

Finite Differences

For a polynomial function of degree n, where n is a positive integer, the n^{th} differences…

- are equal
- have the same sign as the leading coefficient
- are equal to $a \cdot n!$, where a is the leading coefficient

Note:

$n!$ is read as n factorial.

$n! = n\times(n-1)\times(n-2)\times \ldots \times 2\times 1$

$5! = 5\times 4\times 3\times 2\times 1 = 120$

Example 4: The table of values represents a polynomial function. Use finite differences to determine

a) the degree of the polynomial function
b) the sign of the leading coefficient
c) the value of the leading coefficient

x	y	First differences	Second differences	Third differences
-3	-36			
-2	-12			
-1	-2			
0	0			
1	0			
2	4			
3	18			
4	48			

a)

b)

c)

Example 5: For the function $2x^4 - 4x^2 + x + 1$ what is the value of the constant finite differences?

$Finite\ differences =$

L3 – 1.3 – Factored Form Polynomial Functions Lesson
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In this section, you will investigate the relationship between the factored form of a polynomial function and the x-intercepts of the corresponding graph, and you will examine the effect of repeated factor on the graph of a polynomial function.

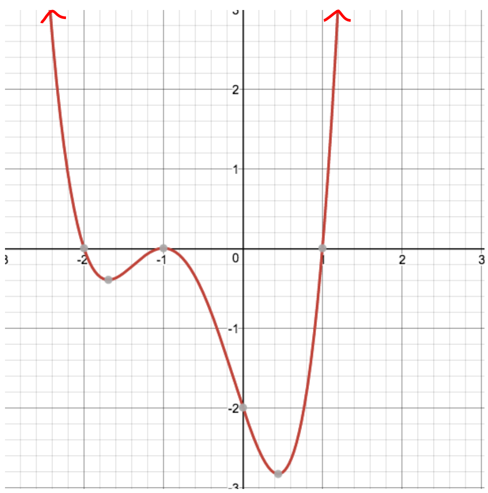
Factored Form Investigation

If we want to graph the polynomial function $f(x) = x^4 + 3x^3 + x^2 - 3x - 2$ accurately, it would be most useful to look at the factored form version of the function:

$f(x) = (x+1)^2(x+2)(x-1)$

Let's start by looking at the graph of the function and making connections to the factored form equation.

Graph of $f(x)$:



From the graph, answer the following questions…

a) What is the degree of the function?

b) What is the sign of the leading coefficient?

c) What are the x-intercepts?

d) What is the y-intercept?

e) The x-intercepts divide the graph in to into four intervals. Write the intervals in the first row of the table. In the second row, choose a test point within the interval. In the third row, indicate whether the function is positive (above the x-axis) or negative (below the y-axis).

Interval				
Test Point				
Sign of $f(x)$				

f) What happens to the sign of the of $f(x)$ near each x-intercept?

Conclusions from investigation:

The x-intercepts of the graph of the function correspond to the roots (zeros) of the corresponding equation. For example, the function $f(x) = (x-2)(x+1)$ has x-intercepts at ___ and ___. These are the roots of the equation $(x-2)(x+1) = 0$.

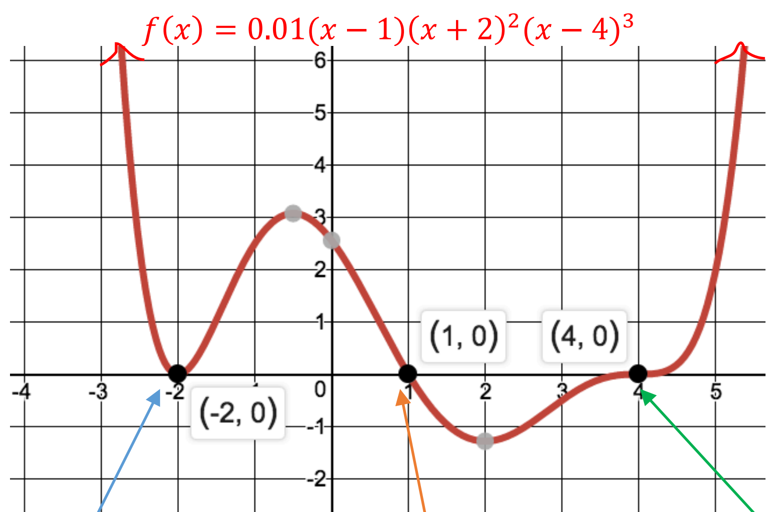
If a polynomial function has a factor $(x-a)$ that is repeated n times, then $x = a$ is a zero of __________ n.

Order – the exponent to which each factor in an algebraic expression is raised.

For example, the function $f(x) = (x-3)^2(x-1)$ has a zero of order _______ at $x = 3$ and a zero of order _______ at $x = 1$.

The graph of a polynomial function changes sign at zeros of __________ order but does not change sign at zeros of __________ order.

Shapes based on order of zero:



ORDER 2

(-2, 0) is an x-intercept of order 2. Therefore, it doesn't change sign.

"Bounces off" x-axis.

Parabolic shape.

ORDER 1

(1, 0) is an x-intercept of order 1. Therefore, it changes sign.

"Goes straight through" x-axis.

Linear Shape

ORDER 3

(4, 0) is an x-intercept of order 3. Therefore, it changes sign.

"S-shape" through x-axis.

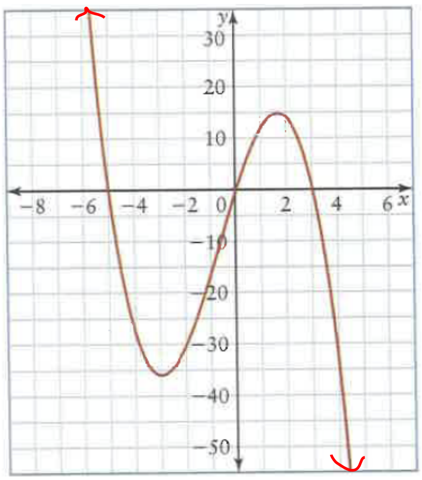
Cubic shape.

Example 1: Analyzing Graphs of Polynomial Functions

For each graph,

i) the least possible degree and the sign of the leading coefficient
ii) the x-intercepts and the factors of the function
iii) the intervals where the function is positive/negative

a)



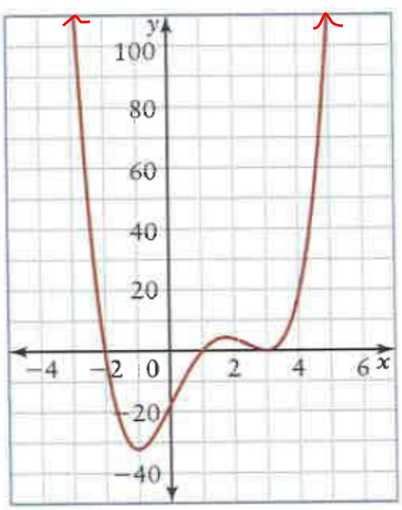
i)

ii)

iii)

Interval				
Sign of $f(x)$				

b)



i)

ii)

iii)

Interval				
Sign of $f(x)$				

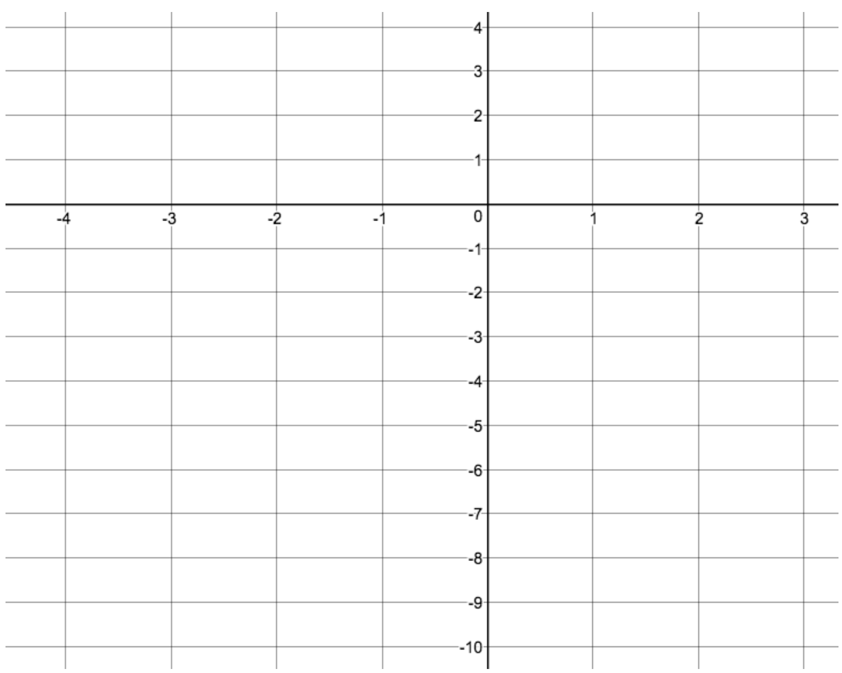
Example 2: Analyze Factored Form Equations to Sketch Graphs

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
The exponent on x when all factors of x are multiplied together OR Add the exponents on the factors that include an x.	The product of all the x coefficients	Use degree and sign of leading coefficient to determine this	Set each factor equal to zero and solve for x	Set $x = 0$ and solve for y

Sketch a graph of each polynomial function:

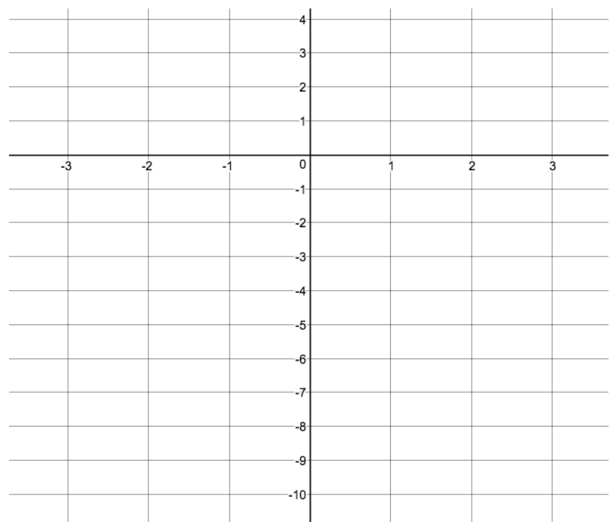
a) $f(x) = (x - 1)(x + 2)(x + 3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept



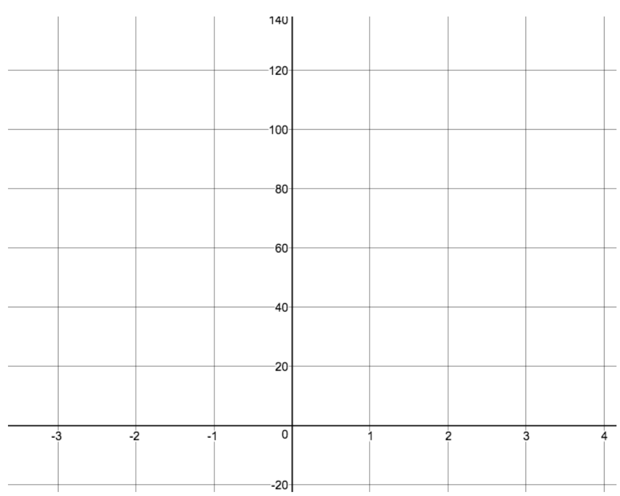
b) $g(x) = -2(x-1)^2(x+2)$

Degree	**Leading Coefficient**	**End Behaviour**	**x-intercepts**	**y-intercept**



c) $h(x) = -(2x+1)^3(x-3)$

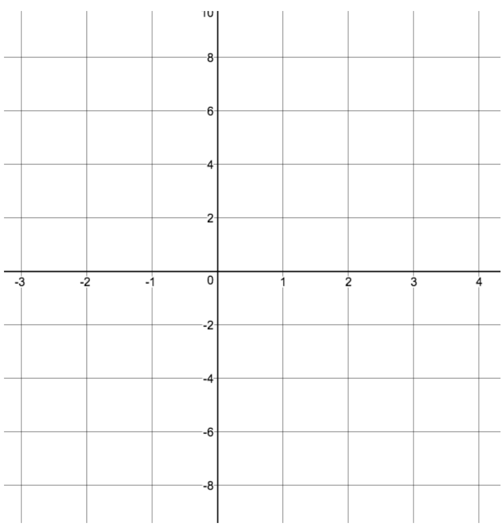
Degree	**Leading Coefficient**	**End Behaviour**	**x-intercepts**	**y-intercept**



d) $j(x) = x^4 - 4x^3 + 3x^2$

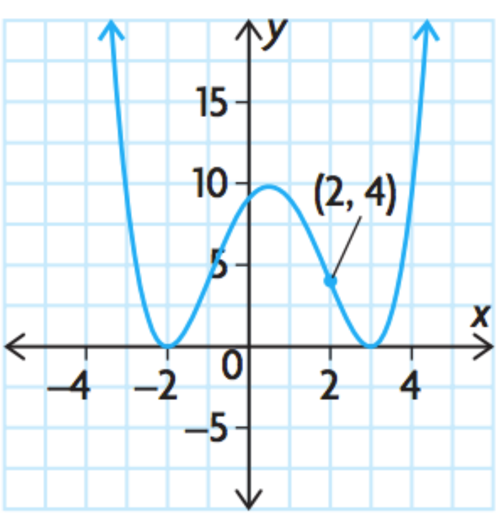
> ***Note:*** *must put in to factored form to find x-intercepts*

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept



Example 3: Representing the Graph of a Polynomial Function with its Equation

a) Write the equation of the function shown below:



> **Steps:**
>
> **1)** Write the equation of the family of polynomials using factors created from x-intercepts
>
> **2)** Substitute the coordinates of another point (x, y) into the equation.
>
> **3)** Solve for a
>
> **4)** Write the equation in factored form

b) Find the equation of a polynomial function that is degree 4 with zeros -1 (order 3) and 1, and with a y-intercept of -2.

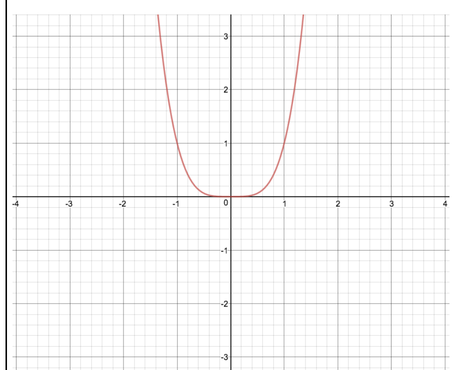
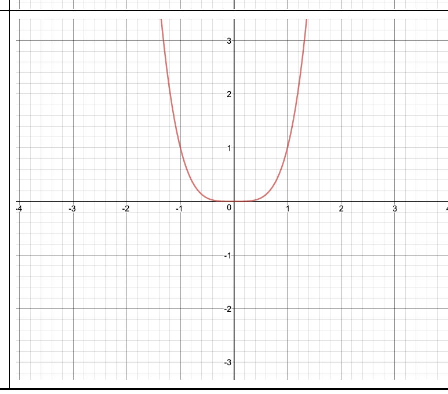
L4 – 1.4 – Transformations Lesson
MHF4U
Jensen

In this section, you will investigate the roles of the parameters a, k, d, and c in polynomial functions of the form $\boldsymbol{f(x) = a[k(x-d)]^n + c}$. You will apply transformations to the graphs of basic power functions to sketch the graph of its transformed function.

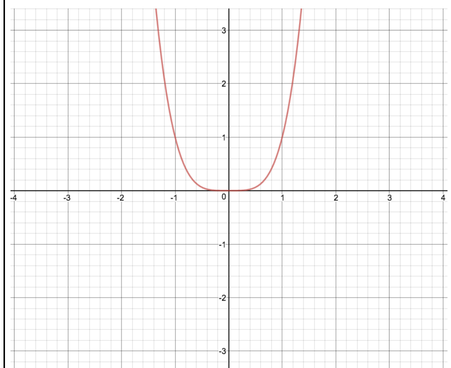
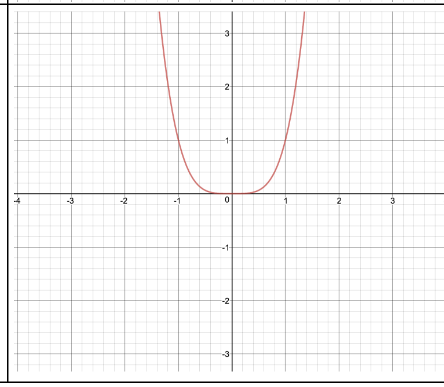
Part 1: Transformations Investigation

In this investigation, you will be looking at transformations of the power function $y = x^4$. Complete the following table using graphing technology to help. The graph of $y = x^4$ is given on each set of axes; sketch the graph of the transformed function on the same set of axes. Then comment on how the value of the parameter a, k, d, or c transforms the parent function.

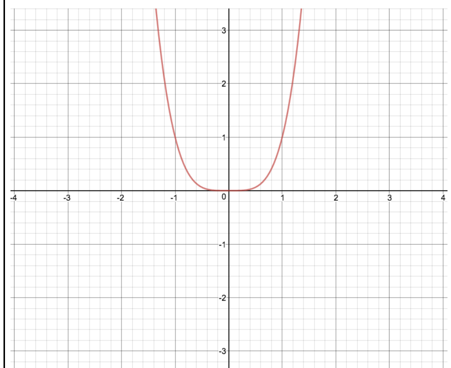
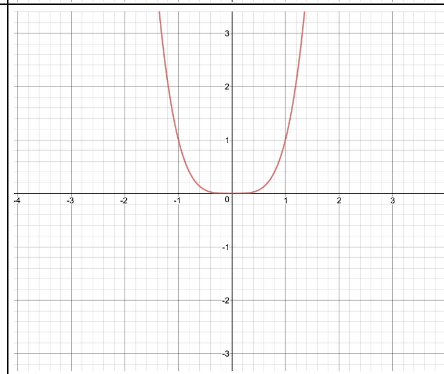
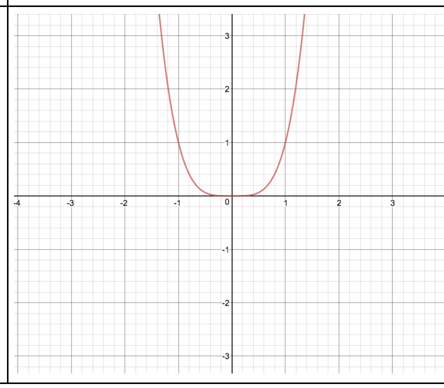
Effects of c on $y = x^4 + c$

Transformed Function	Value of c	Transformations to $y = x^4$	Graph of transformed function compared to $y = x^4$
$y = x^4 + 1$			
$y = x^4 - 2$			

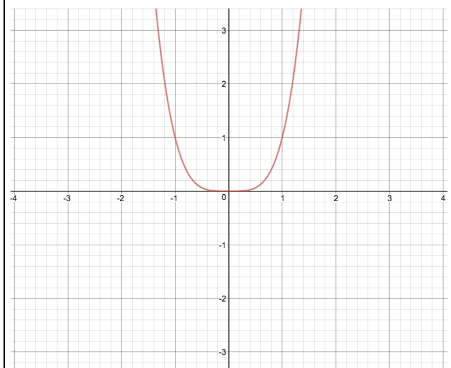
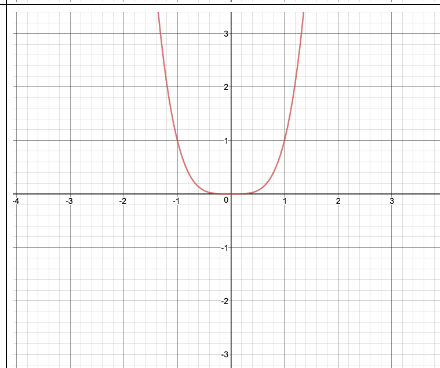
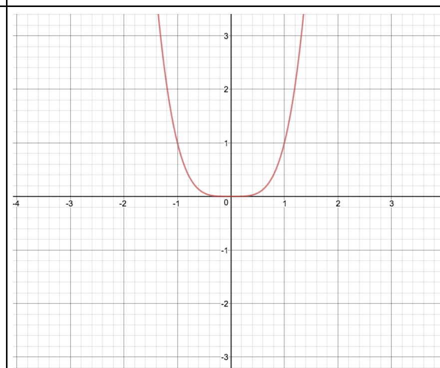
Effects of d on $y = (x - d)^4$

Transformed Function	**Value of d**	**Transformations to $y = x^4$**	**Graph of transformed function compared to $y = x^4$**
$y = (x - 2)^4$			
$y = (x + 3)^4$			

Effects of a on $y = ax^4$

Transformed Function	**Value of a**	**Transformations to $y = x^4$**	**Graph of transformed function compared to $y = x^4$**
$y = 2x^4$			
$y = \frac{1}{2}x^4$			
$y = -2x^4$			

Effects of k on $y = (kx)^4$

Transformed Function	**Value of k**	**Transformations to $y = x^4$**	**Graph of transformed function compared to $y = x^4$**
$y = (2x)^4$			
$y = \left(\frac{1}{3}x\right)^4$			
$y = (-2x)^4$			

Summary of effects of a, k, d, and c in polynomial functions of the form $f(x) = a[k(x-d)]^n + c$

Value of c in $f(x) = a[k(x-d)]^n + c$	
$c > 0$	
$c < 0$	

Value of d in $f(x) = a[k(x-d)]^n + c$	
$d > 0$	
$d < 0$	

Value of a in $f(x) = a[k(x-d)]^n + c$	
$a > 1$ or $a < -1$	
$-1 < a < 1$	
$a < 0$	

Value of k in $f(x) = a[k(x-d)]^n + c$	
$k > 1$ or $k < -1$	
$-1 < k < 1$	
$k < 0$	

Note:

a and c cause ________________ transformations and therefore effect the y-coordinates of the function.

k and d cause ________________ transformations and therefore effect the x-coordinates of the function.

When applying transformations to a parent function, make sure to apply the transformations represented by a and k BEFORE the transformations represented by d and c.

Part 2: Describing Transformations from an Equation

Example 1: Describe the transformations that must be applied to the graph of each power function, $f(x)$, to obtain the transformed function, $g(x)$. Then, write the corresponding equation of the transformed function. Then, state the domain and range of the transformed function.

a) $f(x) = x^4, g(x) = 2f\left[\frac{1}{3}(x - 5)\right]$

b) $f(x) = x^5, g(x) = \frac{1}{4}f[-2(x - 3)] + 4$

Part 3: Applying Transformations to Sketch a Graph

Example 2: The graph of $f(x) = x^3$ is transformed to obtain the graph of $g(x) = 3[-2(x+1)]^3 + 5$.

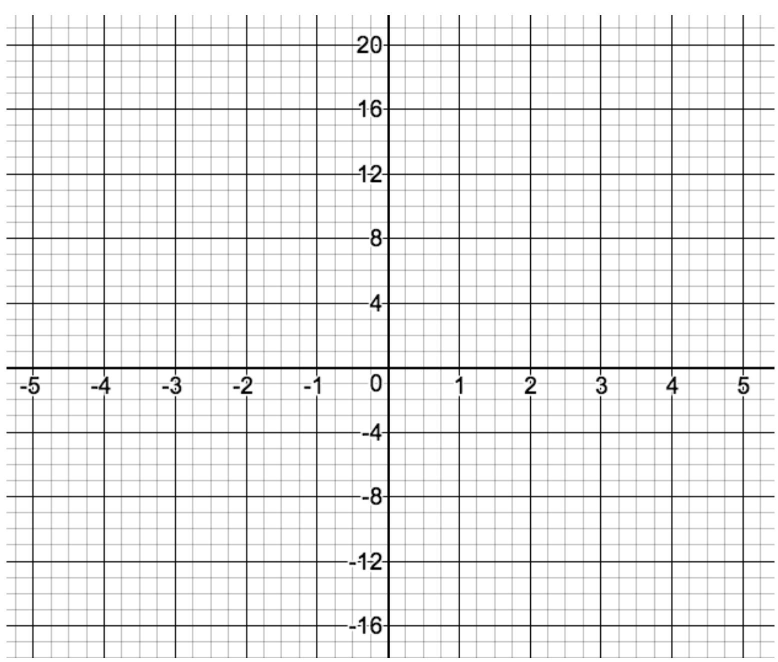
a) State the parameters and describe the corresponding transformations

b) Make a table of values for the parent function and then use the transformations described in part a) to make a table of values for the transformed function.

$f(x) = x^3$	
x	y

Note: When choosing key points for the parent function, always choose x-values between -2 and 2 and calculate the corresponding values of y.

c) Graph the parent function and the transformed function on the same grid.



Example 3: The graph of $f(x) = x^4$ is transformed to obtain the graph of $g(x) = -\left(\frac{1}{3}x + 2\right)^4 - 1$.

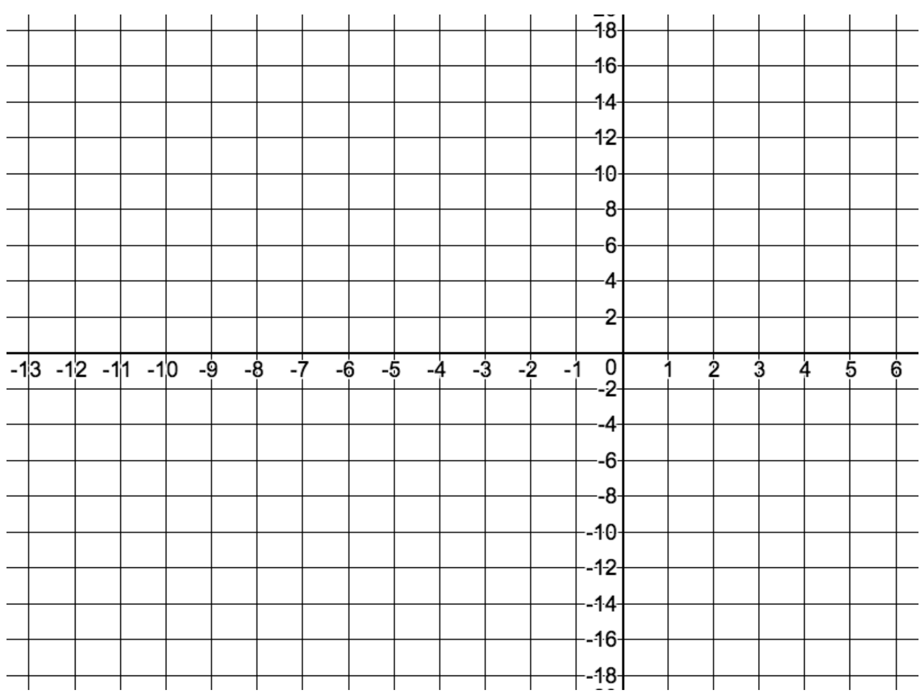
a) State the parameters and describe the corresponding transformations

Note: k value must be factored out in to the form $[k(x + d)]$

b) Make a table of values for the parent function and then use the transformations described in part a) to make a table of values for the transformed function.

$f(x) = x^4$	
x	y

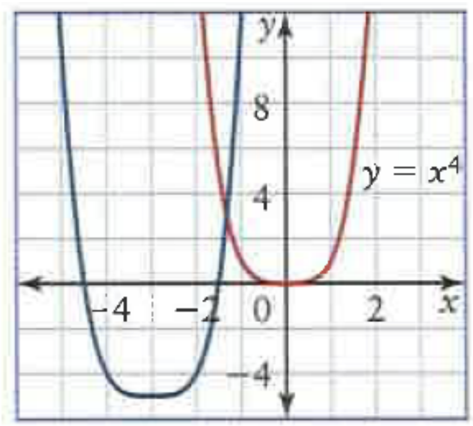
c) Graph the parent function and the transformed function on the same grid.



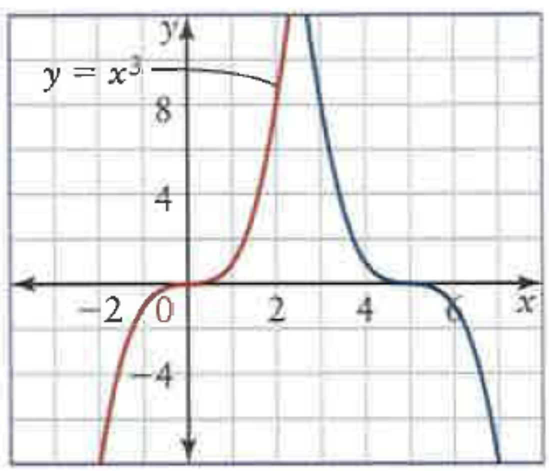
Part 4: Determining an Equation Given the Graph of a Transformed Function

Example 4: Transformations are applied to each power function to obtain the resulting graph. Determine an equation for the transformed function. Then state the domain and range of the transformed function.

a)



b)



L5 – 1.3 – Symmetry in Polynomial Functions
MHF4U
Jensen

In this section, you will learn about the properties of even and odd polynomial functions.

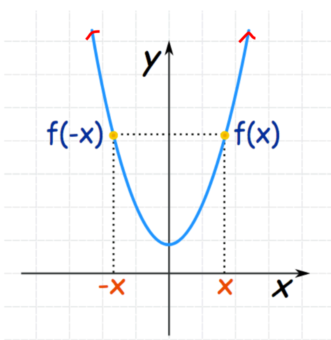
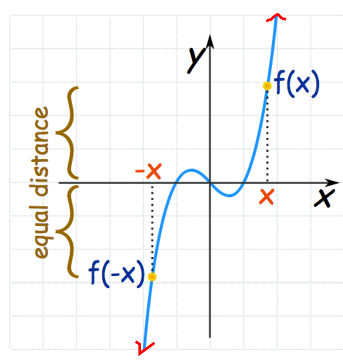
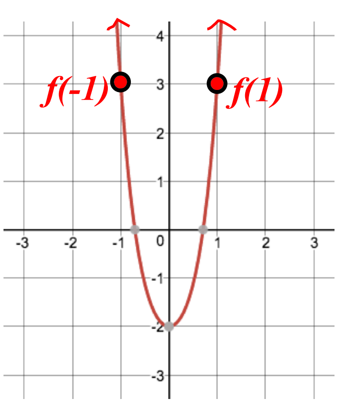
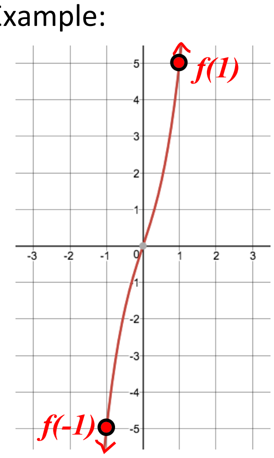
Symmetry in Polynomial Functions

____________________ – there is a vertical line over which the polynomial remains unchanged when reflected.

____ – there is a point about which the polynomial remains unchanged when rotated $180°$

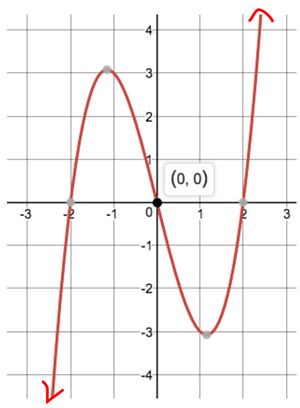
Section 1: Properties of Even and Odd Functions

A polynomial function of even or odd degree is NOT necessarily and even or odd function. The following are properties of all even and odd functions:

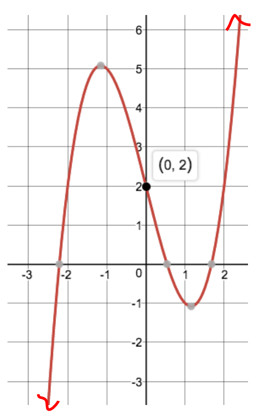
Even Functions	Odd Functions
An even degree polynomial function is an EVEN FUNCTION if: • Line symmetry over the ___________ • The exponent of each term is ________ • May have a constant term	An odd degree polynomial function is an ODD FUNCTION if: • Point symmetry about the ___________ • The exponent of each term is ______ • No constant term
Rule: 	Rule: 
Example:  $f(x) = 2x^4 + 3x^2 - 2$ Notice: $f(1) =$ $f(-1) =$ $\therefore$	Example:  $f(x) = 2x^3 + 3x$ Notice: $f(1) =$ $f(-1) =$ $\therefore$

Example 1: Identify each function as an even function, odd function, or neither. Explain how you can tell.

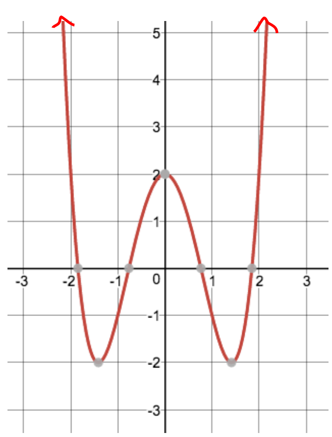
a) $y = x^3 - 4x$



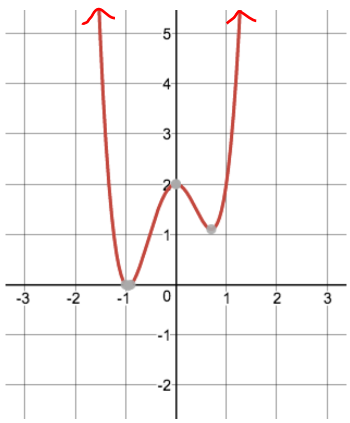
b) $y = x^3 - 4x + 2$



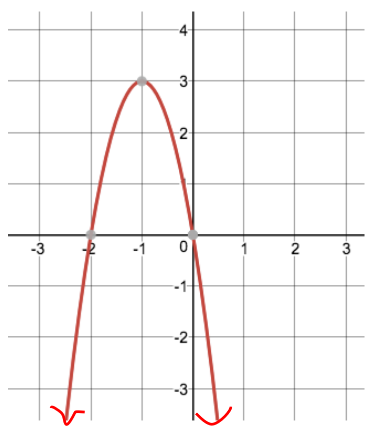
c) $y = x^4 - 4x^2 + 2$



d) $y = 3x^4 + x^3 - 4x^2 + 2$

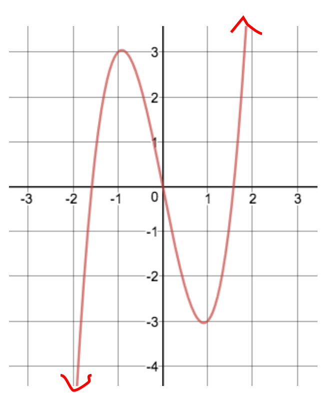


e) $y = -3x^2 - 6x$



<u>Example 2:</u> Choose all that apply for each function

a)



i) no symmetry

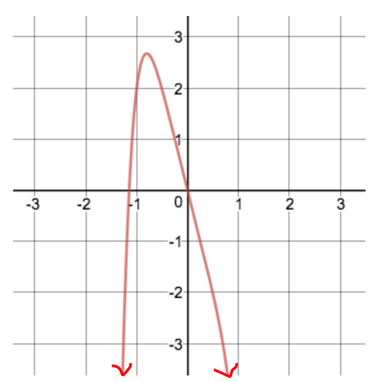
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

b)



i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

c) $P(x) = 5x^3 + 3x^2 + 2$

Note:

i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

d) $P(x) = x^6 + x^2 - 11$

i) no symmetry

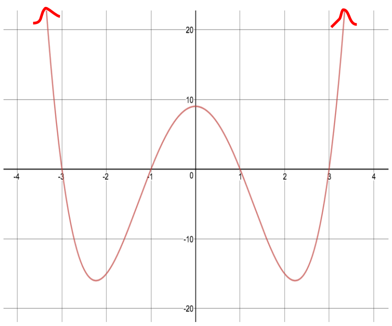
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

e)



i) no symmetry

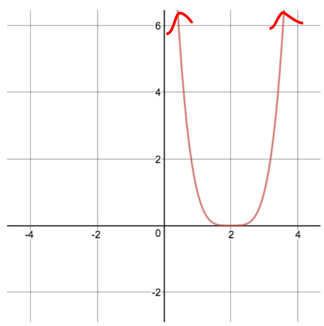
ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

f)



i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

g) $P(x) = 5x^5 - 4x^3 + 8x$

i) no symmetry

ii) point symmetry

iii) line symmetry

iv) odd function

v) even function

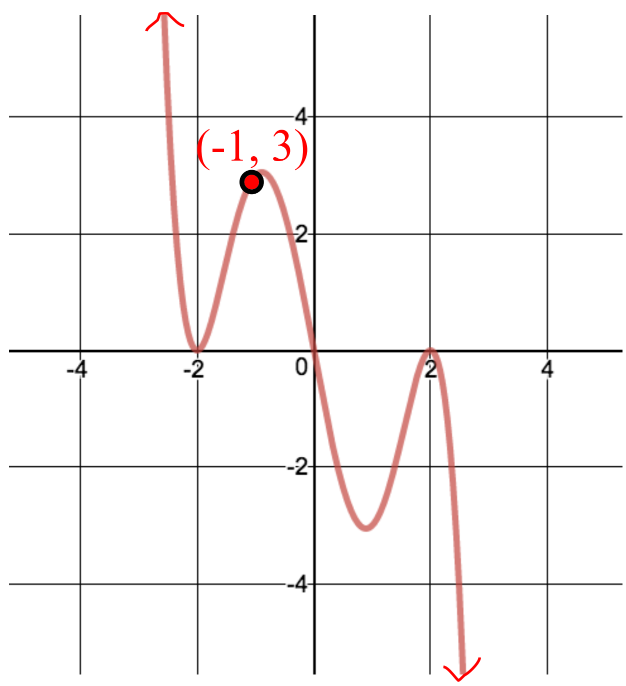
Example 3: Without graphing, determine if each polynomial function has line symmetry about the y-axis, point symmetry about the origin, or neither. Verify your response algebraically.

a) $f(x) = 2x^4 - 5x^2 + 4$

b) $f(x) = -3x^5 + 9x^3 + 2x$

c) $x^6 - 4x^3 + 6x^2 - 4$

Section 2: Connecting from throughout the unit



Example 4: Use the given graph to state:

a) x-intercepts

b) number of turning points

c) least possible degree

b) any symmetry present

c) the intervals where $f(x) < 0$

d) Find the equation in factored form