# Chapter 3 Exam Review – Summarizing Univariate Data

MDM4U
Jensen

## Section 3.1: Shapes of Distributions

**1)** Using the following data:

24, 25, 25, 25, 26, 27, 29, 30, 32, 32, 32, 32, 34, 34, 35, 36, 38, 39, 41, 43, 44, 45, 46, 47, 48, 49, 51, 54, 55, 57, 58, 60, 65

**a)** Calculate a bin width that would form six uniform intervals

$range = 65 - 24 = 41$

$$bin\ width = \frac{rounded\ range}{number\ of\ intervals} = \frac{42}{6} = 7$$

**b)** Calculate the starting and end point for each of the five intervals. Then complete the frequency distribution.

*starting point*

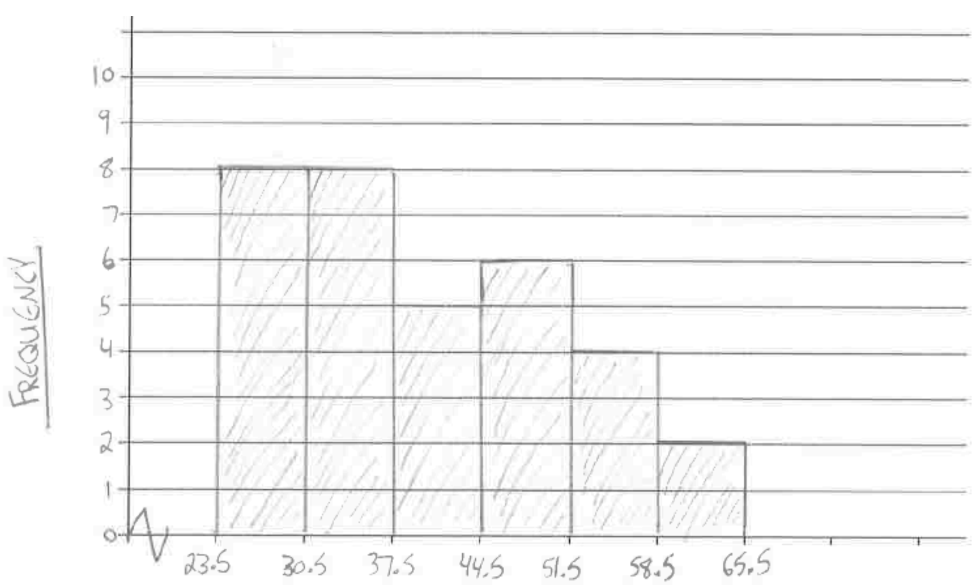
$$= 24 - \frac{42 - 41}{2}$$

$= 24 - 0.5$

$= 23.5$

| Interval | Frequency |
|---|---|
| 23.5 – 30.5 | 8 |
| 30.5 – 37.5 | 8 |
| 37.5 – 44.5 | 5 |
| 44.5 – 51.5 | 6 |
| 51.5 – 58.5 | 4 |
| 58.5 – 65.5 | 2 |

**c)** Create an appropriate histogram.



**2)** State the shape of distribution that occurs when the mean, median and mode are equal.

**MOUND SHAPED**

**3)** What shape of distribution occurs when the height of each bar is roughly equal?

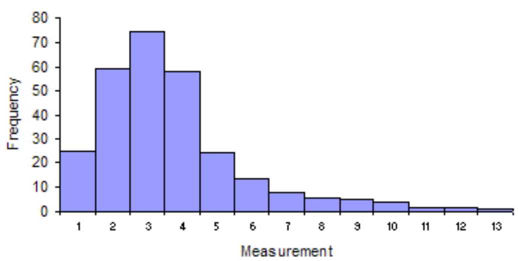
**UNIFORM**

**4)** What shape of distribution occurs when there are peaks at both ends of the range?

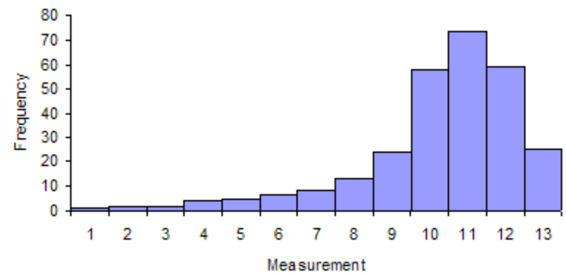
**U-SHAPED OR BIMODAL**

**5)** What shape of distribution is represented in each of the following graphs?

**a)**




**b)**




**RIGHT SKEWED** **LEFT SKEWED**

**<u>Section 3.2/3.3: Measures of Central Tendency/Measures of Spread</u>**

**6)** The number of patients treated in a dental office on Mondays was recorded for 15 weeks.

5, 17, 28, 28, 28, 15, 13, 18, 10, 16, 20, 7, 20 22, 15

Use the above **sample** data to complete the chart below (you may use your graphing calculator).

| **Mean** | **Mode** | $Q_1$ | $Q_2$ | $Q_3$ | $IQR$ | **Standard Deviation** |
|---|---|---|---|---|---|---|
| $\bar{x} = 17.47$ | **28** | **13** | **17** | **22** | **9** | $s = 7.18$ |

**7)** Listed below are the points scored in the 2009 playoffs for **all** 20 players on the Stanley Cup winning Pittsburgh Penguins.

0, 1, 1, 2, 4, 4, 5, 5, 6, 7, 7, 9, 9, 13, 14, 14, 14, 15, 31, 36

Use the above **population** data to complete the chart below (you may use your graphing calculator).

| **Mean** | **Mode** | $Q_1$ | $Q_2$ | $Q_3$ | $IQR$ | **Standard Deviation** |
|---|---|---|---|---|---|---|
| $\mu = 9.85$ | **14** | **4** | **7** | **14** | **10** | $\sigma = 9.17$ |

**8)** Given the following distribution of mathematics marks on a test out of 25…

| Score | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 1 | 1 | 2 | 0 | 4 | 2 | 3 | 3 | 4 | 6 | 3 | 4 | 4 | 1 | 2 |

Use your calculator to…

**a)** Calculate the population mean score on the test for the class.

$\mu = 18.975$

**b)** Calculate the population standard deviation of the test scores.

$\sigma = 3.52$

**9)** A sample of 10 quiz marks of two students are compared.

Sue: 75, 59, 58, 72, 80, 66, 71, 79, 68, 55 Leopold: 90, 83, 55, 84, 72, 63, 50, 65, 52, 91

**a)** Which student has the higher sample average?

Sue: $\bar{x} = 68.3$

***Leopold has the higher average***

Leopold: $\bar{x} = 70.5$

**b)** Which student gets the more consistent mark? (calculate the sample standard deviation)

Sue: $s = 8.77$

***Sue is more consistent because her standard deviation is lower.***

Leopold: $s = 15.74$

**10)** You are taking a class in which your grade is determined from 5 sources: 15% from your homework, 30% from your quizzes (15% per quiz), 25% from your final exam, 15% from your culminating project, and 15% from your speech. Based on the following results, what is the weighted mean of your scores?

| Source | Score, $x$ | Weight, $w$ | $xw$ |
|---|---|---|---|
| **Homework** | 80 | 15 | 1200 |
| **Quiz #1** | 85 | 15 | 1275 |
| **Quiz #2** | 76 | 15 | 1140 |
| **Project** | 95 | 15 | 1425 |
| **Speech** | 90 | 15 | 1350 |
| **Final Exam** | 84 | 25 | 2100 |

Mean:

$$\bar{x} = \frac{\sum xw}{\sum w} = \frac{8490}{100} = 84.9\%$$

**11)** The following data represent the salaries of a sample of employees at RIM Corporation

| Salary (in thousands) | Frequency, $f$ | Midpoint, $m$ | $f \times m$ |
|---|---|---|---|
| 30-39 | 18 | 34.5 | 621 |
| 40-49 | 15 | 44.5 | 667.5 |
| 50-59 | 10 | 54.5 | 545 |
| 60-69 | 5 | 64.5 | 322.5 |
| 70-79 | 3 | 74.5 | 223.5 |

Mean:

$$\bar{x} = \frac{\sum mf}{\sum f} = \frac{2379.5}{51} = 46.657$$

The mean salary is about $46 657

**12)** The middle 50% of the data surrounding the median is called the **INTERQUARTILE RANGE**.

**13)** Jordan's term mark is 82. The term mark counts for 70% of the final mark. What mark must Jordan achieve on the final exam to earn a final mark of 75%?

$$\bar{x} = \frac{\sum xw}{\sum w}$$

$$75 = \frac{82(0.7) + x(0.3)}{1}$$

$$75 = 57.4 + 0.3x$$

$$17.6 = 0.3x$$

$$x = 58.7$$

Jordan must get about a 58.7% on the exam to earn a final mark of 75%.

## Section 3.4: The Normal Distribution

**14)** Fill in the blanks:

Normal distributions are symmetrical and approach **zero** at the extremes. Of the data, **68%** is within one standard deviation of the mean, **95%** is within two standard deviations, and **99.7%** is within three standard deviations of the mean. The area under any normal curve is **1**.

**15)** The temperatures in Florida for the month of December can be represented by the normal distribution $X \sim N(24, 4.8^2)$

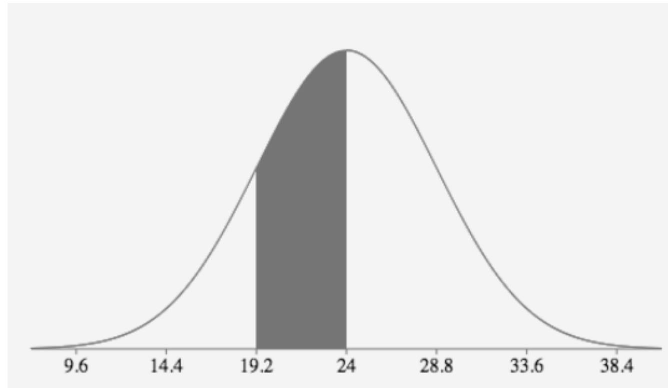
**a)** What range of temperatures would you expect 68% of the days in December to fall between?

68% of the data in a normal distribution is within 1 standard deviation of the mean…

$$\mu \pm 1\sigma = 24 \pm 4.8 = 19.2\ °\text{C}\ to\ 28.8\ °\text{C}$$

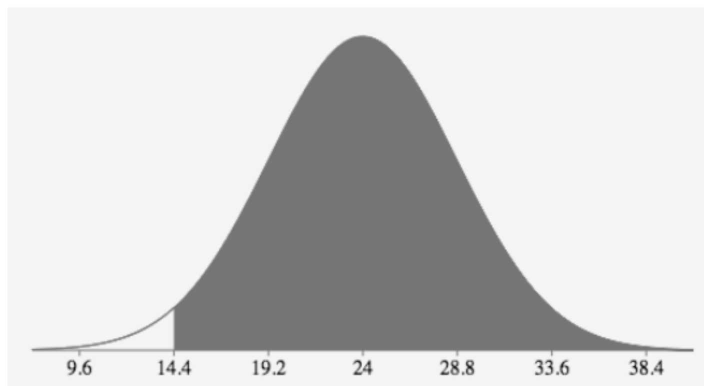
**b)** In what percent of the days will the temperature be between 19.2 and 24 degrees Celcius?

$19.2 < X < 24$
$= normalcdf(19.2{,}24{,}24{,}2.8)$
$= 0.3413$
$= 34.13\%$




**c)** In what percent of the days will the temperature be greater than 14.4 degrees Celcius?

$X > 14.4$
$= normalcdf(14.4, E99{,}24{,}4.8)$
$= 0.9772$
$= 97.72\%$




**d)** 99.7% of the days will be between what two temperatures?

99.7% of data in a normal distribution is within 3 standard deviations of the mean…

$\mu \pm 3\sigma = 24 \pm 3(4.8) = 9.6\ ℃\ to\ 38.4\ ℃$

**e)** Calculate the z-score for a temperature of 30 degrees Celsius. What does this z-score mean?

$$z = \frac{x - \mu}{\sigma} = \frac{30 - 24}{4.8} = 1.25$$

$Z < 1.25 = normalcdf(-E99, 1.25) = 0.894$

A z-score of 1.25 means that the temperature of 30 ℃ is 1.25 standard deviations above the mean temperature. Using a z-score table or a graphing calculator, we can use the z-score to determine that a temperature of 30 ℃ is in the 89$^{th}$ percentile.

**16)** If the mass of children in Oakville were normally distributed, with a mean of 11.2 kg and a standard deviation of 2.8 kg, determine:

**a)** The percent of children with a mass less than or equal to 6.1 kg.

$X \leq 6.1 = normalcdf(-E99,6.1,11.2,2.8) = 0.03427 \cong 3.4\%$

**b)** The percent of children with a mass between 7.3 kg and 14 kg.

$7.3 < X < 14 = normalcdf(7.3,14,11.2,2.8) = 0.7595 \cong 75.95\%$

**c)** The percent of children with a mass greater than 10.5 kg.

$X > 10.5 = normalcdf(10.5, E99,11.2,2.8) = 0.5987 \cong 59.87\%$

**d)** What percentile would a child weighing 15 kg be in?

$X < 15 = normalcdf(-E99, 15, 11.2, 2.8) = 0.9126 \cong the\ 91st\ percentile$

**17)** Calculate the z-score, to two decimal places, of $x = 87.2$ if $\mu = 89.1$ and $\sigma = 3$

$$z = \frac{87.2 - 89.1}{3} \cong -0.63$$

**18)** Suppose $X \sim N(50, 4^2)$. What value of $x$ would have a z-score of 2.10?

$$2.1 = \frac{x - 50}{4}$$

$8.4 = x - 50$

$x = 58.4$

**19)** In $X \sim N(12, 2^2)$, what percent of the data is between 10 and 13?

$10 < X < 13 = normalcdf(10, 13, 12, 2) = 0.5328 \cong 53.28\%$

**20)** In $X \sim N(12, 2^2)$, what value of $x$ corresonds to the 87th percentile?

$x = invnorm(0.87, 12, 2) \cong 14.25$

**21)** For the distribution $N(16, 3.5^2)$, determine the percent of the data that is within the given interval.

**a)** $X > 12$

$= normalcdf(12, E99,16,3.5)$
$\cong 87.3\%$

**b)** $10 < X < 15$

$= normalcdf(10,15,16,3.5)$
$\cong 34.4\%$

**c)** $X < 18.7$

$= normalcdf(-E99,18.7,16,3.5)$
$\cong 78.0\%$

**22)** A group of students wrote an entrance Math exam and the scores were normally distributed. The mean score was 750 and there was a standard deviation of 95. If Johnny wants to score in the 94th percentile, what score must he get?

$score = invnorm(0.94, 750, 95) \cong 897.7$

Johnny must get a score of about 898 to be in the 94th percentile.

**23)** Mr. Jensen's Average golf score is 80 with a standard deviation of 3. In what percent of his golf rounds will he score less than 72?

$X < 72 = normalcdf(-E99,72,80,3) \cong 0.38\%$

**24)** The lengths of nails, in millimeters, at a certain plant are normally distributed with a mean of 20.00 and a standard deviation of 0.21. Nails produced will be rejected unless their lengths are between 19.71 mm and 20.42 mm. What percent of the nails are accepted?

$19.71 < X < 20.42 = normalcdf(19.71{,}20.42{,}20{,}0.21) \cong 0.8936$

About 89.36% of nails are accepted.

**25)** The masses, in grams, of 750 packages of cheese are normally distributed. A package will be rejected if its z-score is -2.57 or less. How many of these packages face rejection?

$z < -2.57 = normalcdf(-E99, -2.57) = 0.0050849541$

$Number\ of\ packages\ facing\ rejection = 0.0050849541(750) \cong 3.8$

About 4 packages face rejection.