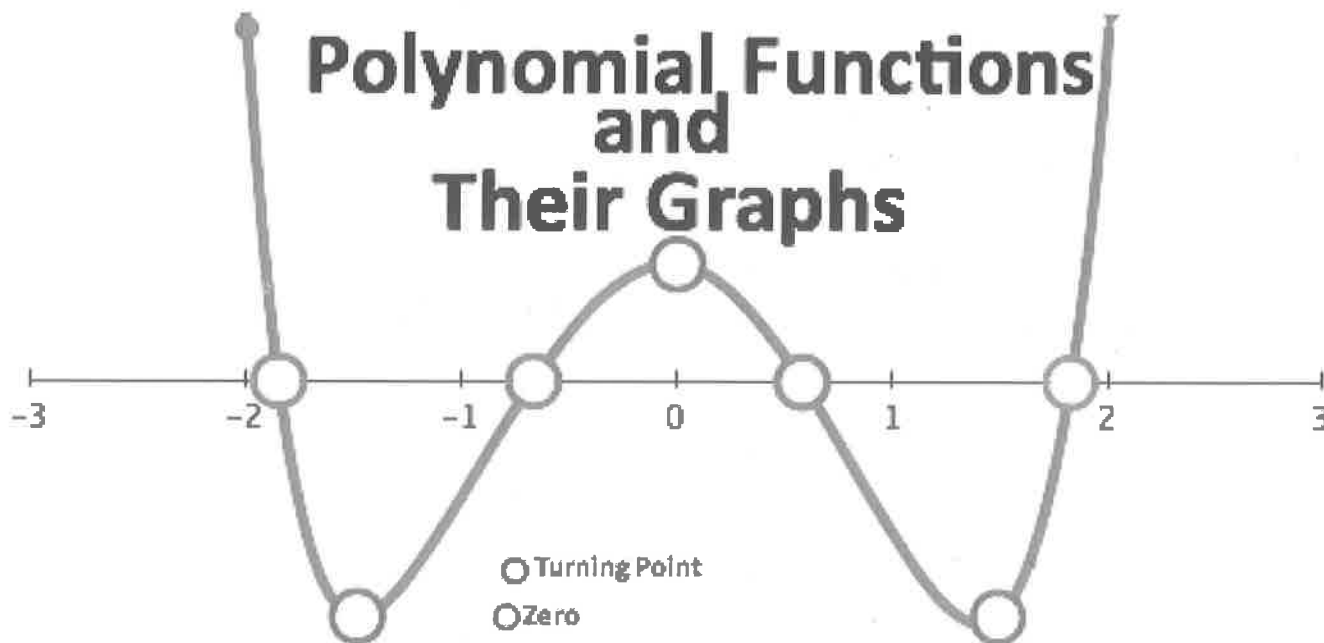


Chapter 1- Polynomial Functions

WORKBOOK SOLUTION MANUAL
MHF4U

Polynomial Functions and Their Graphs



W1 – 1.1 – Power Functions

MHF4U

Jensen

ANSWERS

1) Identify which of the following are polynomial functions:

a) $p(x) = \cos x$

b) $h(x) = -7x$

c) $f(x) = 2x^4$

d) $y = 3x^5 - 2x^3 + x^2 - 1$

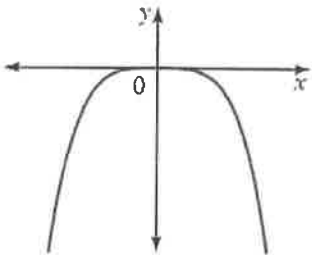
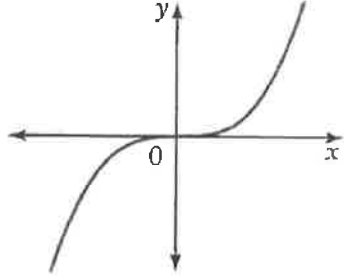
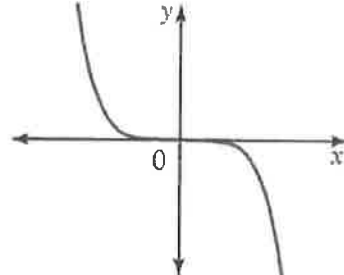
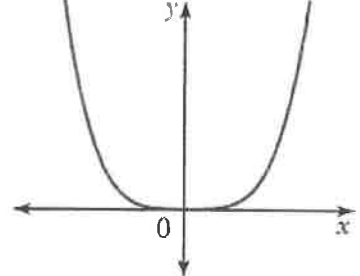
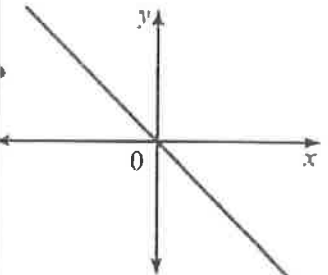
e) $k(x) = 8^x$

f) $y = x^{-3}$

2) State the degree and the leading coefficient of each polynomial

Polynomial	Degree	Leading Coefficient
$y = 5x^4 - 3x^3 + 4$	4	5
$y = -x + 2$	1	-1
$y = 8x^2$	2	8
$y = -\frac{x^3}{4} + 4x - 3$	3	$-\frac{1}{4}$
$y = -5$	0	-5
$y = x^2 - 3x$	2	1

3) Complete the following table

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	Even	-	$D: \{x \in \mathbb{R}\}$ $R: \{y \in \mathbb{R} \mid y \leq 0\}$	Line	Q3 to Q4
	Odd	+	$D: \{x \in \mathbb{R}\}$ $R: \{y \in \mathbb{R}\}$	Point	Q3 to Q1
	Odd	-	$D: \{x \in \mathbb{R}\}$ $R: \{y \in \mathbb{R}\}$	Point	Q2 to Q4
	Even	+	$D: \{x \in \mathbb{R}\}$ $R: \{y \in \mathbb{R} \mid y \geq 0\}$	Line	Q2 to Q1
	Odd	-	$D: \{x \in \mathbb{R}\}$ $R: \{y \in \mathbb{R}\}$	Point	Q2 to Q4

4) Match each function to its end behavior

$$y = -x^3 \checkmark$$

$$y = \frac{3}{7}x^2 \checkmark$$

$$y = 5x \checkmark$$

$$y = 4x^5 \checkmark$$

$$y = -x^6 \checkmark$$

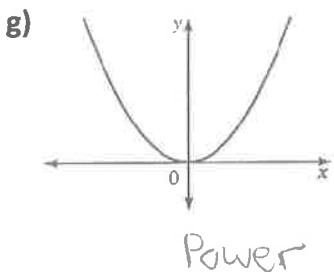
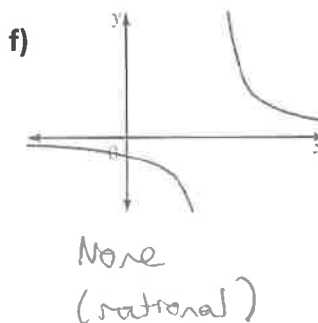
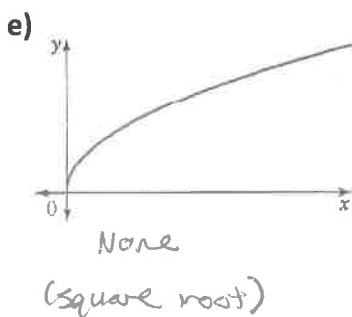
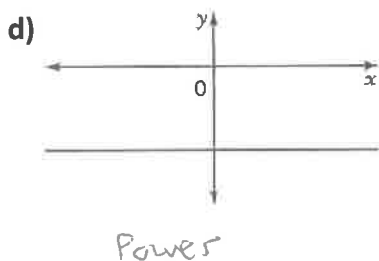
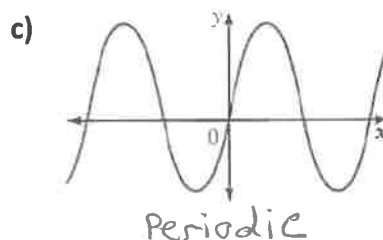
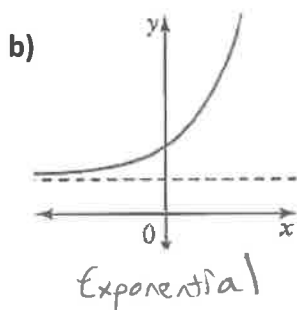
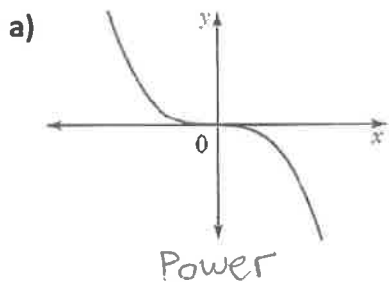
$$y = -0.1x^{11} \checkmark$$

$$y = 2x^4 \checkmark$$

$$y = -9x^{10} \checkmark$$

End Behaviour	Functions
Q3 to Q1	$y = 4x^5$, $y = 5x$
Q2 to Q4	$y = -x^3$, $y = -0.1x^{11}$
Q2 to Q1	$y = 2x^4$, $y = \frac{3}{7}x^2$
Q3 to Q4	$y = -x^6$, $y = -9x^{10}$

5) Determine whether each graph represents a power function, exponential function, a periodic function, or none of these.



Answer Key

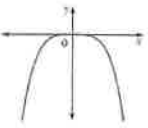
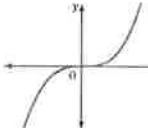
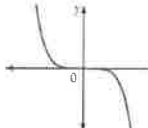
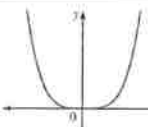
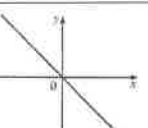
W1

1) a) No b) Yes c) Yes d) Yes e) No f) No

2)

Polynomial	Degree	Leading Coefficient
$y = 5x^4 - 3x^3 + 4$	4	5
$y = -x + 2$	1	-1
$y = 8x^2$	2	8
$y = -\frac{x^3}{4} + 4x - 3$	3	$-\frac{1}{4}$
$y = -5$	0	-5
$y = x^2 - 3x$	2	1

3)

Graph of Function	Even or Odd Degree?	Sign of Leading Coefficient	Domain and Range	Symmetry	End Behaviour
	EVEN	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} y \leq 0\}$	Line	Q3 to Q4
	ODD	POSITIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q3 to Q1
	ODD	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q2 to Q4
	EVEN	POSITIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} y \geq 0\}$	Line	Q2 to Q1
	ODD	NEGATIVE	D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$	Point	Q2 to Q4

4)

End Behaviour	Functions
Q3 to Q1	$y = 4x^5, y = 5x$
Q2 to Q4	$y = -x^3, y = -0.1x^{11}$
Q2 to Q1	$y = 2x^4, y = \frac{3}{7}x^2$
Q3 to Q4	$y = -x^6, y = -9x^{10}$

5) a) power b) exponential c) periodic d) power e) none (square root) f) none (rational) g) power

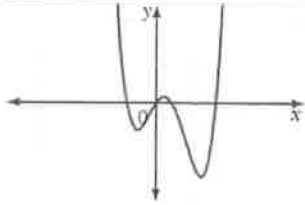
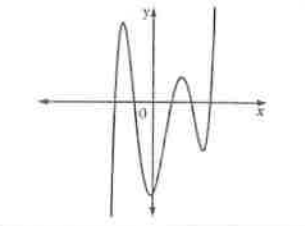
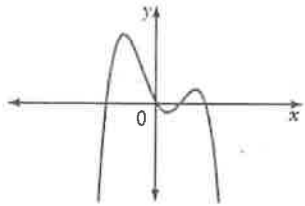
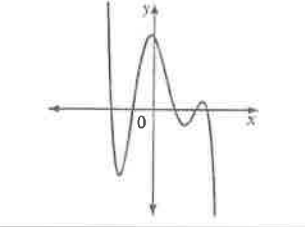
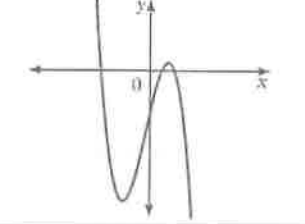
W2 – 1.2 – Characteristics of Polynomial Functions

MHF4U

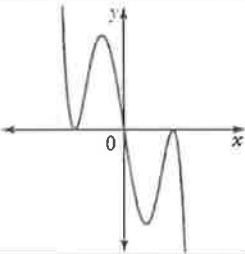
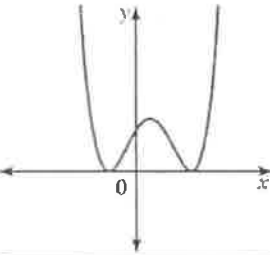
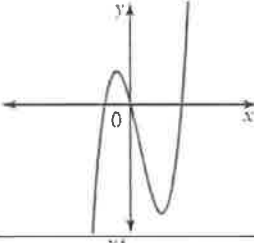
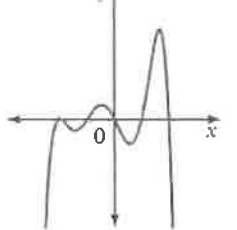
Jensen

ANSWERS

1) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	+	Even	Q2 to Q1	None	3	4	4
	+	Odd	Q3 to Q1	None	4	5	5
	-	Even	Q3 to Q4	None	3	4	4
	-	Odd	Q2 to Q4	None	4	5	5
	-	Odd	Q2 to Q4	Point	2	3	3

2) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	-	Odd	Q2 to Q4	Point	4	3	5
	+	Even	Q2 to Q1	Line	3	2	4
	+	Odd	Q3 to Q1	Point	2	3	3
	-	Even	Q3 to Q4	None	5	5	6

3) Complete the following table

Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	-	Even	Q3 to Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	+	Odd	Q3 to Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	-	Odd	Q2 to Q4	2, 0	3, 2, 1
$h(x) = 2x(x - 5)(3x + 2)(4x - 3)$	4	+	Even	Q2 to Q1	3, 1	4, 3, 2, 1, 0

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

$$A) y = 2x^3 - 4x^2 + 3x + 2$$

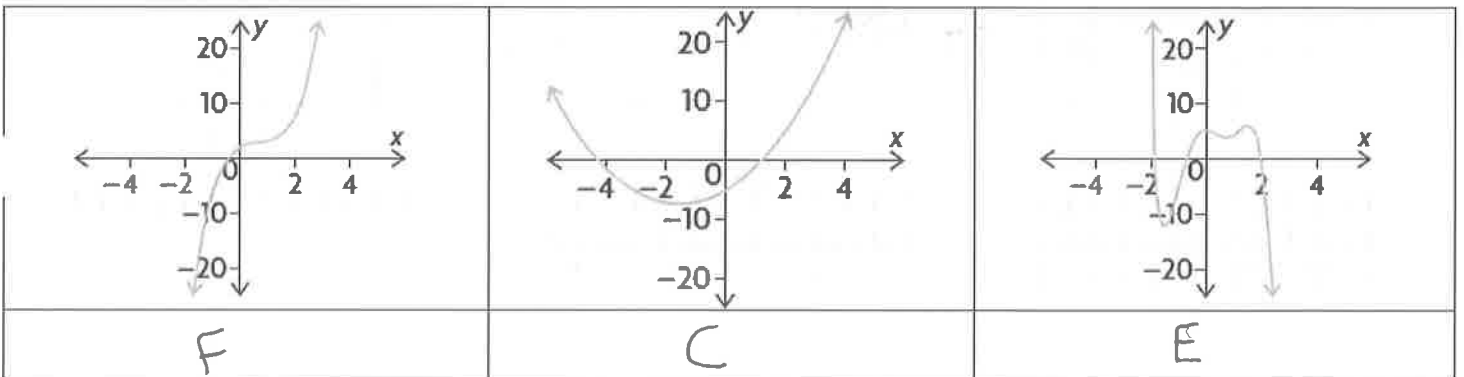
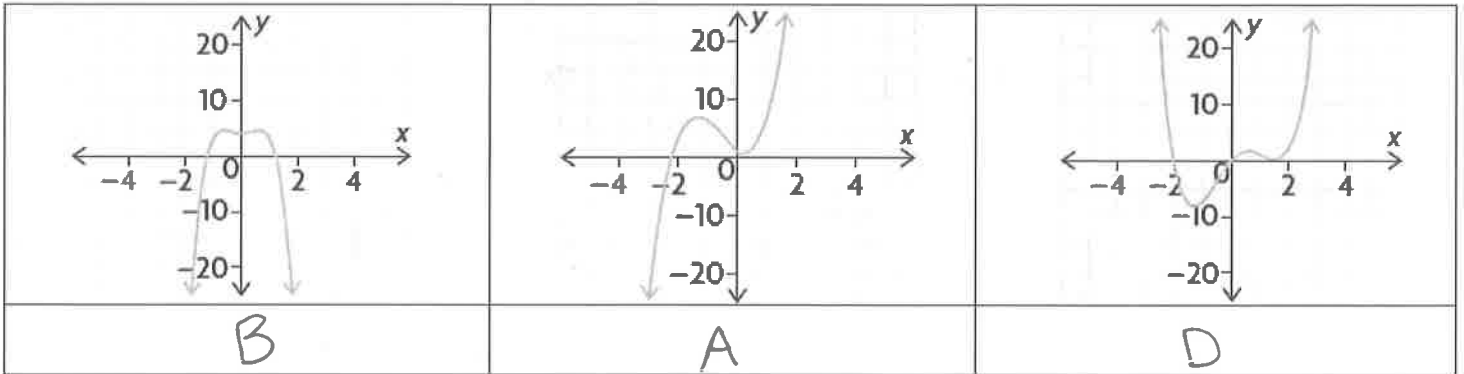
$$B) y = -4x^4 + 3x^2 + 4$$

$$C) y = x^2 + 3x - 5$$

$$D) y = x^4 - x^3 - 4x^2 + 5x$$

$$E) y = -2x^5 + 3x^4 + 6x^3 - 10x^2 + 2x + 5$$

$$F) y = 3x^3 + 5x^2 - 3x + 1$$



5) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) ^{degree 2}
second differences = -8

$$-8 = a(2!)$$

$$-8 = 2a$$

$$\boxed{-4 = a}$$

b) ^{degree 4}
fourth differences = 24

$$24 = a(4!)$$

$$24 = 24a$$

$$\boxed{1 = a}$$

6) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y
-3	-45
-2	-16
-1	-3
0	0
1	-1
2	0
3	9
4	32

1st 2nd 3rd

29 }
 13 }
 3 }
 -1 }
 1 }
 9 }
 23 }

-16 }
 -10 }
 -4 }
 2 }
 8 }
 14 }

6 }
 6 }
 6 }
 6 }
 6 }

Degree = 3

$6 = a(3!)$

$6 = 6a$

$1 = a$

b)

x	y
-2	-40
-1	12
0	20
1	26
2	48
3	80
4	92
5	30

1st 2nd 3rd 4th

52 }
 8 }
 6 }
 22 }
 32 }
 12 }
 -62 }

-44 }
 -2 }
 16 }
 10 }
 -20 }
 -24 }

42 }
 18 }
 -6 }
 -30 }
 54 }

-24 }
 -24 }
 -24 }
 -24 }

Degree = 4

$-24 = a(4!)$

$-24 = 24a$

$-1 = a$

7) By analyzing the impact of growing economic conditions, a demographer establishes that the predicted population, P , of a town t years from now can be modelled by the function

$$P(t) = 6t^4 - 5t^3 + 200t + 12000$$

a) What is the value of the constant finite differences

$$\begin{aligned} \text{Finite Differences} &= a(n!) \\ &= 6(4!) \\ &= 144 \end{aligned}$$

b) What is the current population of the town $P(0) = 6(0)^4 - 5(0)^3 + 200(0) + 12000$

$$= 12000$$

c) What will the population of the town be 10 years from now

$$\begin{aligned} P(10) &= 6(10)^4 - 5(10)^3 + 200(10) + 12000 \\ &= 69000 \end{aligned}$$

ANSWER KEY

1)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	POS	EVEN	Q2 to Q1	NONE	3	4	4
	POS	ODD	Q3 to Q1	NONE	4	5	5
	NEG	EVEN	Q3 to Q4	NONE	3	4	4
	NEG	ODD	Q2 to Q4	NONE	4	5	5
	NEG	ODD	Q2 to Q4	POINT	2	3	3

2)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	NEG	ODD	Q2 to Q4	Point	4	3	5
	POS	EVEN	Q2 to Q1	Line	3	2	4
	POS	ODD	Q3 to Q1	Point	2	3	3
	NEG	EVEN	Q3 to Q4	None	5	5	6

3)

Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	NEG	EVEN	Q3 → Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	POS	ODD	Q3 → Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	NEG	ODD	Q2 → Q4	2, 0	3, 2, 1
$h(x) = 2x(x - 5)(3x + 2)(4x - 3)$	4	POS	EVEN	Q2 → Q1	3, 1	4, 3, 2, 1, 0

4) B F D
A C E

5) a) degree 2, $a = -4$ b) degree 4, $a = 1$

6) a) degree 3, $a = 1$ b) degree 4, $a = -1$

7) a) 144 b) 12 000 c) 69 000

W3 – 1.3 – Factored Form Polynomial Functions

MHF4U

Jensen

ANSWERS

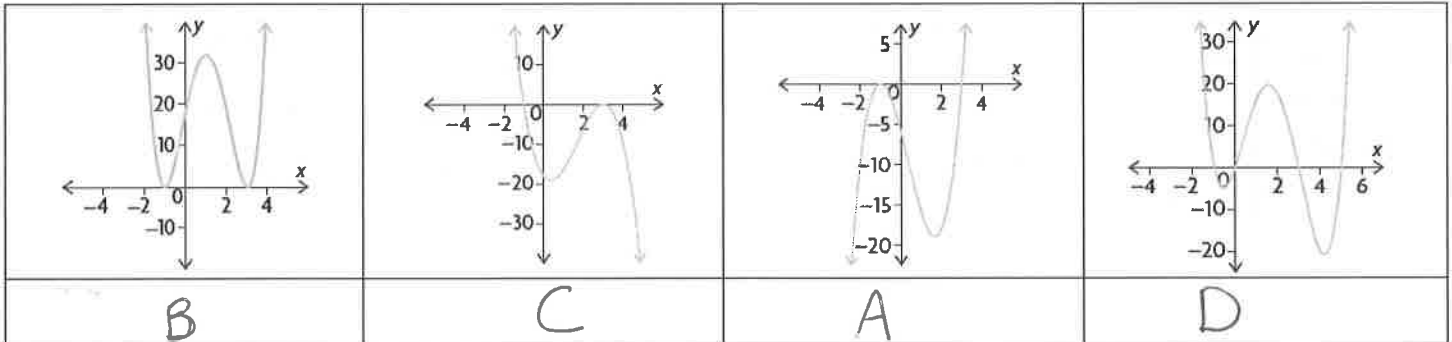
1) Match each equation with the most suitable graph. Write the letter of the equation beneath the matching graph.

A) $f(x) = 2(x + 1)^2(x - 3)$

B) $f(x) = (x + 1)^2(x - 3)^2$

C) $f(x) = -2(x + 1)(x - 3)^2$

D) $f(x) = x(x + 1)(x - 3)(x - 5)$

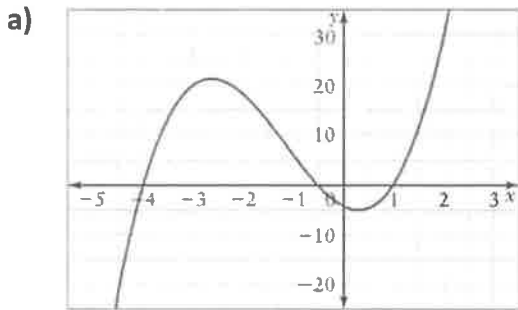


2) Complete the table

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x - 4)(x + 3)(2x - 1)$	$(x)(x)(x) = x^3$ ③	$(1)(1)(2) = 2$	Q3 to Q1	$(4, 0)$ $(-3, 0)$ $(\frac{1}{2}, 0)$
$g(x) = -2(x + 2)(x - 2)(1 + x)(x - 1)$	$(x)(x)(x)(x) = x^4$ ④	$-2(1)(1)(1)(1) = -2$	Q3 to Q4	$(-2, 0)$ $(-1, 0)$ $(2, 0)$ $(1, 0)$
$h(x) = (3x + 2)^2(x - 4)(x + 1)(2x - 3)$	$(x^2)(x)(x)(x) = x^5$ ⑤	$(3^2)(1)(1)(2) = 18$	Q3 to Q1	$(-\frac{2}{3}, 0)$ $(\frac{3}{2}, 0)$ $(4, 0)$ $(-1, 0)$
$p(x) = -(x + 5)^3(x - 5)^3$	$(x^3)(x^3) = x^6$ ⑥	$-1(1^3)(1^3) = -1$	Q3 to Q4	$(-5, 0)$ $(5, 0)$

3) For each graph, state...

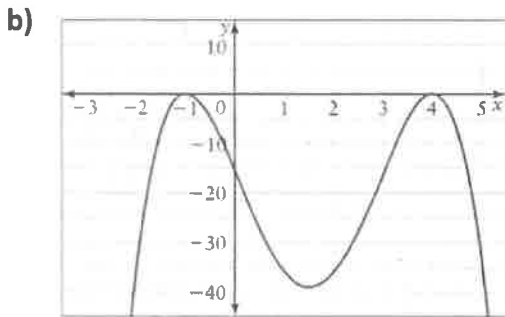
- i) the least possible degree and the sign of the leading coefficient
- ii) the x -intercepts (specify order of zero) and the factors of the function
- iii) the intervals where the function is positive/negative



- i) degree: 3
leading coefficient: Positive
- ii) x -intercepts: $-4, -\frac{1}{2}, 1$
factors: $(x+4), (2x+1), (x-1)$

iii)

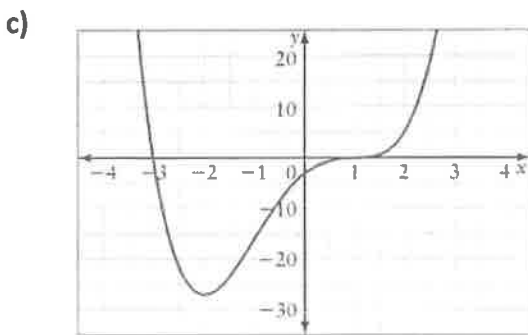
Interval	$(-\infty, -4)$	$(-4, -\frac{1}{2})$	$(-\frac{1}{2}, 1)$	$(1, \infty)$
Sign	-	+	-	+



- i) degree: 4
leading coefficient: Negative
- ii) x -intercepts: -1 (order 2), 4 (order 2)
factors: $(x+1)^2, (x-4)^2$

iii)

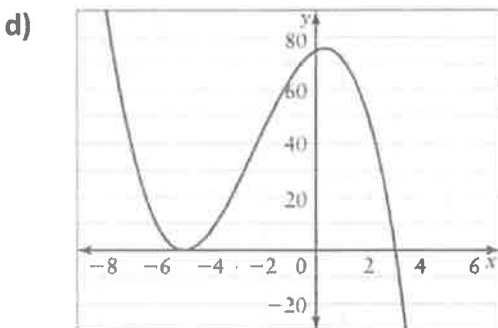
Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign	-	-	-



- i) degree: 4
leading coefficient: Positive
- ii) x -intercepts: $-3, 1$ (order 3)
factors: $(x+3), (x-1)^3$

iii)

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign	+	-	+



- i) degree: 3
leading coefficient: Negative
- ii) x -intercepts: -5 (order 2), 3
factors: $(x+5)^2, (x-3)$

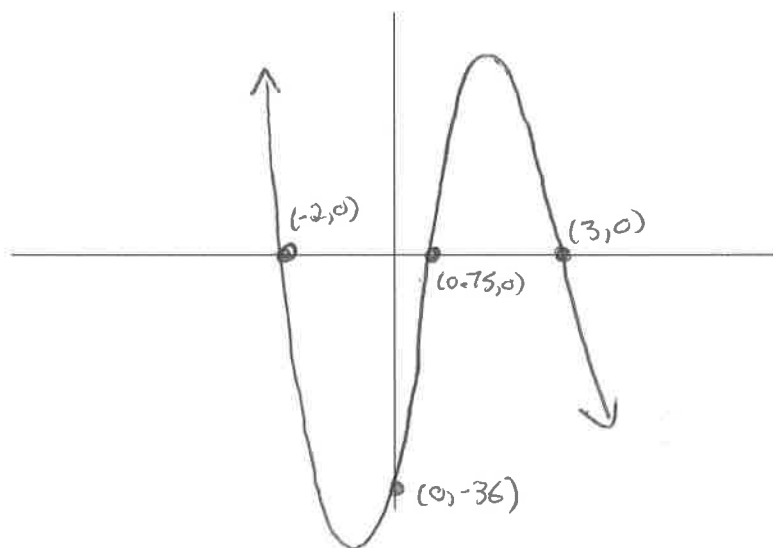
iii)

Interval	$(-\infty, -5)$	$(-5, 3)$	$(3, \infty)$
Sign	+	+	-

4) For each function, complete the chart and sketch a possible graph of the function labelling key points.

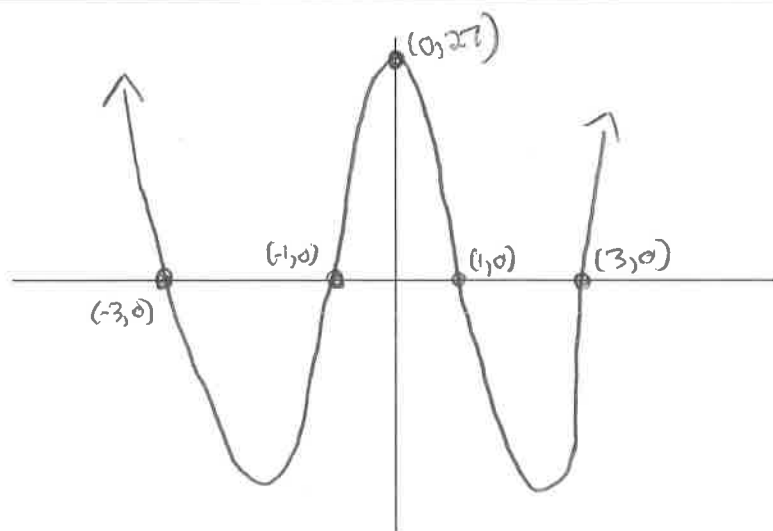
a) $f(x) = -2(x - 3)(x + 2)(4x - 3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$-2(1)(1)(4)$ $= -8$	Q2 to Q4	$(3, 0)$ $(-2, 0)$ $(\frac{3}{4}, 0)$	$f(0) = -2(-3)(2)(-3)$ $= -36$ $(0, -36)$



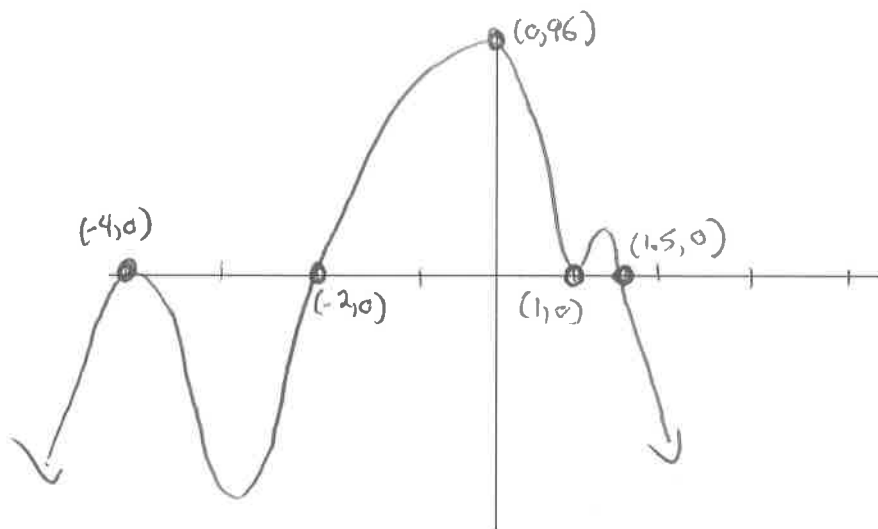
b) $g(x) = (x - 1)(x + 3)(1 + x)(3x - 9)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)(x)$ $= x^4$ Degree 4	$(1)(1)(1)(3)$ $= 3$	Q2 to Q1	$(1, 0)$ $(3, 0)$ $(-3, 0)$ $(-1, 0)$	$g(0) = (-1)(3)(1)(-9)$ $= 27$



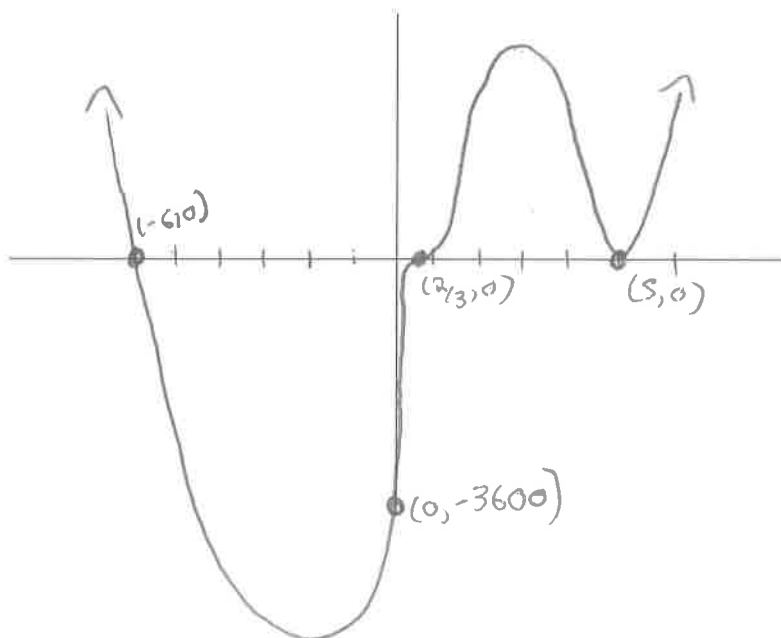
c) $h(x) = -(x + 4)^2(x - 1)^2(x + 2)(2x - 3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x^2)(x^2)(x)(x)$ $= x^6$ Degree 6	$-1(1^2)(1^2)(1)(2)$ $= -2$	Q3 to Q4	$(-4, 0)$ order 2 $(1, 0)$ order 2 $(-2, 0)$ $(3/2, 0)$	$h(0) = -1(4)^2(-1)^2(2)(-3)$ $= 96$



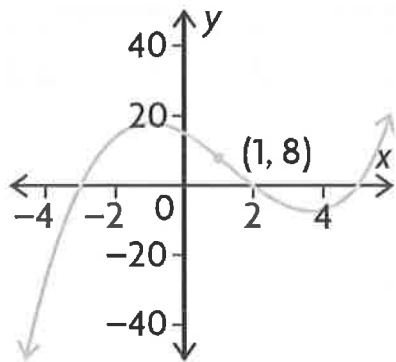
d) $p(x) = 3(x + 6)(x - 5)^2(3x - 2)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x^2)(x^3)$ $= x^6$ Degree 6	$3(1)(1)^2(3)^3$ $= 81$	Q2 to Q1	$(-6, 0)$ $(5, 0)$ order 2 $(2/3, 0)$ order 3	$p(0) = 3(6)(-5)^2(-2)^3$ $= -3600$



5) Write the equation of each function

a)



$$f(x) = k(x+3)(x-2)(x-5)$$

$$8 = k(1+3)(1-2)(1-5)$$

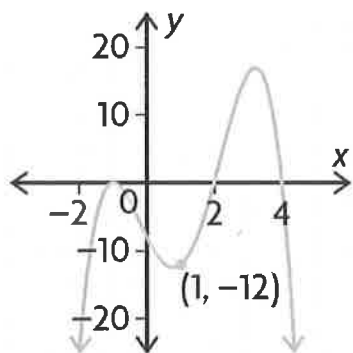
$$8 = k(4)(-1)(-4)$$

$$8 = 16k$$

$$k = \frac{1}{2}$$

$$f(x) = \frac{1}{2}(x+3)(x-2)(x-5)$$

b)



$$g(x) = k(x+1)^2(x-2)(x-4)$$

$$-12 = k(1+1)^2(1-2)(1-4)$$

$$-12 = k(4)(-1)(-3)$$

$$-12 = 12k$$

$$k = -1$$

$$g(x) = -(x+1)^2(x-2)(x-4)$$

6) Determine an equation for a quintic function with zeros -1 (order 3) and 3 (order 2) that passes through the point (-2, 50)

$$h(x) = k(x+1)^3(x-3)^2$$

$$50 = k(-2+1)^3(-2-3)^2$$

$$50 = k(-1)^3(-5)^2$$

$$50 = k(-1)(25)$$

$$50 = -25k$$

$$k = -2$$

$$h(x) = -2(x+1)^3(x-3)^2$$

7) Determine the zeros of $f(x) = (2x^2 - x - 1)(x^2 - 3x - 4)$

$$2x^2 - x - 1 \quad p: -2 \quad s: -1 \quad (-2 \text{ and } 1)$$

$$= 2x^2 - 2x + 1x - 1$$

$$= 2x(x-1) + 1(x-1)$$

$$= (x-1)(2x+1)$$

$$x^2 - 3x - 4 \quad p: -4 \quad s: -3 \quad (-4 \text{ and } 1)$$

$$= (x-4)(x+1)$$

$$f(x) = (x-1)(2x+1)(x-4)(x+1)$$

$$0 = (x-1)(2x+1)(x-4)(x+1)$$

$$x_1 = 1 \quad x_2 = -\frac{1}{2} \quad x_3 = 4 \quad x_4 = -1$$

Answer Key

1) B C A D

Equation	Degree	Leading Coefficient	End Behaviour	x-Intercepts
$f(x) = (x - 4)(x + 3)(2x - 1)$	3	2	Q3 → Q1	$(4, 0)$ $(-3, 0)$ $(\frac{1}{2}, 0)$
$g(x) = -2(x + 2)(x - 2)(1 + x)(x - 1)$	4	-2	Q3 → Q4	$(-2, 0)$ $(-1, 0)$ $(1, 0)$ $(2, 0)$
$h(x) = (3x + 2)^2(x - 4)(x + 1)(2x - 3)$	5	18	Q3 → Q1	$(4, 0)$ $(-1, 0)$ $(-\frac{2}{3}, 0)$ $(\frac{3}{2}, 0)$
$p(x) = -(x + 5)^3(x - 5)^2$	6	-1	Q3 → Q4	$(-5, 0)$ $(5, 0)$

3) a) i) degree: 3
leading coefficient: positive

ii) x-intercepts: -4, -0.5, 1
factors: $(x + 4)$, $(2x + 1)$, and $(x - 1)$

iii)

Interval	$(-\infty, -4)$	$(-4, -0.5)$	$(-0.5, 1)$	$(1, \infty)$
Sign	-	+	-	+

b) i) degree: 4
leading coefficient: negative

ii) x-intercepts: -1 (order 2), 4 (order 2)
factors: $(x + 1)^2$, and $(x - 4)^2$

iii)

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign	-	-	-

c) i) degree: 4
leading coefficient: positive

ii) x-intercepts: -3, 1 (order 3)
factors: $(x + 3)$, and $(x - 1)^3$

iii)

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign	+	-	+

d) i) degree: 3
leading coefficient: negative

ii) x-intercepts: -5 (order 2), 3
factors: $(x + 5)^2$, and $(x - 3)$

iii)

Interval	$(-\infty, -5)$	$(-5, 3)$	$(3, \infty)$
Sign	+	+	-

4) a)

Degree	Leading Coefficient	End Behaviour	x-Intercepts	y-Intercept
3	-8	Q2 → Q4	$(3, 0)$ $(-2, 0)$ $(\frac{3}{4}, 0)$	$(0, -36)$

b)

Degree	Leading Coefficient	End Behaviour	x-Intercepts	y-Intercept
4	3	Q2 → Q1	$(1, 0)$ $(-3, 0)$ $(-1, 0)$ $(3, 0)$	$(0, 27)$

c)

Degree	Leading Coefficient	End Behaviour	x-Intercepts	y-Intercept
6	-2	Q3 → Q4	$(-4, 0)$ order 2 $(1, 0)$ order 2 $(-2, 0)$ $(1.5, 0)$	$(0, 96)$

d)

Degree	Leading Coefficient	End Behaviour	x-Intercepts	y-Intercept
6	81	Q2 → Q1	$(-6, 0)$ $(5, 0)$ order 2 $(\frac{2}{3}, 0)$ order 3	$(0, -3600)$

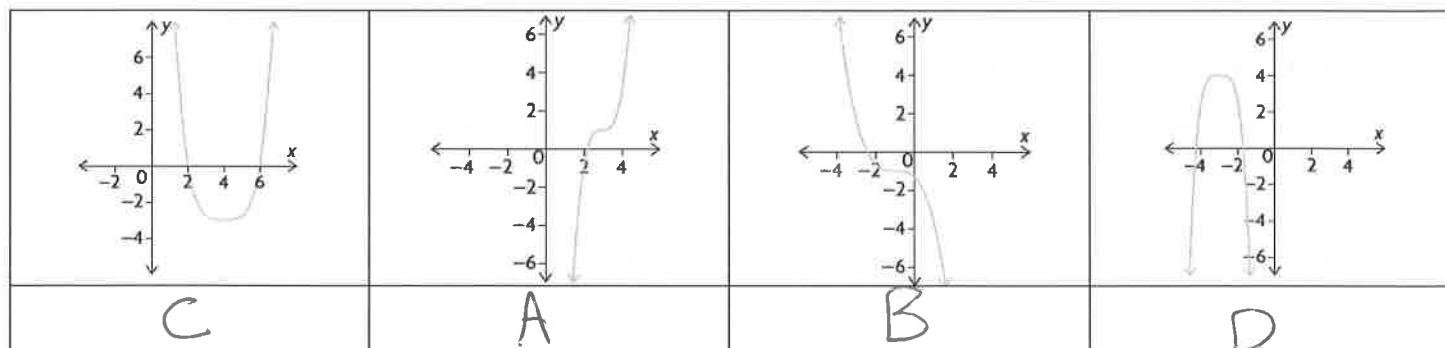
5) a) $y = 0.5(x + 3)(x - 2)(x - 5)$ b) $y = -(x + 1)^2(x - 2)(x - 4)$

$$y = -2(x + 1)^3(x - 3)^2$$

7) 4, 1, -1, and -0.5

1) Match each graph with the corresponding function.

A) $y = 2(x - 3)^3 + 1$ B) $y = -\frac{1}{3}(x + 1)^3 - 1$ C) $y = 0.2(x - 4)^4 - 3$ D) $y = -1.5(x + 3)^4 + 4$



2) List a good set of key points for the following parent functions:

$f(x) = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

$f(x) = x^5$	
x	y
-2	-32
-1	-1
0	0
1	1
2	32

3) Identify the a , k , d and c values and explain what transformation is occurring to the parent function:

a) $f(x) = -2(x - 1)^2$

$a = -2$; vertical reflection
vertical stretch factor 2 ($-2y$)

$d = 1$; shift 1 unit RIGHT ($x+1$)

b) $g(x) = [-\frac{1}{3}(x + 5)]^4 - 1$

$k = -\frac{1}{3}$; horizontal reflection
horizontal stretch factor 3 ($-3x$)

$d = -5$; shift 5 units LEFT ($x-5$)

$c = -1$; shift down 1 unit ($y-1$)

4) Write the full equation given the parent function and the transforming function:

a) $f(x) = x^5$, $g(x) = -3f[2(x + 5)] - 1$

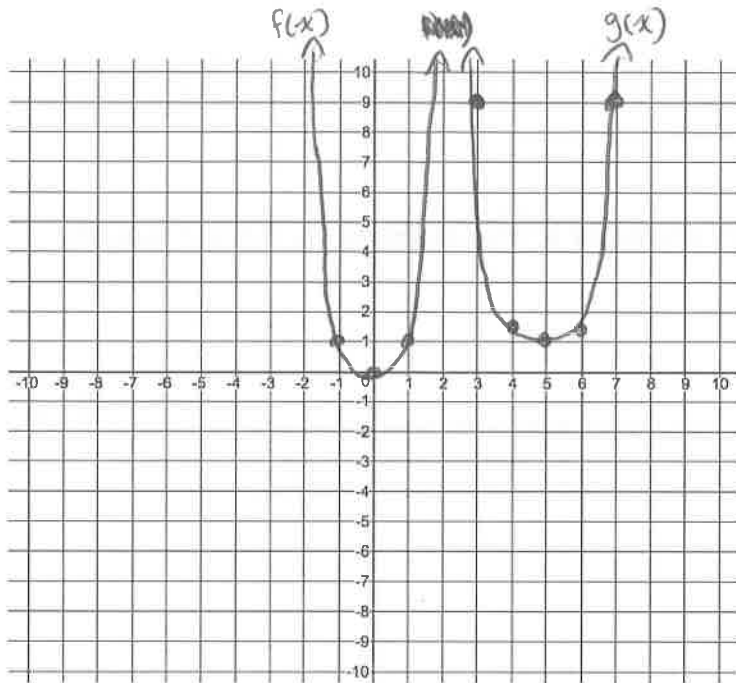
$$g(x) = -3[2(x+5)]^5 - 1$$

b) $f(x) = x^3$, $g(x) = \frac{1}{2}f[-\frac{1}{4}(x - 4)] + 7$

$$g(x) = \frac{1}{2}[-\frac{1}{4}(x-4)]^3 + 7$$

5) For the following questions, use the key points of the parent function to perform transformations. Graph the parent and transformed function. Write the equation of the transformed function.

a) $f(x) = x^4$ $g(x) = \frac{1}{2}f[-(x-5)] + 1$



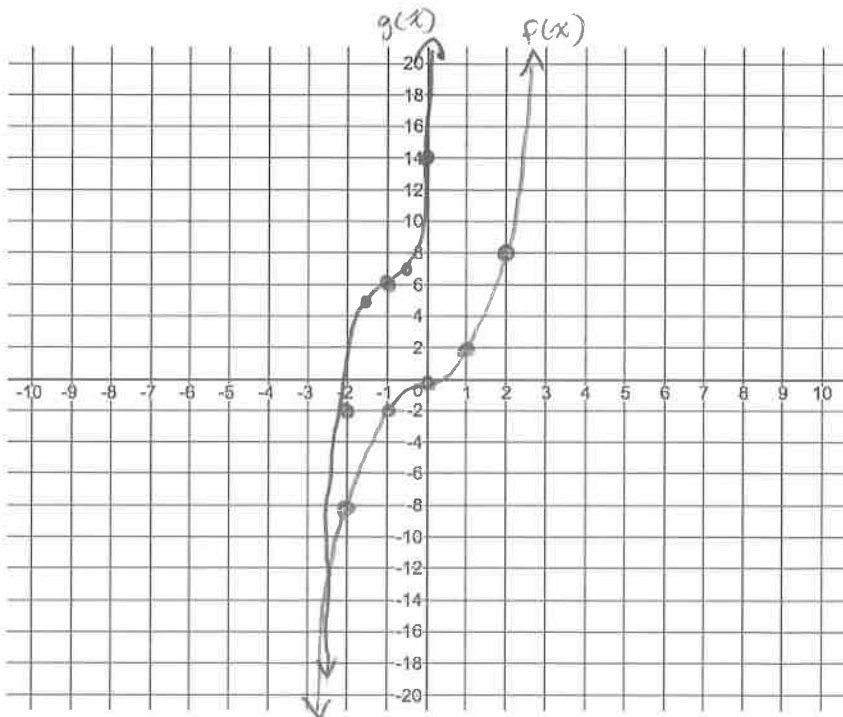
$f(x) = x^4$

x	y
-2	16
-1	1
0	0
1	1
2	16

$g(x) = \frac{1}{2}[-(x-5)]^4 + 1$

$-x+5$	$\frac{y}{2}+1$
7	9
6	10.5
5	1
4	10.5
3	9

b) $f(x) = x^3$ $g(x) = -f[-2(x+1)] + 6$



$f(x) = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

$g(x) = -[-2(x+1)]^3 + 6$

$\frac{x}{-2}-1$	$-y+6$
0	14
-0.5	7
-1	6
-1.5	5
-2	-2

6) Write an equation for the function that results from the given transformations.

a) The function $f(x) = x^4$ is translated 2 units to the left and 3 units up. $d = -2$ $c = 3$

$$g(x) = (x+2)^4 + 3$$

b) The function $f(x) = x^5$ is stretched horizontally by a factor of 5 and translated 12 units to the left. $k = 1/5$ $d = -12$

$$g(x) = \left[\frac{1}{5}(x+12)\right]^5$$

c) The function $f(x) = x^4$ is stretched vertically by a factor of 3, reflected vertically in the x -axis, and translated 6 units down and 1 unit to the left. $a = -3$ $c = -6$ $d = -1$

$$g(x) = -3(x+1)^4 - 6$$

d) The function $f(x) = x^6$ is reflected vertically in the x -axis, stretched horizontally by a factor of 5, reflected horizontally in the y -axis, and translated 3 units down and 1 unit to the right. $k = 1/5$ $a = -1$ $c = -3$ $d = 1$

$$g(x) = -\left[-\frac{1}{5}(x-1)\right]^6 - 3$$

ANSWER KEY

1) C A B D

2)

$f(x) = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

$f(x) = x^5$	
x	y
-2	-32
-1	-1
0	0
1	1
2	32

3) a) $a = -2$; vertical reflection and vertical stretch by a factor of 2 ($-2y$)
 $d = 1$; shift right 1 unit ($x + 1$)

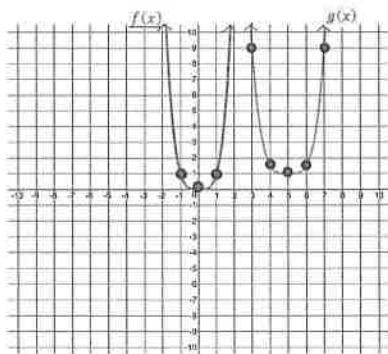
b) $k = -\frac{1}{3}$; horizontal reflection and horizontal stretch by a factor of 3 ($-3x$)
 $d = -5$; shift left 5 units ($x - 5$)
 $c = -1$; shift down 1 unit ($y - 1$)

4) a) $g(x) = -3[2(x+5)]^5 - 1$ b) $g(x) = \frac{1}{2}\left[-\frac{1}{4}(x-4)\right]^3 + 7$

5) a)

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

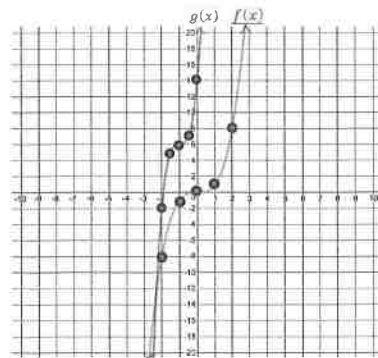
$g(x) = \frac{1}{2}[-(x-5)]^4 + 1$	
$-x+5$	$\frac{1}{2}y+1$
7	9
6	1.5
5	1
4	1.5
3	9



b)

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

$g(x) = -[-2(x+1)]^3 + 6$	
$\frac{x}{-2}-1$	$-y+6$
0	14
-0.5	7
-1	6
-1.5	5
-2	-2



6) a) $g(x) = (x+2)^4 + 3$ b) $g(x) = \left[\frac{1}{5}(x+12)\right]^5$ c) $g(x) = -3(x+1)^4 - 6$ d) $g(x) = -\left[-\frac{1}{5}(x-1)\right]^6 - 3$

W5 - 1.3 - Symmetry in Polynomial Functions

MHF4U

Jensen

ANSWERS

1) Determine whether each function is even, odd, or neither. Does it have line symmetry about the y-axis, point symmetry about the origin, or neither?

a) $y = x^4 - x^2$

Even

Line symmetry about y-axis

b) $y = -2x^3 + 5x$

Odd

Point symmetry about origin

c) $y = -4x^5 + 2x^2$

Neither

d) $y = x(2x + 1)^2(x - 4)$

$$\begin{aligned} &= x(4x^2 + 4x + 1)(x - 4) \\ &= (4x^3 + 4x^2 + x)(x - 4) \\ &= 4x^4 - 16x^3 + 4x^3 - 16x^2 + x^2 - 4x \\ &= 4x^4 - 12x^3 - 15x^2 - 4x \end{aligned}$$

Neither

e) $y = -2x^6 + x^4 + 8$

Even

Line symmetry about the y-axis

2) State whether each function is even or odd. Verify algebraically.

a) $f(x) = x^4 - 13x^2 + 36$ Even

$$\begin{aligned} f(-x) &= (-x)^4 - 13(-x)^2 + 36 \\ &= (-1)^4(x)^4 - 13(-1)^2(x)^2 + 36 \\ &= x^4 - 13x^2 + 36 \end{aligned}$$

$f(x) = f(-x)$

b) $g(x) = 6x^5 - 7x^3 - 3x$ ODD

$$\begin{aligned} -g(x) &= -1[6x^5 - 7x^3 - 3x] \\ &= -6x^5 + 7x^3 + 3x \end{aligned} \quad \& \quad -g(x) = g(-x)$$

$$\begin{aligned} g(-x) &= 6(-x)^5 - 7(-x)^3 - 3(-x) \\ &= 6(-1)^5(x)^5 - 7(-1)^3(x)^3 - 3(-1)(x) \\ &= -6x^5 + 7x^3 + 3x \end{aligned}$$

3) Use the given graph to state:

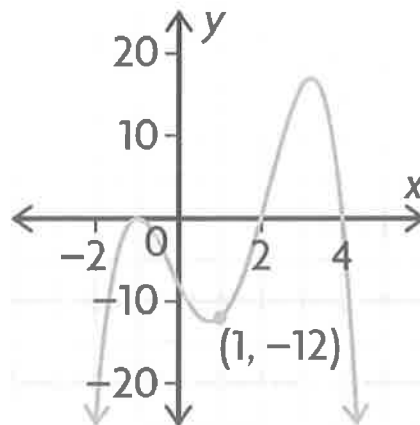
a) x-intercepts -1 (order 2), 2 , and 4

b) number of turning points 3

c) least possible degree 4

d) any symmetry present; even or odd function?

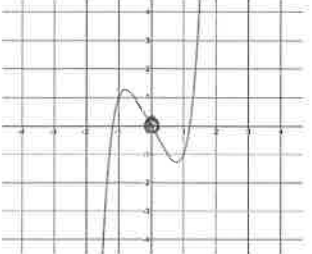
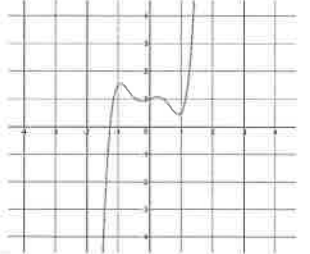
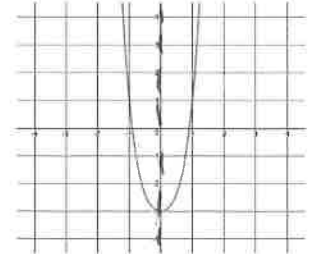
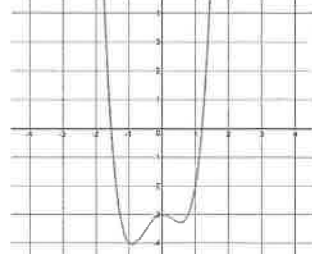
None, Neither



the intervals where $f(x) < 0$

$$f(x) < 0 \text{ when } x \in (-\infty, -1) \cup (-1, 2) \cup (4, \infty)$$

4) Label each function as even, odd, or neither

			
ODD	NEITHER	EVEN	NEITHER

ANSWER KEY

1) a) even, line symmetry about y-axis b) odd, point symmetry about origin c) neither

d) neither e) even, line symmetry about y-axis

2) a) even, $f(-x) = f(x)$ b) odd, $f(-x) = -f(x)$

3) a) -1 (order 2), 2, and 4 b) 3 c) 4 d) no symmetry, neither e) $X \in (-\infty, -1) \cup (-1, 2) \cup (4, \infty)$

4) odd, neither, even, neither