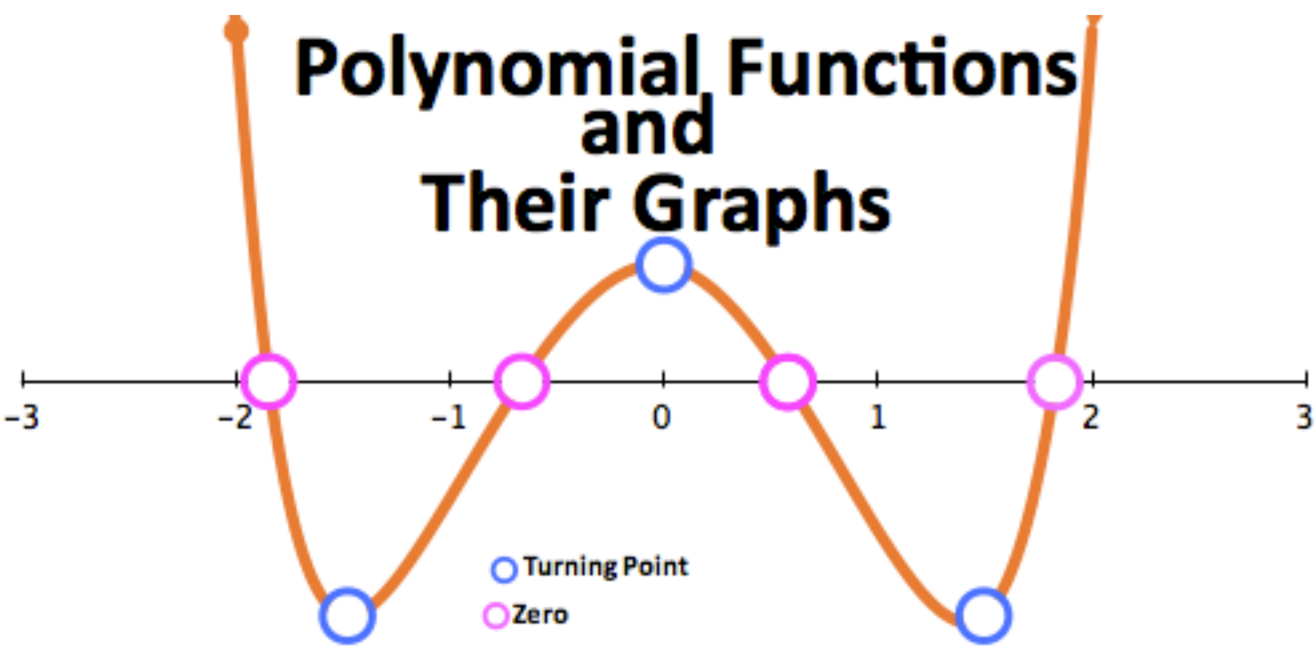
UNIT 1

Chapter 1- Polynomial Functions

WORKBOOK

MHF4U



W1 – 1.1 – Power Functions
MHF4U
Jensen

1) Identify which of the following are polynomial functions:

a) $p(x) = \cos x$

b) $h(x) = -7x$

c) $f(x) = 2x^4$

d) $y = 3x^5 - 2x^3 + x^2 - 1$

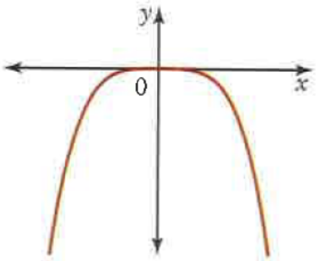
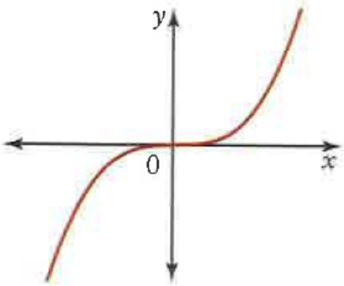
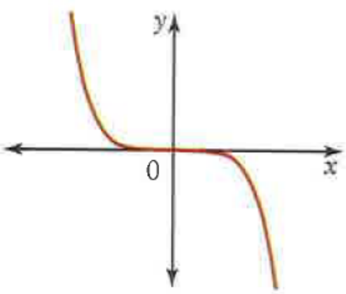
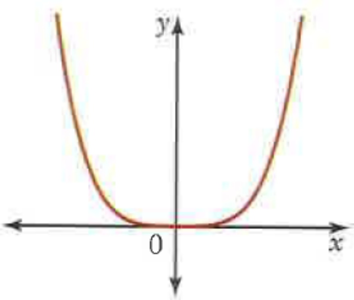
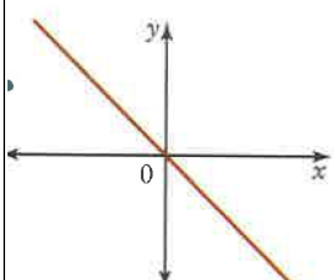
e) $k(x) = 8^x$

f) $y = x^{-3}$

2) State the degree and the leading coefficient of each polynomial

Polynomial	Degree	Leading Coefficient
$y = 5x^4 - 3x^3 + 4$		
$y = -x + 2$		
$y = 8x^2$		
$y = -\frac{x^3}{4} + 4x - 3$		
$y = -5$		
$y = x^2 - 3x$		

3) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
					
					
					
					
					

4) Match each function to its end behavior

$y = -x^3$ $\quad\quad$ $y = \frac{3}{7}x^2$ $\quad\quad$ $y = 5x$

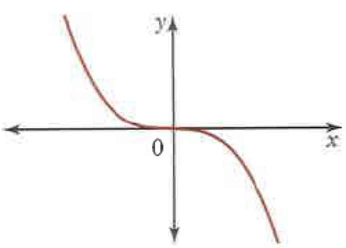
$y = 4x^5$ $\quad\quad$ $y = -x^6$ $\quad\quad$ $y = -0.1x^{11}$

$y = 2x^4$ $\quad\quad$ $y = -9x^{10}$

End Behaviour	Functions
Q3 to Q1	
Q2 to Q4	
Q2 to Q1	
Q3 to Q4	

5) Determine whether each graph represents a power function, exponential function, a periodic function, or none of these.

a)



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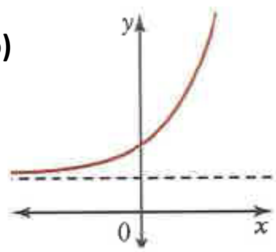
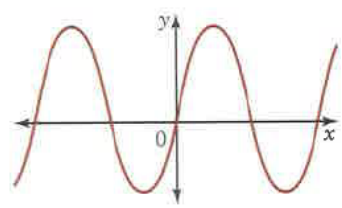


c)



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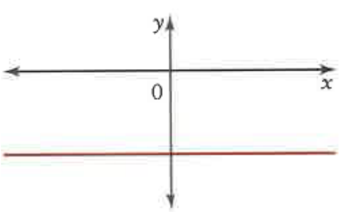
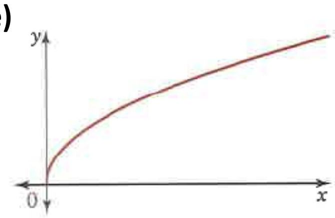
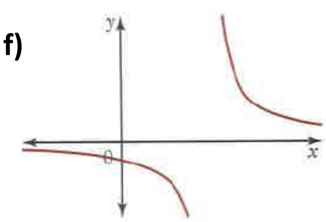


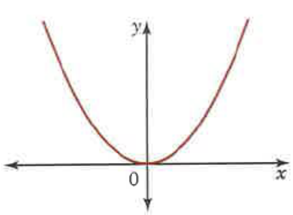
e)



f)



g)



Answer Key

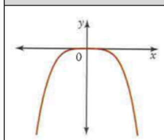
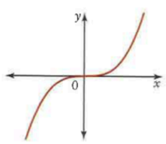
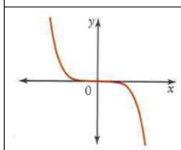
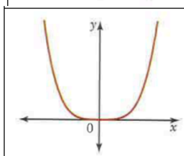
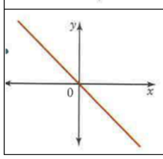
W1

1) a) No **b)** Yes **c)** Yes **d)** Yes **e)** No **f)** No

2)

Polynomial	Degree	Leading Coefficient
$y = 5x^4 - 3x^3 + 4$	4	5
$y = -x + 2$	1	-1
$y = 8x^2$	2	8
$y = -\frac{x^3}{4} + 4x - 3$	3	$-\frac{1}{4}$
$y = -5$	0	-5
$y = x^2 - 3x$	2	1

3)

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	EVEN	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} \mid y \leq 0\}$	Line	Q3 to Q4
	ODD	POSITIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q3 to Q1
	ODD	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q2 to Q4
	EVEN	POSITIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} \mid y \geq 0\}$	Line	Q2 to Q1
	ODD	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q2 to Q4

4)

End Behaviour	Functions
Q3 to Q1	$y = 4x^5, y = 5x$
Q2 to Q4	$y = -x^3, y = -0.1x^{11}$
Q2 to Q1	$y = 2x^4, y = \frac{3}{7}x^2$
Q3 to Q4	$y = -x^6, y = -9x^{10}$

5) a) power **b)** exponential **c)** periodic **d)** power **e)** none (square root) **f)** none (rational) **g)** power

W2 – 1.2 – Characteristics of Polynomial Functions
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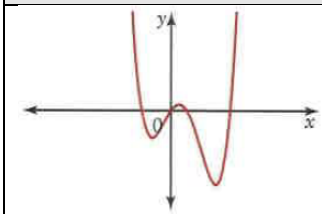
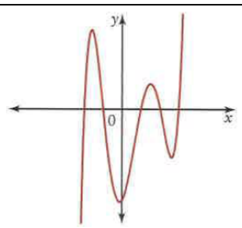
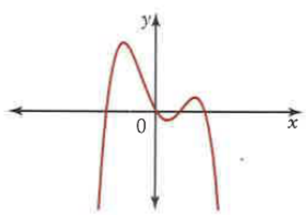
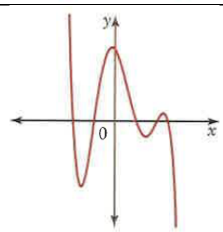
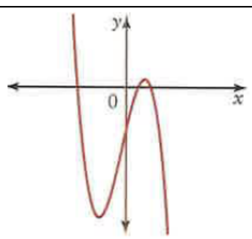
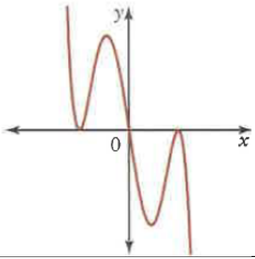
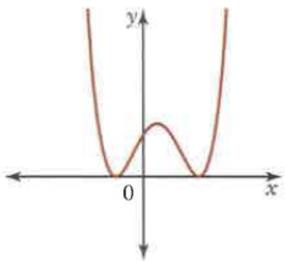
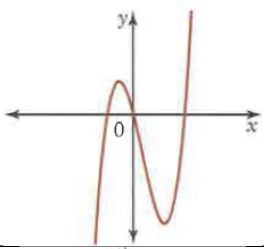
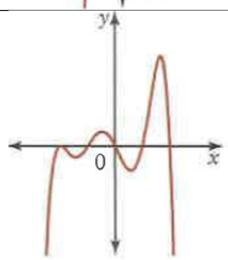
Jensen

1) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
							
							
							
							
							

2) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
							
							
							
							

3) Complete the following table

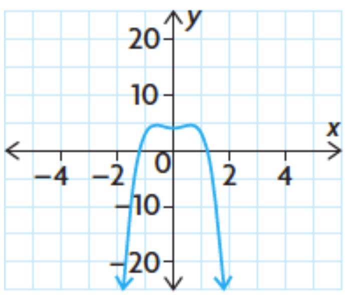
Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$						
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$						
$p(x) = 4 - 5x + 4x^2 - 3x^3$						
$h(x) = 2x(x-5)(3x+2)(4x-3)$						

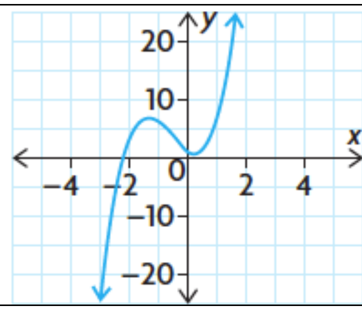
4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

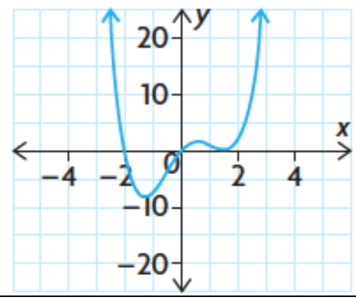
A) $y = 2x^3 - 4x^2 + 3x + 2$ **B)** $y = -4x^4 + 3x^2 + 4$ **C)** $y = x^2 + 3x - 5$

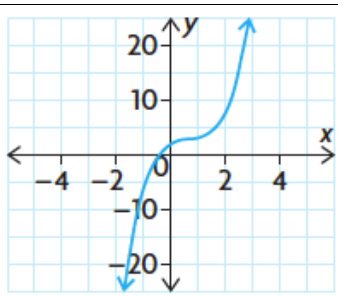
D) $y = x^4 - x^3 - 4x^2 + 5x$ **E)** $y = -2x^5 + 3x^4 + 6x^3 - 10x^2 + 2x + 5$

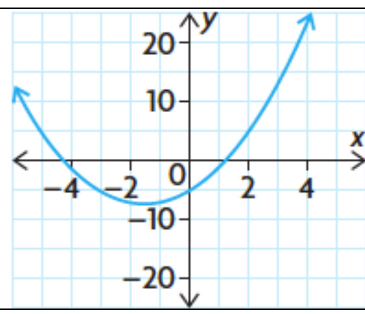
F) $y = 3x^3 + 5x^2 - 3x + 1$

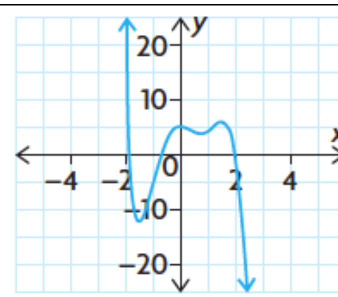












5) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) second differences = -8

b) fourth differences = 24

6) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y
−3	−45
−2	−16
−1	−3
0	0
1	−1
2	0
3	9
4	32

b)

x	y
−2	−40
−1	12
0	20
1	26
2	48
3	80
4	92
5	30

7) By analyzing the impact of growing economic conditions, a demographer establishes that the predicted population, P, of a town t years from now can be modelled by the function
$P(t) = 6t^4 - 5t^3 + 200t + 12000$

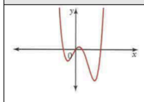
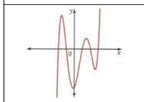
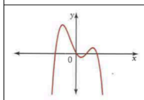
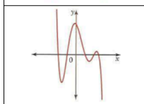
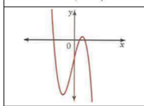
a) What is the value of the constant finite differences

b) What is the current population of the town

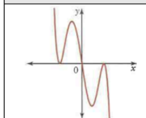
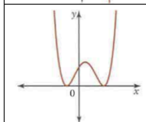
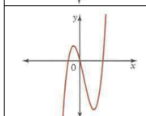
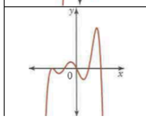
c) What will the population of the town be 10 years from now

ANSWER KEY

1)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	POS	EVEN	Q2 to Q1	NONE	3	4	4
	POS	ODD	Q3 to Q1	NONE	4	5	5
	NEG	EVEN	Q3 to Q4	NONE	3	4	4
	NEG	ODD	Q2 to Q4	NONE	4	5	5
	NEG	ODD	Q2 to Q4	POINT	2	3	3

2)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	NEG	ODD	Q2 to Q4	Point	4	3	5
	POS	EVEN	Q2 to Q1	Line	3	2	4
	POS	ODD	Q3 to Q1	Point	2	3	3
	NEG	EVEN	Q3 to Q4	None	5	5	6

3)

Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	NEG	EVEN	Q3 → Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	POS	ODD	Q3 → Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	NEG	ODD	Q2 → Q4	2, 0	3, 2, 1
$h(x) = 2x(x-5)(3x+2)(4x-3)$	4	POS	EVEN	Q2 → Q1	3, 1	4, 3, 2, 1, 0

4) B F D
A C E

5) a) degree 2, $a = -4$ **b)** degree 4, $a = 1$

6) a) degree 3, $a = 1$ **b)** degree 4, $a = -1$

7) a) 144 **b)** 12 000 **c)** 69 000

W3 – 1.3 – Factored Form Polynomial Functions
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Jensen

1) Match each equation with the most suitable graph. Write the letter of the equation beneath the matching graph.

A) $f(x) = 2(x+1)^2(x-3)$ **B)** $f(x) = (x+1)^2(x-3)^2$

C) $f(x) = -2(x+1)(x-3)^2$ **D)** $f(x) = x(x+1)(x-3)(x-5)$

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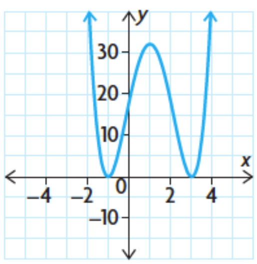
<td>
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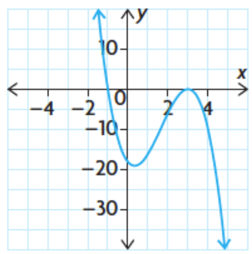
<td>
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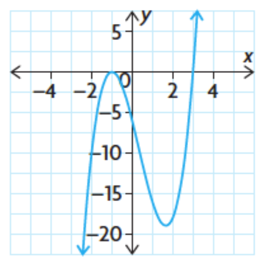
<td>
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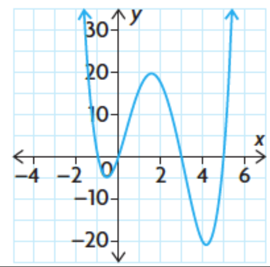
<td>
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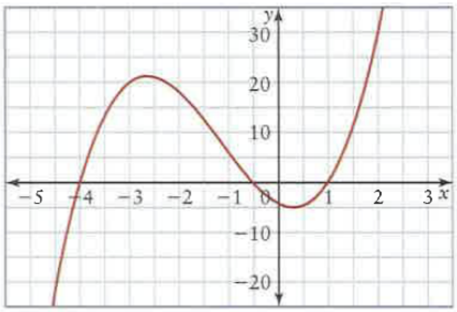
2) Complete the table

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x-4)(x+3)(2x-1)$				
$g(x) = -2(x+2)(x-2)(1+x)(x-1)$				
$h(x) = (3x+2)^2(x-4)(x+1)(2x-3)$				
$p(x) = -(x+5)^3(x-5)^3$				

3) For each graph, state…

i) the least possible degree and the sign of the leading coefficient
ii) the x-intercepts (specify order of zero) and the factors of the function
iii) the intervals where the function is positive/negative

a)



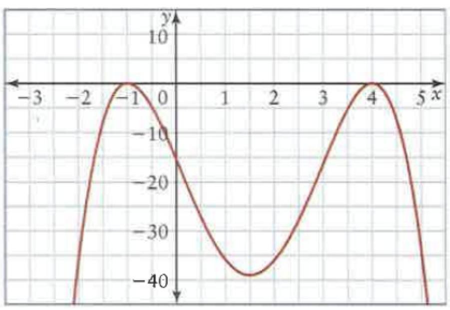
i) degree:
leading coefficient:

ii) x-intercepts:
factors:

iii)

Interval				
Sign				

b)



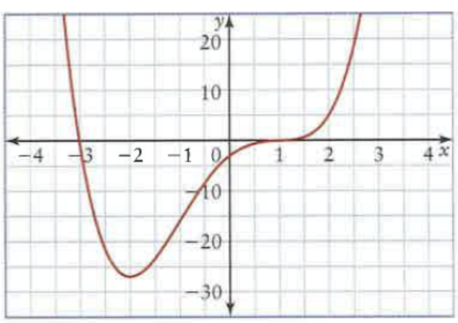
i) degree:
leading coefficient:

ii) x-intercepts:
factors:

iii)

Interval			
Sign			

c)



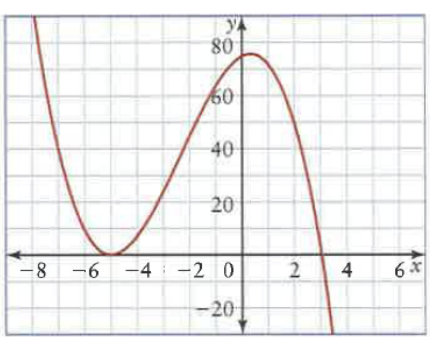
i) degree:
leading coefficient:

ii) x-intercepts:
factors:

iii)

Interval			
Sign			

d)



i) degree:
leading coefficient:

ii) x-intercepts:
factors:

iii)

Interval			
Sign			

4) For each function, complete the chart and sketch a possible graph of the function labelling key points.

a) $f(x) = -2(x-3)(x+2)(4x-3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept

b) $g(x) = (x-1)(x+3)(1+x)(3x-9)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept

c) $h(x) = -(x+4)^2(x-1)^2(x+2)(2x-3)$

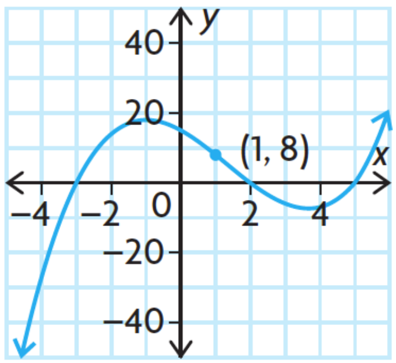
Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept

d) $p(x) = 3(x+6)(x-5)^2(3x-2)^3$

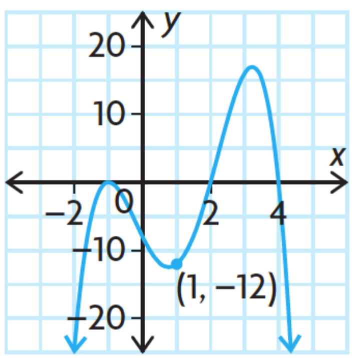
Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept

5) Write the equation of each function

a)



b)



6) Determine an equation for a quintic function with zeros -1 (order 3) and 3 (order 2) that passes through the point (-2, 50)

7) Determine the zeros of $f(x) = (2x^2 - x - 1)(x^2 - 3x - 4)$

Answer Key

1) B C A D

2)

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x-4)(x+3)(2x-1)$	3	2	Q3 → Q1	(4, 0) (-3, 0) $\left(\frac{1}{2}, 0\right)$
$g(x) = -2(x+2)(x-2)(1+x)(x-1)$	4	-2	Q3 → Q4	(-2, 0) (-1, 0) (1, 0) (2, 0)
$h(x) = (3x+2)^2(x-4)(x+1)(2x-3)$	5	18	Q3 → Q1	(4, 0) (-1, 0) $\left(-\frac{2}{3}, 0\right)$ $\left(\frac{3}{2}, 0\right)$
$p(x) = -(x+5)^3(x-5)^3$	6	-1	Q3 → Q4	(-5, 0) (5, 0)

3) a) **i)** degree: 3
leading coefficient: positive

ii) x-intercepts: -4, -0.5, 1
factors: $(x+4)$, $(2x+1)$, and $(x-1)$

iii)

Interval	$(-\infty, -4)$	$(-4, -0.5)$	$(-0.5, 1)$	$(1, \infty)$
Sign	-	+	-	+

b) **i)** degree: 4
leading coefficient: negative

ii) x-intercepts: -1 (order 2), 4 (order 2)
factors: $(x+1)^2$, and $(x-4)^2$

iii)

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign	-	-	-

c) **i)** degree: 4
leading coefficient: positive

ii) x-intercepts: -3, 1 (order 3)
factors: $(x+3)$, and $(x-1)^3$

iii)

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign	+	-	+

d) **i)** degree: 3
leading coefficient: negative

ii) x-intercepts: -5 (order 2), 3
factors: $(x+5)^2$, and $(x-3)$

iii)

Interval	$(-\infty, -5)$	$(-5, 3)$	$(3, \infty)$
Sign	+	+	-

4) a)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
3	-8	Q2 → Q4	(3, 0) (-2, 0) $\left(\frac{3}{4}, 0\right)$	(0, -36)

b)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
4	3	Q2 → Q1	(1, 0) (-3, 0) (-1, 0) (3,0)	(0, 27)

c)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
6	-2	Q3 → Q4	(-4, 0) order 2 (1, 0) order 2 (-2, 0) (1.5,0)	(0, 96)

d)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
6	81	Q2 → Q1	(-6, 0) (5, 0) order 2 $\left(\frac{2}{3}, 0\right)$ order 3	(0, -3600)

5) a) $y = 0.5(x+3)(x-2)(x-5)$ **b)** $y = -(x+1)^2(x-2)(x-4)$

6) $y = -2(x+1)^3(x-3)^2$

7) 4, 1, -1, and -0.5

W4 – 1.4 – Transformations
MHF4U
Jensen

1) Match each graph with the corresponding function.

A) $y = 2(x-3)^3 + 1$ **B)** $y = -\frac{1}{3}(x+1)^3 - 1$ **C)** $y = 0.2(x-4)^4 - 3$ **D)** $y = -1.5(x+3)^4 + 4$

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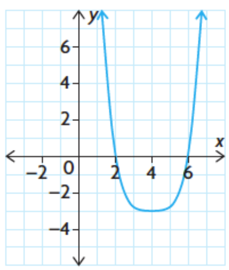
<td>
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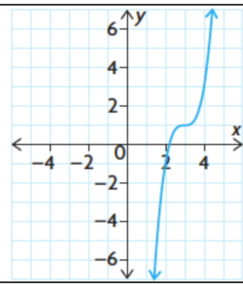
<td>
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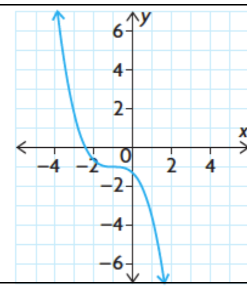
<td>
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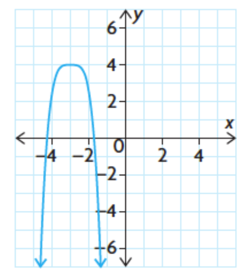
<td>
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2) List a good set of key points for the following parent functions:

$f(x) = x^2$	
x	y

$f(x) = x^3$	
x	y

$f(x) = x^4$	
x	y

$f(x) = x^5$	
x	y

3) Identify the a, k, d and c values and explain what transformation is occurring to the parent function:

a) $f(x) = -2(x-1)^2$

b) $g(x) = [-\frac{1}{3}(x+5)]^4 - 1$

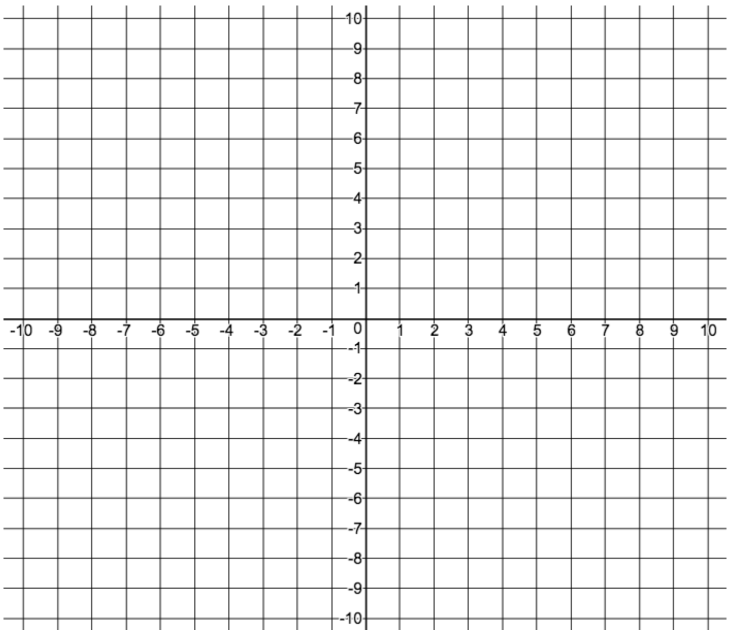
4) Write the full equation given the parent function and the transforming function:

a) $f(x) = x^5$, $g(x) = -3f[2(x+5)] - 1$

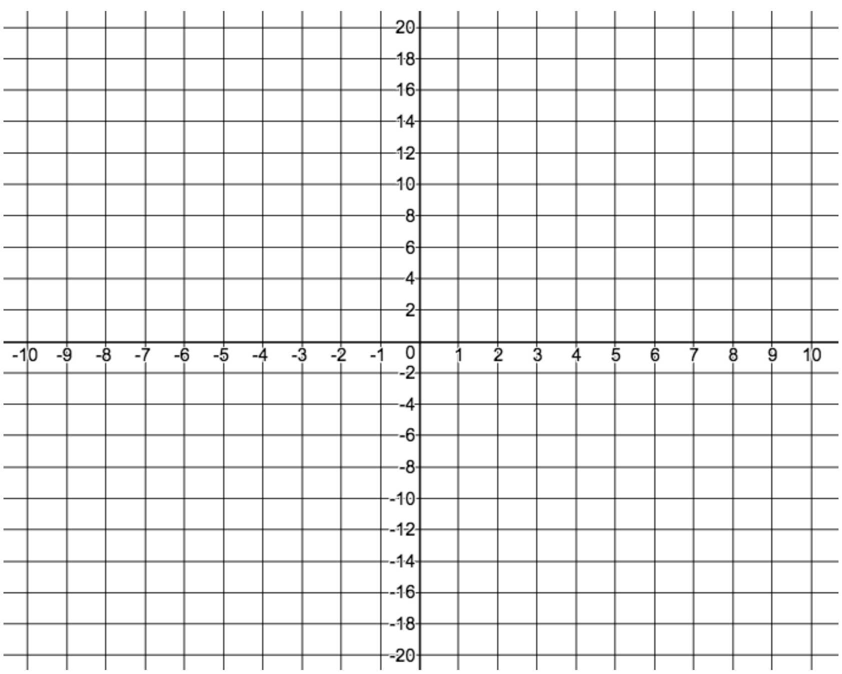
b) $f(x) = x^3$, $g(x) = \frac{1}{2}f\left[-\frac{1}{4}(x-4)\right] + 7$

5) For the following questions, use the key points of the parent function to perform transformations. Graph the parent and transformed function. Write the equation of the transformed function.

a) $f(x) = x^4$ $\qquad g(x) = \frac{1}{2}f[-(x-5)] + 1$



b) $f(x) = x^3$ $\qquad g(x) = -f[-2(x+1)] + 6$



6) Write an equation for the function that results from the given transformations.

a) The function $f(x) = x^4$ is translated 2 units to the left and 3 units up.

b) The function $f(x) = x^5$ is stretched horizontally by a factor of 5 and translated 12 units to the left.

c) The function $f(x) = x^4$ is stretched vertically by a factor of 3, reflected vertically in the x-axis, and translated 6 units down and 1 unit to the left.

d) The function $f(x) = x^6$ is reflected vertically in the x-axis, stretched horizontally by a factor of 5, reflected horizontally in the y-axis, and translated 3 units down and 1 unit to the right.

ANSWER KEY

1) C A B D

2)

$f(x) = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

$f(x) = x^5$	
x	y
-2	-32
-1	-1
0	0
1	1
2	32

3) a) $a = -2$; vertical reflection and vertical stretch by a factor of 2 $(-2y)$
$d = 1$; shift right 1 unit $(x + 1)$

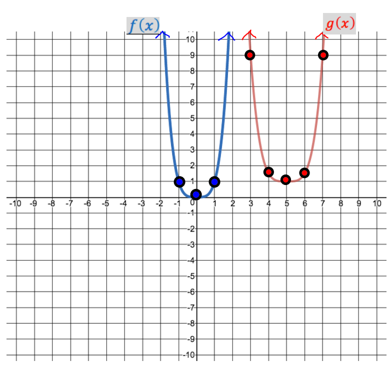
b) $k = -\frac{1}{3}$; horizontal reflection and horizontal stretch by a factor of 3 $(-3x)$
$d = -5$; shift left 5 units $(x - 5)$
$c = -1$; shift down 1 unit $(y - 1)$

4) a) $g(x) = -3[2(x + 5)]^5 - 1$ **b)** $g(x) = \frac{1}{2}[-\frac{1}{4}(x - 4)]^3 + 7$

5) a)

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

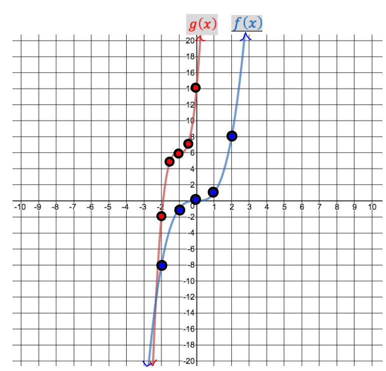
$g(x) = \frac{1}{2}[-(x-5)]^4 + 1$	
$-x + 5$	$\frac{y}{2} + 1$
7	9
6	1.5
5	1
4	1.5
3	9



b)

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$g(x) = -[-2(x+1)]^3 + 6$	
$\frac{x}{-2} - 1$	$-y + 6$
0	14
-0.5	7
-1	6
-1.5	5
-2	-2



6) a) $g(x) = (x + 2)^4 + 3$ **b)** $g(x) = \left[\frac{1}{5}(x + 12)\right]^5$ **c)** $g(x) = -3(x + 1)^4 - 6$ **d)** $g(x) = -\left[-\frac{1}{5}(x - 1)\right]^6 - 3$

W5 – 1.3 – Symmetry in Polynomial Functions
MHF4U
Jensen

1) Determine whether each function is even, odd, or neither. Does it have line symmetry about the y-axis, point symmetry about the origin, or neither?

a) $y = x^4 - x^2$

b) $y = -2x^3 + 5x$

c) $y = -4x^5 + 2x^2$

d) $y = x(2x + 1)^2(x - 4)$

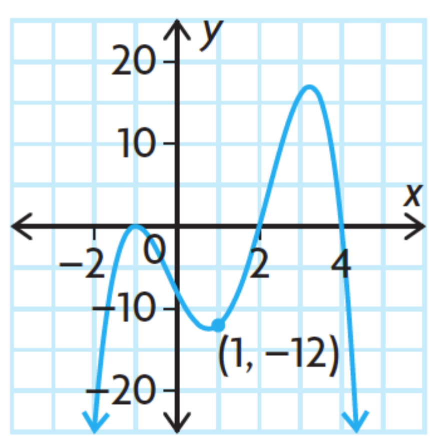
e) $y = -2x^6 + x^4 + 8$

2) State whether each function is even or odd. Verify algebraically.

a) $f(x) = x^4 - 13x^2 + 36$

b) $g(x) = 6x^5 - 7x^3 - 3x$

3) Use the given graph to state:



a) x-intercepts

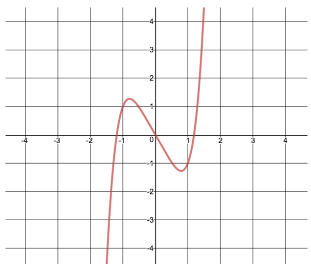
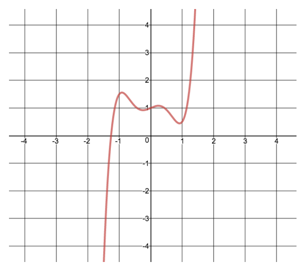
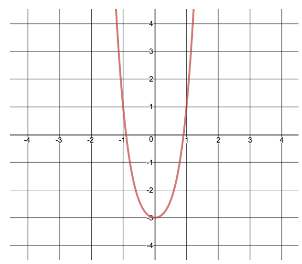
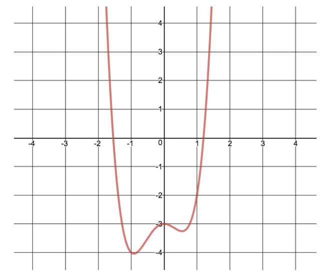
b) number of turning points

c) least possible degree

d) any symmetry present; even or odd function?

e) the intervals where $f(x) < 0$

4) Label each function as even, odd, or neither

ANSWER KEY

1) a) even, line symmetry about y-axis **b)** odd, point symmetry about origin **c)** neither

d) neither **e)** even, line symmetry about y-axis

2) a) even, $f(-x) = f(x)$ **b)** odd, $f(-x) = -f(x)$

3) a) -1 (order 2), 2, and 4 **b)** 3 **c)** 4 **d)** no symmetry, neither **e)** $X \in (-\infty, -1) \cup (-1,2) \cup (4, \infty)$

4) odd, neither, even, neither