Chapter 4 Exam Review - Probability

MDM4U
Jensen

Section 4.2: Theoretical Probability

1) Determine the theoretical probability for each of the following events.

a) rolling a 1 on a die

$$P(1) = \frac{1}{6}$$

b) drawing a face card from a well-shuffled deck

$$P(face\ card) = \frac{12}{52} = \frac{3}{13}$$

c) drawing a red Queen from a well-shuffled deck

$$P(red\ queen) = \frac{2}{52} = \frac{1}{26}$$

d) rolling a "Q" on a die with each side containing a letter of the English alphabet

$$P(Q) = \frac{1}{26}$$

e) rolling a sum of 10 when two dice are rolled

$$P(10) = \frac{3}{36} = \frac{1}{12}$$

2) An experiment consists of taking one card from a regular 52 card deck. What is the probability that:

a) the card chosen will be a diamond.

$$P(diamond) = \frac{13}{52} = \frac{1}{4}$$

b) the card chosen will not be a jack or a king

$$P(jack\ or\ king') = 1 - P(jack\ or\ king) = 1 - \frac{8}{52} = \frac{44}{52} = \frac{11}{13}$$

c) the card chosen will be a diamond or an ace.

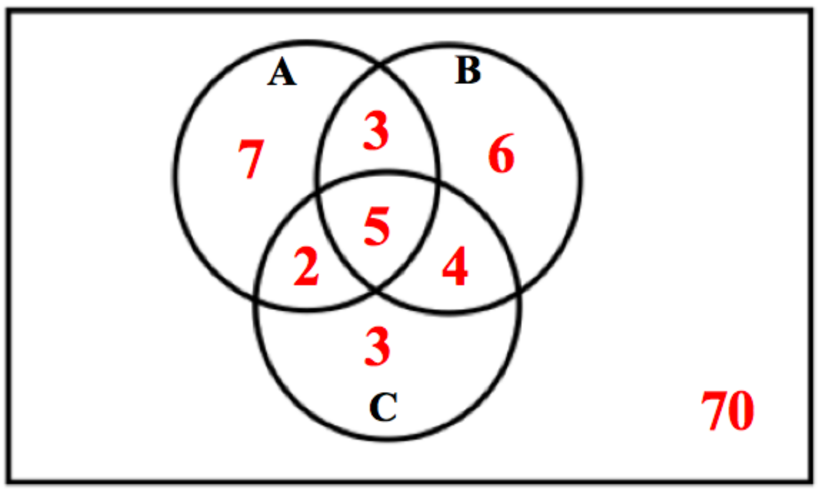
$$P(diamond \cup ace) = P(diamond) + P(ace) - P(diamond \cap ace) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Section 4.3: Probability Using Sets

3) A magazine poll sampling 100 people gives the following results:

17 read magazine A
18 read magazine B
14 read magazine C
8 read magazines A and B
7 read magazines A and C
9 read magazines B and C
5 read all three magazines

a) Illustrate this information on a Venn Diagram:



b) Use your Venn Diagram to calculate the following:

i) How many of the people polled do not read any of the three magazines?

$n(\textit{none of the three}) = 70$

ii) $n(\textit{only } C) = 3$

iii) $P(A \cup B) = \frac{27}{100}$

iv) $P(B \cap C) = \frac{9}{100}$

v) $P(A') = 1 - P(A) = 1 - \frac{17}{100} = \frac{83}{100}$

4) The probability that a student has a cold is 3/7 and the probability that a student has the flu is 4/9. The probability that a student has both a cold and the flu is 1/3. If a student is picked from the school, at random, determine the probability that the student has a cold or the flu.

$$\boldsymbol{P(cold \cup flu) = P(cold) + P(flu) - P(cold \cap flu) = \frac{3}{7} + \frac{4}{9} - \frac{1}{3} = \frac{34}{63}}$$

5) A card is randomly selected from a standard deck of cards. What is the probability that either a red card or a face card (jack, queen, or king) is selected?

$$\boldsymbol{P(red \cup face) = P(red) + P(face) - P(red \cap face) = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13}}$$

<u>Section 4.4 and 4.5: Conditional Probability and Multiplication of Independent Events</u>

6) In a sales effectiveness seminar, a group of sales representatives tried two approaches to selling a customer a new automobile: the aggressive approach and the passive approach. For 1160 customers, the following record was kept:

	Sale	**No Sale**	**Total**
Aggressive	117	50	167
Passive	130	91	221
Total	247	141	388

Suppose a customer is selected at random from the 1160 participating customers. Calculate the following:

a) $P(no\ sale) = \frac{141}{388}$

b) $P(aggressive \cap no\ sale) = \frac{50}{388} = \frac{25}{194}$

c) What is the probability that the aggressive approach is used or there is no sale? are these events mutually exclusive?

$$P(aggresive \cup no\ sale)$$
$$= P(aggresive) + P(no\ sale) - P(aggresive \cap no\ sale)$$
$$= \frac{167}{388} + \frac{141}{388} - \frac{50}{388}$$
$$= \frac{258}{388}$$
$$= \frac{129}{194}$$

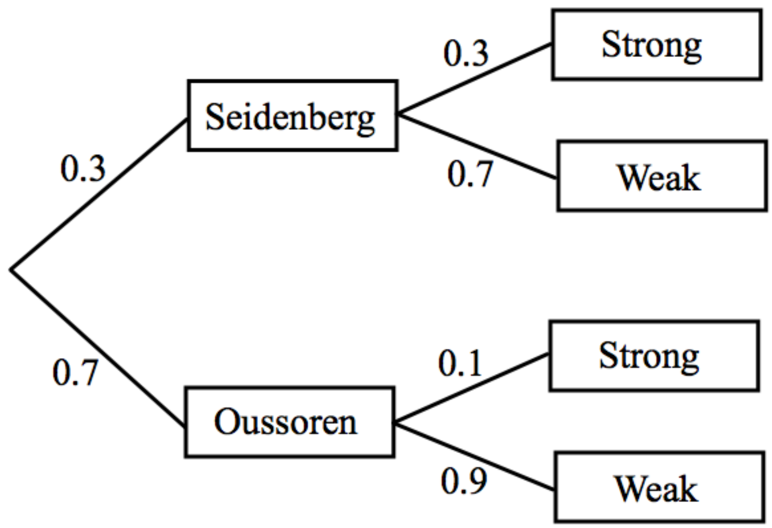
d) $P(no\ sale|passive) = \frac{P(no\ sale \cap passive)}{P(passive)} = \frac{91}{221} = \frac{7}{17}$

e) Given that there is a sale, what is the probability that the aggressive approach was used?

$$P(agressive|sale) = \frac{P(aggresive \cap sale)}{P(sale)} = \frac{117}{247} = \frac{9}{19}$$

7) Each morning, coffee is brewed in the school work-room by one of two faculty members, depending on who arrives first at work. Mr. Seidenberg arrives first 30% of the time, and Mr. Oussoren arrives first 70% of the time. The probability that the coffee is strong when brewed by Mr. Oussoren is 0.1, while the probability that it is strong when Mr. Seidenberg brews the coffee is 0.3.

a) Start by creating a tree diagram to model the situation.



b) What is the probability that Mr. Seidenberg arrives first and the coffee is weak?

$P(Seidenberg \cap weak) = (0.3)(0.7) = 0.21$

c) On a randomly chosen day, what is the probability that the coffee is weak?

$P(weak) = (0.3)(0.7) + (0.7)(0.9) = 0.21 + 0.63 = 0.84$

d) Given that the coffee is strong, what is the probability that Mr. Oussoren arrived first?

$$P(Oussoren|strong) = \frac{P(Oussoren \cap strong)}{P(strong)} = \frac{(0.7)(0.1)}{(0.7)(0.1) + (0.3)(0.3)} = \frac{0.07}{0.16} = 0.4375$$

8) A bag contains 2 red, 4 white and 6 black jelly beans.

a) One jelly bean is drawn at random. What is the probability of drawing a red jelly bean?

$$P(red) = \frac{2}{12} = \frac{1}{6}$$

b) One jelly bean is drawn and then replaced. A second jelly bean is drawn. What is the probability of drawing a red bean and then a black one?

$$P(red, black) = \left(\frac{2}{12}\right)\left(\frac{6}{12}\right) = \frac{12}{144} = \frac{1}{12}$$

c) One jelly bean is drawn but not replaced. Then a second jelly bean is drawn. What is the probability of drawing a red jelly bean and then a black one?

$$P(red \cap black) = \left(\frac{2}{12}\right)\left(\frac{6}{11}\right) = \frac{12}{132} = \frac{1}{11}$$

9) An experiment consists of drawing three cards one after another with replacement between draws. What is the probability of drawing a spade, a five, and a red card in that order?

$$P(spade, 5, red) = \left(\frac{13}{52}\right)\left(\frac{4}{52}\right)\left(\frac{26}{52}\right) = \left(\frac{1}{4}\right)\left(\frac{1}{13}\right)\left(\frac{1}{2}\right) = \frac{1}{104}$$

10) In an archery tournament the probability that Sandy will hit the bullseye is 0.85 and the probability that Adam will hit it is 0.70. Find each of the following:

a) the probability that Adam hits the bullseye and Sandy doesn't

$$P(Adam\ hits \cap Sandy\ misses) = (0.7)(0.15) = 0.105$$

b) the probability that neither hits the bullseye

$$P(both\ miss) = (0.3)(0.15) = 0.045$$

11) There are 3 twenty dollar bills, 6 ten dollar bills and 9 five dollar bills in a box. Suppose a game is played in which a bill is randomly taken from the box, replaced, and then a second bill is drawn from the box. If you are allowed to keep the second bill only if it was the same type as the one that was drawn the first time, calculate the probability that you will be able to keep any bill.

$$P(win) = P(win\ \$20) + P(win\ \$10) + P(win\ \$5) = \left(\frac{3}{18}\right)^2 + \left(\frac{6}{18}\right)^2 + \left(\frac{9}{18}\right)^2 = \frac{9+36+81}{324} = \frac{126}{324} = \frac{7}{18}$$

12) A golfer has 2 pairs of shoes, 7 belts, 7 shirts, 4 pants, and 2 hats. How many different outfits can he make?

$n(outfits) = (2)(7)(7)(4)(2) = 784$

<u>Section 4.6/4.7: Permutations and Combinations</u>

13) Evaluate each of the following

a) $7! = 5040$ **b)** ${}_8P_2 = 56$ **c)** $\frac{5!}{3!} = (5)(4) = 20$ **d)** $P(10,3) = 720$

e) $C(11,3) = 165$ **f)** $\binom{7}{2} = 21$

14) How many 4-digit numbers can be made from the digits 1 to 9 if:

a) Repetition of digits is not allowed?

$$P(9,4) = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9\times8\times7\times6 = 3024$$

b) Repetition is allowed

$= 9\times9\times9\times9 = 6561$

15) In how many ways can the letters of the word "accuracy" be arranged if:

a) There are no restrictions?

$$n(arrangements) = \frac{8!}{3!\,2!} = 3360$$

b) The arrangement must end with a "y"?

$$n(arrangements\ ending\ in\ y) = \frac{7!}{3!\,2!} = 420$$

16) How many arrangements are there for the letters in the word GOALIE ?

$$n(arrangements) = 6! = 720$$

17) All 12 members of a Student Parliament had their picture taken.

a) In how many ways can the 12 pictures be hung in a row outside the student parliament office?

$$n(arrangements) = 12! = 479001600$$

b) In how many ways can 5 of the 12 pictures be hung in a row?

$$n(arrangements\ of\ 5) = P(12,5) = 95040$$

c) In how many ways can 7 of the 12 pictures be hung in a row if Patrick's picture must be first?

$$n(arrangements\ of\ 7\ with\ Patrick\ first) = P(11,6) = 332640$$

d) In how many ways can all 12 pictures be hung if Lisa and Vince's pictures must be hung beside each other?

$$n(arrangements\ with\ Lisa\ and\ Vince\ beside\ each\ other) = 11!\,2! = 79833600$$

18) A 5-letter/digit computer password is given to all the employees of RIM Corporation. How many different passwords can be formed using any of the 26 letters of the alphabet and the 10 numerical digits if no repetition is allowed. Only lower case letters are used.

$n(passwords) = P(36,5) = 45239040$

19) In how many ways can 4 people be selected from a group of 15 to work on a committee?

$n(committees) = \binom{15}{4} = 1365$

20) There are 10 males and 18 females in the Data Management class. How many different committees of 5 students can be formed if:

a) There are no restrictions?

$= \binom{28}{5}$

$=98280$

b) There must be 3 males and 2 females?

$= \binom{10}{3}\binom{18}{2}$

$= (120)(153)$

18360

c) Jessica and Eric must be on the committee?

$= \binom{26}{3}$

$= 2600$

d) There must be a chair, co-chair, secretary, treasurer and speaker?

$= P(28,5)$

$= 11793600$

e) There is at least 1 male on the committee?

$n(at\ least\ 1\ male) = 98280 - n(no\ males)$

$= 98280 - \binom{18}{5}$

$= 98280 - 8568$

$= 89712$

21) Mr. Math is to answer any 8 out of 10 questions on an examination.

a) How many different groups of 8 questions can Mr. Math choose?

$= \binom{10}{8} = 45$

b) How many ways can Mr. Math choose the questions if he must answer at least 4 of the first 5 questions?

$= \binom{5}{4}\binom{5}{4} + \binom{5}{5}\binom{5}{3} = (5)(5) + (1)(10) = 35$

22) A committee of 6 is to be chosen from the 28 students in a class. If there are 10 males and 18 females in the class, in how many ways can this be done if there must be at least three females on the committee?

$n(committees\ with\ at\ least\ 3\ females)$

$= \binom{18}{3}\binom{10}{3} + \binom{18}{4}\binom{10}{2} + \binom{18}{5}\binom{10}{1} + \binom{18}{6}\binom{10}{0}$

$= (816)(120) + (3060)(45) + (8568)(10) + (18564)(1)$

$= 339864$