Chapter 4 Review - Probability

MDM4U
Jensen

Section 4.2: Theoretical Probability

1) Determine the theoretical probability for each of the following events.

a) rolling a 6 on a die

$$P(6) = \frac{n(6)}{n(S)} = \frac{1}{6}$$

b) drawing a black card from a well-shuffled deck

$$P(black) = \frac{n(black)}{n(cards)} = \frac{26}{52} = \frac{1}{2}$$

c) drawing a red 2 from a well-shuffled deck

$$P(red\ 2) = \frac{n(red\ 2)}{n(cards)} = \frac{2}{52} = \frac{1}{26}$$

d) rolling a "Q" on a die with each side containing a letter of the English alphabet

$$P(Q) = \frac{1}{26}$$

2) Each of the letters of the word PROBABILITY is printed on same-sized pieces of paper and placed in a bag. The bag is shaken and one piece of paper is drawn. (Consider Y as a vowel.)

a) What is the probability that the letter A is selected?

$$P(A) = \frac{1}{11}$$

b) What is the probability that the letter B is selected?

$$P(B) = \frac{2}{11}$$

c) What is the probability that a vowel is selected?

$$P(vowel) = \frac{n(vowels)}{n(letters)} = \frac{5}{11}$$

d) What is the probability that a vowel is *not* selected?

$$P(vowel') = 1 - P(vowel)$$
$$= 1 - \frac{5}{11}$$
$$= \frac{6}{11}$$

3) A lottery-mixing bin contains 149 lottery balls numbered 1 to 149. If a winning ball is drawn at random, find the probability that the winning ball is between 10 and 20 inclusive.

$$P(10 \leq lottery\ ball \leq 20) = \frac{11}{149}$$

4) Suppose you roll a pair of six-sided die.

a) How many elements are in the sample space of this experiment?

$$n(S) = 36$$

b) What is the probability of rolling an 8?

$$P(8) = \frac{n(8)}{n(S)} = \frac{5}{36}$$

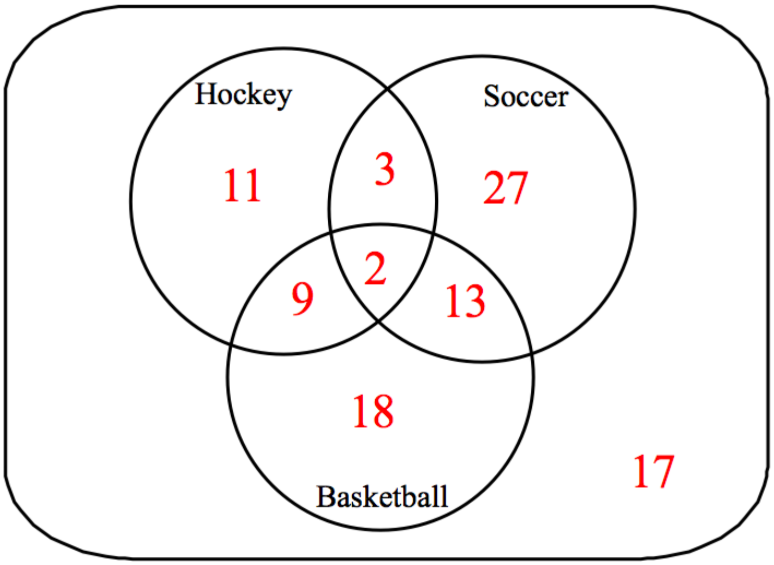
c) What is the probability of not rolling doubles?

$$P(doubles') = 1 - P(doubles)$$
$$= 1 - \frac{6}{36}$$
$$= 1 - \frac{1}{6}$$
$$= \frac{5}{6}$$

Section 4.3: Probability Using Sets

5) 100 students at King's were asked which professional sports they like to watch.

a) Draw a Venn diagram to show student viewing preferences given the following information:



Sports Watched	# of Students
Hockey	25
Soccer	45
Basketball	42
Hockey and Soccer	5
Hockey and Basketball	11
Soccer and Basketball	15
Like to watch all three	2

b) Use your Venn Diagram to calculate the following:

i) $P(only\ hockey)$

$$P(only\ hockey) = \frac{n(only\ hockey)}{n(S)} = \frac{11}{100}$$

ii) $n(hockey \cup basketball)$

$$n(hockey \cup basketball) = n(hockey) + n(basketball) - n(hockey \cap basketball) = 25 + 42 - 11 = 56$$

iii) $P(soccer \cap basketball)$

$$P(soccer \cap basketball) = \frac{n(soccer \cap basketball)}{n(S)} = \frac{15}{100} = \frac{3}{20}$$

iv) $n(none\ of\ the\ three)$

$$n(none\ of\ the\ three) = 17$$

v) $P(soccer')$

$$P(soccer') = 1 - P(soccer) = 1 - \frac{45}{100} = \frac{55}{100} = \frac{11}{20}$$

6) For each of the following, find the indicated probability and state whether A and B are mutually exclusive.

a) $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cup B) = 0.5$, $P(A \cap B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.5 = 0.2 + 0.4 - P(A \cap B)$
$P(A \cap B) = 0.6 - 0.5$
$P(A \cap B) = 0.1$

Not mutually exclusive

b) $P(A) = 0.8$, $P(B) = 0.1$, $P(A \cup B) = ?$, $P(A \cap B) = 0.25$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cup B) = 0.8 + 0.1 - 0.25$
$P(A \cup B) = 0.65$

Not mutually exclusive

c) $P(A) = ?$, $P(B) = 0.15$, $P(A \cup B) = 0.75$, $P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.75 = P(A) + 0.15 - 0$
$0.75 - 0.15 = P(A)$
$P(A) = 0.6$

Mutually exclusive

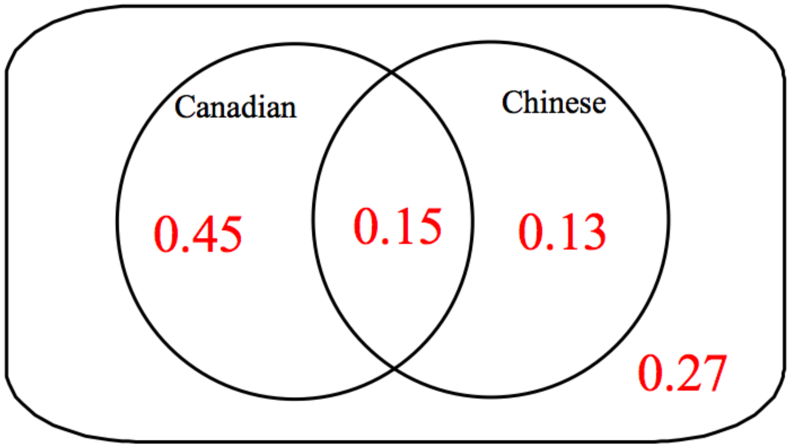
7) Rory applies for two jobs. The probability that she will get Job A is $\frac{5}{20}$ and the probability that she will get Job B is $\frac{2}{5}$. If the probability that she will get both jobs is $\frac{1}{10}$, what is the probability that she will get at least one of the jobs?

$$P(job\ A\ \cup job\ B) = P(job\ A) + P(job\ B) - P(job\ A\ \cap job\ B) = \frac{5}{20} + \frac{2}{5} - \frac{1}{10} = \frac{11}{20}$$

8) In Ms. Tanti's geography class,

• 60% of the students are Canadian
• 28% of the students are of Chinese origin
• 15% are Canadian and of Chinese origin.

a) Construct a Venn diagram to represent this information



b) What is the chance that a student drawn at random would be:

i) Canadian or of Chinese origin?

$P(Canadian \cup Chinese) = P(Canadian) + P(Chinese) - P(Canadian \cap Chinese)$
$= 0.6 + 0.28 - 0.15$
$= 0.73$

ii) neither Canadian nor of Chinese origin?

$P(not\ Canadian\ or\ Chinese) = 1 - P(Canadian \cup Chinese)$
$= 1 - 0.73$
$= 0.27$

iii) of Chinese origin, but not Canadian?

$P(Chinese\ but\ not\ Canadian) = 0.13$

9) A card is randomly selected from a standard deck of cards. What is the probability that either a red card or a face card (jack, queen, or king) is selected?

$P(red \cup face) = P(red) + P(face) - P(red \cap face)$

$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52}$

$= \frac{32}{52}$

$= \frac{8}{13}$

Section 4.4 and 4.5: Conditional Probability and Multiplication of Independent Events

10) Two psychologists surveyed 478 children in elementary schools in Michigan. Among other questions, they asked the students whether their primary goal was to get good grades, to be popular, or to be good at sports. The results are shown in the following table:

	Grades	**Popular**	**Sports**	**Total**
Boy	117	50	60	227
Girl	130	91	30	251
Total	247	141	90	478

Calculate the following probabilities:

a) $P(girl)$

$$P(girl) = \frac{251}{478}$$

b) $P(sports)$

$$P(sports) = \frac{90}{478} = \frac{45}{239}$$

c) $P(girl \cap sports)$

$$P(girl \cap sports) = \frac{n(girl \cap sports)}{n(S)} = \frac{30}{478} = \frac{15}{239}$$

d) $P(grades|girl)$

$$P(grades|girl) = \frac{n(grades \cap girl)}{n(girl)} = \frac{130}{251}$$

e) $P(boy|popular)$

$$P(boy|popular) = \frac{n(boy \cap popular)}{n(popular)} = \frac{50}{141}$$

11) A student is chosen at random in Kim's school. If the probability that the student is taking math this semester is $\frac{37}{50}$, the probability that the student is on the school's soccer team is $\frac{2}{125}$, and the probability that the student is doing both is $\frac{4}{305}$, determine the probability a student on the soccer team is taking math.

$$P(math|soccer\ team) = \frac{P(math \cap soccer\ team)}{P(soccer\ team)} = \frac{\left(\frac{4}{305}\right)}{\left(\frac{2}{125}\right)} = \frac{50}{61}$$

12) What is the probability of drawing two queens in a row (without replacement) from a well-shuffled deck of 52 playing cards?

$$P(2\ queens) = P(1st\ queen) \times P(2nd\ queen|1st\ queen) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$$

13) What is the probability of drawing two queens in a row (with replacement) from a well-shuffled deck of 52 playing cards?

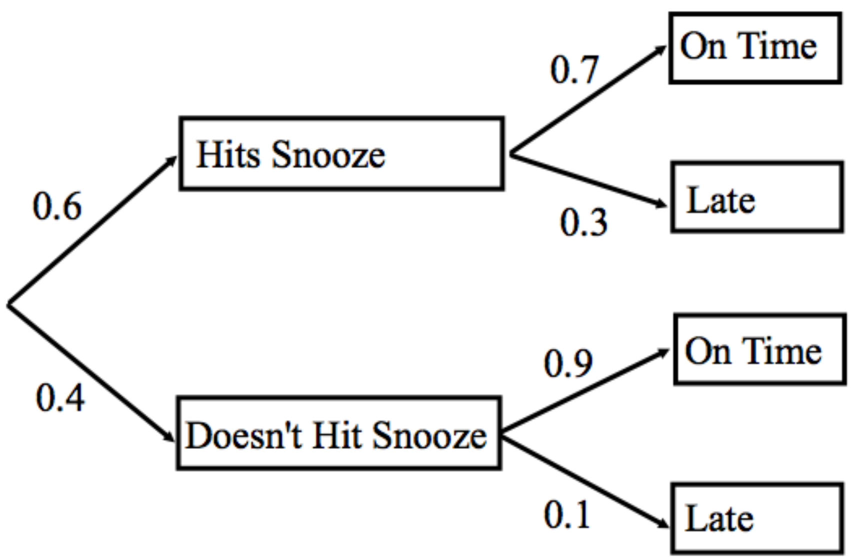
$$P(queen, queen) = P(queen) \times P(queen) = \frac{4}{52} \times \frac{4}{52} = \frac{16}{2704} = \frac{1}{169}$$

14) In a bag of candy canes, there are 14 broken candies mixed in with 20 good ones. If Neha chooses three candies at random from the bag (without replacement), what is the probability that all three are good?

$$P(all\ 3\ good) = \frac{20}{34} \times \frac{19}{33} \times \frac{18}{32} = \frac{6840}{35904} = \frac{285}{1496}$$

15) Shannon hits the snooze bar on her alarm clock on 60% of school days. If she doesn't hit the snooze bar, there is a 90% chance that she makes it to class on time. However, if she hits the snooze bar, there is only a 70% chance that she makes it to class on time.

a) Draw a tree diagram to model this situation



b) On a randomly chosen day, what is the probability that Shannon is late for class?

$P(late) = P(snooze \cap late) + P(no\ snooze \cap late)$
$= (0.6)(0.3) + (0.4)(0.1)$
$= 0.18 + 0.04$
$= 0.22$

There is a 22% chance Shannon is late for class

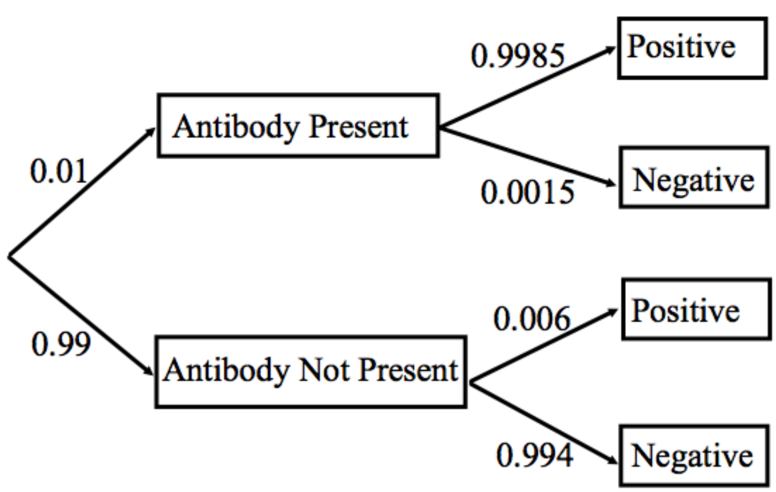
c) Suppose that Shannon is late for school. What is the probability that she hit the snooze bar that morning?

$$P(snooze|late) = \frac{P(snooze \cap late)}{P(late)} = \frac{(0.6)(0.3)}{(0.6)(0.3) + (0.4)(0.1)} = \frac{0.18}{0.22} \cong 0.82$$

There is about an 82% chance she hit the snooze if she is late for school.

16) Enzyme immunoassay (EIA) tests are used to screen blood specimens for the presence of antibodies to HIV. Antibodies indicate the presence of the virus. The test is quite accurate but is not always correct. Suppose that 1% of a large population carries antibodies to HIV in their blood. Of the people who carry the antibodies, 99.85% would test positive. Of the employees who don't carry the antibodies, 0.6% would test positive.

a) Start by creating a tree diagram to model the situation.



b) Given that the EIA test is positive, find the probability that the person has the antibody.

$$P(has\ antibody|positive) = \frac{P(has\ antibody \cap positive)}{P(positive)} = \frac{(0.01)(0.9985)}{(0.01)(0.9985) + (0.99)(0.006)} \cong 0.6270$$

There is about a 62.7% chance that a person with a positive test has the antibody.

17) A string of Christmas lights contains 20 lights. The lights are wired in series, so that if any light fails, the whole string will go dark. Each light has 0.02 probability of failing during a 3-year period. The lights fail independently of each other. Find the probability that the string of lights will remain bright for three years.

$P(all\ remain\ bright) = (0.98)^{20} \cong 0.6676$

There is about a 66.76% chance the lights will remain bright for three years.

18) A test has three multiple-choice questions, each question has four possible answers. What is the probability that

a) you get all three questions correct by guessing?

$P(all\ three\ correct) = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$

b) you get at least one correct by guessing?

$P(at\ least\ 1\ correct) = 1 - P(none\ correct) = 1 - \left(\frac{3}{4}\right)^3 = 1 - \frac{27}{64} = \frac{37}{64}$

19) A box contains a mixture of three types of sporting equipment: eight baseballs, five hockey helmets, and twelve badminton birdies. William randomly takes an item from the box, replaces it, and then takes a second item. He keeps the second item only if he got the same item as the first. Calculate the probability of each of the following.

a) He will be able to keep a hockey helmet.

$P(hockey\ helmet, hockey\ helmet) = \frac{5}{25} \times \frac{5}{25} = \frac{25}{625} = \frac{1}{25}$

b) He will be able to keep a baseball.

$P(baseball, baseball) = \frac{8}{25} \times \frac{8}{25} = \frac{64}{625}$

c) He will not be able to keep a birdie.

$P(not\ keep\ a\ birdie) = 1 - P(birdie, birdie) = 1 - \left(\frac{12}{25} \times \frac{12}{25}\right) = 1 - \frac{144}{625} = \frac{481}{625}$

20) A menu has three choices for salad, six main dishes, and four desserts. How many different meals are available if you select a salad, a main dish and a dessert?

$n(meals) = n(salads) \times n(main\ dishes) \times n(desserts) = 3 \times 6 \times 4 = 72$

21) A baseball player has an on-base percentage of 60%. Calculate the following probabilities.

a) She will not get on base three times in a row.

$P(not\ on\ base\ 3\ times\ in\ a\ row) = (0.4)^3 = 0.064$

b) She will get on base the first attempt and then not get on base the next two.

$P(on\ base, not, not) = 0.6\times0.4\times0.4 = 0.096$

c) She will be on base in two out of three attempts.

$P(on\ base\ twice) = 3(0.6)(0.6)(0.4) = 0.432$

22) A game is played in which a card is drawn from a standard deck of 52 cards and a six-sided die is rolled.

a) Determine the total number of possible outcome for this game.

$n(card, dice) = n(card)\times n(dice) = 52\times6 = 312$

b) Determine the probability that you draw a diamond and an even number.

$$P(diamond, even) = P(diamond)\times P(even) = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

23) A drawer contains five blue, seven pink and three orange scarves. Three scarves are drawn from the drawer, one at a time, without replacement. Determine the probability that the order in which they are selected is blue, pink, orange

$$P(blue, pink, orange) = \frac{5}{15}\times\frac{7}{14}\times\frac{3}{13} = \frac{105}{2730} = \frac{1}{26}$$

Section 4.6: Permutations

24) Evaluate each of the following

a) 7!

$$\begin{aligned}7! &= 7\times6\times5\times4\times3\times2\times1\\ &= 5040\end{aligned}$$

b) ${}_8P_2$

$$P(8,2) = \frac{8!}{(8-2)!}$$
$$= \frac{8!}{6!}$$
$$= 8\times7$$
$$= 56$$

c) $4! \times 3!$

$$\begin{aligned}&4!\times3!\\ &= 24\times6\\ &= 144\end{aligned}$$

d) P(14, 5)

$$P(14,5) = \frac{14!}{(14-5)!}$$
$$= \frac{14!}{9!}$$
$$= 14\times 13\times 12\times 11\times 10$$
$$= 240\ 240$$

e) $\frac{12!}{3!6!5!}$

$$= \frac{479\ 001\ 600}{518400}$$

$$= 924$$

25) Simplify

a) $\frac{r!}{(r-1)!}$

$$= \frac{r(r-1)!}{(r-1)!}$$

$$= r$$

b) $\frac{(n-r)!}{(n-r+1)!}$

$$= \frac{(n-r)!}{(n-r+1)(n-r)!}$$

$$= \frac{1}{n-r+1}$$

26) The nine members of the chess club are standing in a line for a club photo.

a) In how many ways can the 9 students standing in a straight line be arranged?

$$n(arrangements) = 9! = 362\ 880$$

b) In how many ways can 9 students standing in a straight line be arranged if Jill must be first?

$$n(arrangements\ with\ Jill\ first) = 8! = 40\ 320$$

c) What is the probability that Jill and David are standing beside each other?

$$P(Jill\ and\ David\ beside\ each\ other) = \frac{8!\times 2!}{9!} = \frac{80\ 640}{362\ 880} = \frac{2}{9}$$

27) A combination lock opens when the right combination of three numbers from 00 to 59 is entered. The same number may not be used more than once.

a) What is the probability of getting the correct combination by chance?

$$P(guess\ correct) = \frac{1}{P(60,3)} = \frac{1}{205\ 320}$$

b) What is the probability of getting the right combination if you already know the first digit?

$$P(guess\ correct) = \frac{1}{P(59,2)} = \frac{1}{3\ 422}$$

28) The Pittsburgh Penguins have 12 forwards on their roster. In how many ways can they finish first, second, and third in scoring on their team?

$P(12,3) = 1\ 320$

29) How many distinct arrangements of the letters in PERMUTATIONS can you make?

$$n(arrangements) = \frac{12!}{2!} = 239\ 500\ 800$$

30) How many distinct four-digit odd numbers can be formed from the digits in the number 6 738 195?

$n(4\ digit\ odd\ numbers) = 5 \times 6 \times 5 \times 4 = 600$

31) Manpreet has 2 romance novels, 4 fiction novels, and 5 war novels on a shelf. Show a formula to calculate how many ways can she arrange her novels on the shelf if novels of the same genre are to be kept together?

$n(arrangements) = 3! \times 2! \times 4! \times 5! = 34\ 560$

Section 7: Combinations

32) Evaluate each of the following

a) C(8, 3)

$$= \frac{8!}{(8-3)!\,3!}$$

$$= \frac{8!}{5!\,3!}$$

$$= 56$$

b) $_2C_1$

$$= \frac{2!}{(2-1)!\,1!}$$

$$= \frac{2!}{1!\,1!}$$

$$= 2$$

c) $\binom{10}{4}$

$$= \frac{10!}{(10-4)!\,4!}$$

$$= \frac{10!}{6!\,4!}$$

$$= 210$$

d) C(21, 13)

$$= \frac{21!}{(21-13)!\,13}$$

$$= \frac{21!}{8!\,13!}$$

$$= 203\ 490$$

e) $_5C_2$

$$= \frac{5!}{(5-2)!\,2!}$$

$$= \frac{5!}{3!\,2!}$$

$$= 10$$

33) Solve for n in the equation: C(n, 2) = 66

$$\frac{n!}{(n-2)!\,2!} = 66$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 132$$

$$n^2 - n = 132$$

$$n^2 - n - 132 = 0$$

$$(n-12)(n+11) = 0$$

$n = 12$ ~~$n = -11$~~

34) In how many ways can a child select three different ice cream flavours from nine different ice cream flavours?

$$\binom{9}{3} = 84$$

35) How many hands of six cards can be selected from a standard deck of 52 cards?

$$\binom{52}{6} = 20\ 358\ 520$$

36) In how many ways can a group of five people be chosen from seven couples (each of which has one male and one female) to form a club, given each of the following conditions?

a) All are equally eligible for the club.

$n(groups) = \binom{14}{5} = 2\,002$

b) The club must include two females and three males.

$n(2\ females \cap 3\ males) = \binom{7}{2}\binom{7}{3} = 21 \times 35 = 735$

37) Two teachers and six students on a class trip must ride in two four-passenger cars.

a) What is the number of ways that the eight people can be divided into two groups to ride in the two cars?

$\binom{8}{4}\binom{4}{4} = 70 \times 1 = 70$

b) What is the number of ways if only teachers are allowed to drive?

$\binom{2}{1}\binom{6}{3} \times \binom{1}{1}\binom{3}{3} = 2(20) \times 1(1) = 40$

c) What is the probability that the teachers will ride in the same car?

$P(teachers\ in\ same\ car) = 1 - P(teachers\ in\ different\ cars) = 1 - \frac{40}{70} = \frac{30}{70} = \frac{3}{7}$

38) Monique has eight red jelly beans and six purple jelly beans in a jar. She pulls out one jelly bean. What are the odds in favour of the jelly bean being a red one?

$8:6 = 4:3$

39) The weather forecaster predicts that the probability of sun tomorrow is 60%. What are the odds in favour of sun tomorrow?

$60:40 = 3:2$