Section 4.1 Worksheet - Intro to Probability

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1. What is the probability of choosing a King from a standard deck of 52 playing cards?

$$P(King) = \frac{n(Kings)}{n(cards)} = \frac{4}{52} = \frac{1}{13}$$

2. What is the probability of choosing a green marble from a jar containing 5 red, 6 green and 4 blue marbles?

$$P(green) = \frac{n(green)}{n(marbles)} = \frac{6}{15} = \frac{2}{5}$$

3. What is the probability of choosing a marble that is not blue in problem 2?

$$P(not\ blue) = 1 - P(blue) = 1 - \frac{4}{15} = \frac{11}{15}$$

4. What is the probability of getting an odd number when rolling a single 6-sided die?

$$P(odd) = \frac{n(odd)}{n(outcomes)} = \frac{3}{6} = \frac{1}{2}$$

5. What is the probability of choosing a jack or a queen from a standard deck of 52 cards?

$$P(jack\ or\ queen) = P(jack) + P(queen) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

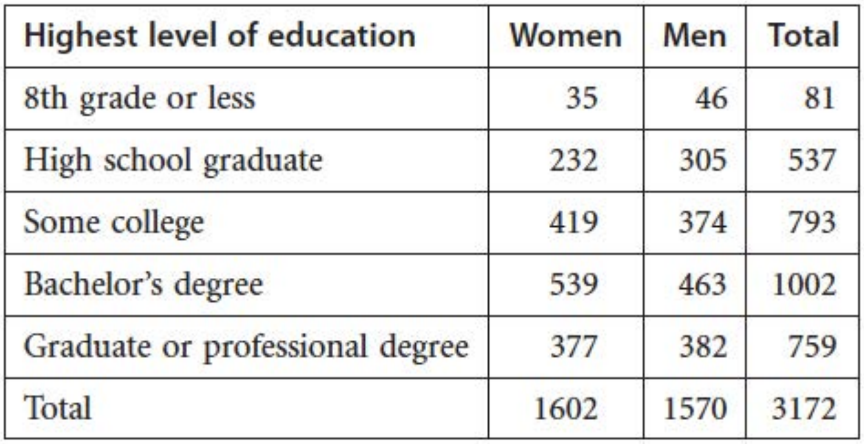
6. What is the probability of landing on an odd number after spinning a spinner with 7 equal sectors numbered 1 through 7?

$$P(odd) = \frac{n(odd)}{n(oucomes)} = \frac{4}{7}$$

7. What is the probability of choosing a queen, a king or an ace from a standard deck of playing cards?

$$P(queen\ or\ king\ or\ ace) = P(queen) + P(king) + P(ace) = \frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{12}{52} = \frac{3}{13}$$

8. A national survey was taken measuring the highest level of educational achievement among adults age 30 and over. Express each probability to the nearest .001.



Highest level of education	Women	Men	Total
8th grade or less	35	46	81
High school graduate	232	305	537
Some college	419	374	793
Bachelor's degree	539	463	1002
Graduate or professional degree	377	382	759
Total	1602	1570	3172

a) What is the probability that a randomly chosen person from the survey group is a man?

$$P(man) = \frac{n(men)}{n(people)} = \frac{1570}{3172} = 0.495$$

b) What is the probability that the highest level of education completed by a randomly chosen person from the survey group is a bachelors degree?

$$P(bachelors\ degree) = \frac{n(bachelors\ degree)}{n(people)} = \frac{1002}{3172} = 0.316$$

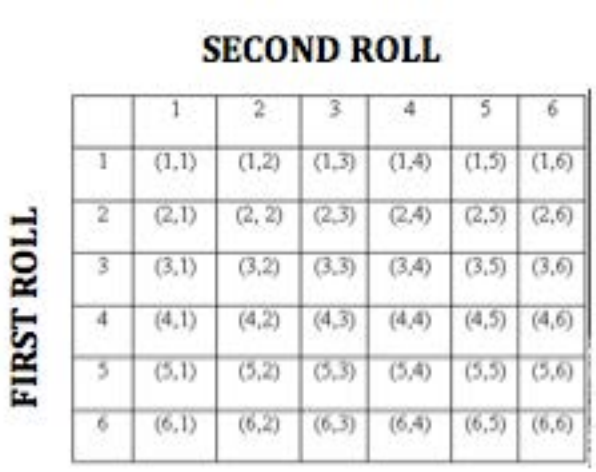
c) What is the probability that a randomly chosen woman has earned a bachelor's or graduate degree?

$$P(bachelors\ given\ woman) = \frac{n(women\ with\ bachelors\ or\ graduate)}{n(women)} = \frac{539 + 377}{1602} = \frac{916}{1602} = 0.572$$

d) What is the probability that a randomly chosen person whose highest level of education is high school is a man?

$$P(man\ given\ high\ school) = \frac{n(men\ with\ high\ school)}{n(high\ school)} = \frac{305}{537} = 0.568$$

9. Two fair dice are rolled.



SECOND ROLL

FIRST ROLL

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2, 2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

a) What is the probability that the second die lands on a higher value than does the first?

$$P(second\ die\ is\ higher) = \frac{n(second\ die\ is\ higher)}{n(outcomes)} = \frac{15}{36}$$

b) What is the probability that the sum of the values is a prime number?

$$P(prime) = \frac{n(prime)}{n(outcomes)} = \frac{15}{36}$$

c) What is the probability the sum of the digits is a prime assuming the first dice rolled a value of either 3 or a 4.

$$P(prime\ given\ a\ 3\ or\ 4\ on\ first\ roll) = \frac{n(prime\ sum\ with\ 3\ or\ 4\ on\ first\ roll)}{n(outcomes\ with\ 3\ or\ 4\ on\ first\ roll)} = \frac{4}{12} = \frac{1}{3}$$

d) What is the probability that the sum of the dice is 9?

$$P(sum\ is\ 9) = \frac{n(sum\ of\ 9)}{n(outcomes)} = \frac{4}{36} = \frac{1}{9}$$

Section 4.2 Worksheet - Theoretical Probability

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1) Suppose you conduct an experiment in which you draw a card from a standard 52-card deck. Compute the theoretical probability of each of the following events.

a) You draw a seven of diamonds

$P(7\ of\ diamonds) = \frac{1}{52}$

b) You draw an ace

$P(ace) = \frac{4}{52} = \frac{1}{13}$

c) You draw a numbered club

$P(numbered\ club) = \frac{9}{52}$

d) You draw an even-numbered card of any suit

$P(even) = \frac{20}{52} = \frac{5}{13}$

2) Three black marbles and two red marbles are in a box. One marble is secretly drawn from the box.

a) What is the total number of possible outcomes?

$n(S) = 5$

b) What is the probability that the marble selected is black?

$P(black) = \frac{n(black)}{n(S)} = \frac{3}{5}$

c) What is the probability that the marble selected is red?

$P(red) = \frac{n(red)}{n(S)} = \frac{2}{5}$

3) Suppose the two joker cards are left in a standard deck of cards. One of the jokers is red and the other is black. A single card is drawn from the deck of 54 cards. Determine the probability of drawing

a) one of the jokers

$P(joker) = \frac{2}{54} = \frac{1}{27}$

b) the red joker

$$P(red\ joker) = \frac{1}{54}$$

c) a queen

$P(queen) = \frac{4}{54} = \frac{2}{27}$

d) any black card

$P(black) = \frac{27}{54} = \frac{1}{2}$

e) any card less than 10 (ace = 1)

$P(< 10) = \frac{36}{54} = \frac{2}{3}$

f) the red joker or a red ace

$P(red\ joker\ or\ red\ ace) = P(red\ joker) + P(red\ ace) = \frac{1}{54} + \frac{2}{54} = \frac{3}{54} = \frac{1}{18}$

4) A spinner is divided into eight equal sectors, numbered 1 through 8.

a) What is the probability of spinning an odd number?

$P(odd) = \frac{n(odd)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

b) What is the probability of spinning a number divisible by 4?

$P(divisible\ by\ 4) = \frac{n(divisible\ by\ 4)}{n(S)} = \frac{2}{8} = \frac{1}{4}$

c) What is the probability of spinning a number less than 3?

$P(< 3) = \frac{n(<3)}{n(S)} = \frac{2}{8} = \frac{1}{4}$

5) A bag contains 12 identically shaped blocks, 3 of which are red and the remainder are green. The bag is well-shaken and a single block is drawn.

a) What is the probability that the block is red?

$P(red) = \frac{n(red)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

b) What is the probability that the block is not red?

$P(red') = 1 - P(red) = 1 - \frac{1}{4} = \frac{3}{4}$

6) Each of the letters for the word 'MATHEMATICS' is printed on same-sized pieces of paper and placed in a hat. That hat is shaken and one piece of paper is drawn.

a) What is the probability that the letters S is selected?

$P(S) = \frac{1}{11}$

b) What is the probability that the letter M is selected?

$P(M) = \frac{2}{11}$

c) What is the probability that a vowel is selected?

$P(vowel) = \frac{4}{11}$

7) Many board games involve a roll of two-six sided dice to see how far you may move your pieces.

a) Copy and complete the following table that shows the totals for all possible rolls of two dice.

		First Die					
		1	**2**	**3**	**4**	**5**	**6**
Second Die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b) What is the probability of rolling a 7?

$P(7) = \frac{6}{36} = \frac{1}{6}$

c) What is the probability of not rolling a 7?

$P(7') = 1 - P(7) = 1 - \frac{1}{6} = \frac{5}{6}$

d) What is the probability of rolling doubles?

$P(doubles) = \frac{6}{36} = \frac{1}{6}$

8) What is the probability that a randomly drawn integer between 1 and 40 is not a perfect square?

Perfect square numbers: {1, 4, 9, 16, 25, 36}

$P(perfect\ square') = 1 - P(perfect\ square) = 1 - \frac{n(perfect\ squares)}{n(S)} = 1 - \frac{6}{40} = \frac{34}{40} = \frac{17}{20}$

9) A picnic cooler contains different types of cola: 12 regular, 8 cherry, 10 diet, 6 diet cherry, 8 caffeine-free, and some caffeine-free diet. You pick a can of cola without looking at its type. There is a 44% chance that the drink selected is diet. How many caffeine-free diet colas are in the cooler?

$P(diet) = \frac{n(diet)}{n(S)}$

$0.44 = \frac{16+x}{44+x}$

$0.44(44 + x) = 16 + x$

$19.36 + 0.44x = 16 + x$

$3.36 = 0.56x$

$x = 6$

There are 6 caffeine-free diet colas in the cooler.

Section 4.3 Worksheet – Probability Using Sets

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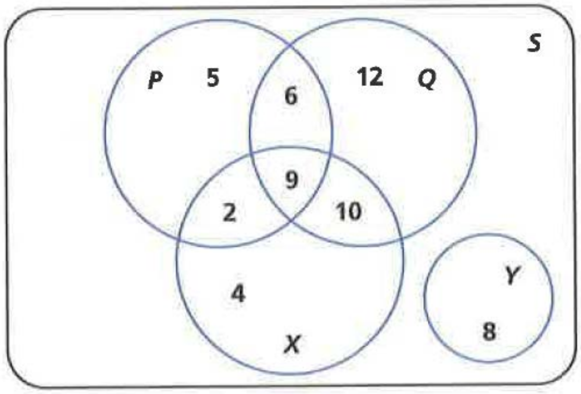
1) Using the Venn diagram below, list the number of elements in each of the following sets.



a) $P \cap Q$

$= 9 + 6$
$= 15$

b) $P \cup Q$

$= 5 + 2 + 9 + 6 + 10 + 12$
$= 44$

c) $X \cap Q$

$= 9 + 10$
$= 19$

d) $X \cup Q$

$= 4 + 2 + 9 + 10 + 6 + 12$
$= 43$

e) $Y \cap Q$

$= 0$

f) $P \cap Q \cap X$

$= 9$

2) For each of the following, find the indicated probability and state whether A and B are mutually exclusive.

a) $P(A) = 0.5,\ \ P(B) = 0.2,\ \ P(A \cup B) = 0.7,\ \ P(A \cap B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.7 = 0.5 + 0.2 - P(A \cap B)$
$P(A \cap B) = 0$

Since $P(A \cap B) = 0$, the events are mutually exclusive.

b) $P(A) = 0.7,\ \ P(B) = 0.2,\ \ P(A \cup B) = ?,\ \ P(A \cap B) = 0.15$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.7 + 0.2 - 0.15$

$P(A \cup B) = 0.75$

Since $P(A \cap B) \neq 0$, the events are not mutually exclusive

c) $P(A) = 0.3,\ \ P(B) = ?,\ \ P(A \cup B) = 0.9,\ \ P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.9 = 0.3 + P(B) - 0$

$P(B) = 0.9 - 0.3$

$P(B) = 0.6$

Since $P(A \cap B) = 0$, the events are mutually exclusive.

3) The probability that Kelly will make the volleyball team is $\frac{2}{3}$ and the probability that she will make the field hockey team is $\frac{3}{4}$. If the probability that she makes both teams is $\frac{1}{2}$, what is the probability that she makes at least one of the teams?

$$P(volleyball\ \cup hockey) = P(volleyball) + P(hockey) - P(volleyball\ \cap hockey)$$

$$P(volleyball\ \cup hockey) = \frac{2}{3} + \frac{3}{4} - \frac{1}{2}$$

$$P(volleyball\ \cup hockey) = \frac{8}{12} + \frac{9}{12} - \frac{6}{12}$$

$$P(volleyball\ \cup hockey) = \frac{11}{12}$$

4) An aquarium at a pet store contains 20 guppies (12 females and 8 males) and 36 tetras (14 females and 22 males). If the clerk randomly nets a fish, what is the probability that it is a female or a tetra?

$$P(female \cup tetra) = \frac{n(female \cup tetra)}{n(fish)}$$

$$P(female \cup tetra) = \frac{n(female) + n(tetra) - n(female \cap tetra)}{n(fish)}$$

$$P(female \cup tetra) = \frac{26 + 36 - 14}{56}$$

$$P(female \cup tetra) = \frac{48}{56}$$

$$P(female \cup tetra) = \frac{6}{7}$$

5) Teri attends a fundraiser at which 15 T-shirts are being given away as door prizes. Door prize winners are randomly given a shirt from a stock of 2 black shirts, 4 blue shirts, and 9 white shirts. Teri really likes the black and blue shirts, but is not too keen on the white ones. Assuming that Teri wins the first door prize, what is the probability that she will get a shirt that she likes?

$$P(black \cup blue) = P(black) + P(blue)$$

$$P(black \cup blue) = \frac{2}{15} + \frac{4}{15}$$

$$P(black \cup blue) = \frac{6}{15}$$

6) A card is randomly selected from a standard deck of cards. What is the probability that either a heart or a face card (jack, queen, or king) is selected?

$$P(heart \cup face\ card) = P(heart) + P(face\ card) - P(heart \cap face\ card)$$

$$P(heart \cup face\ card) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

$$P(heart \cup face\ card) = \frac{22}{52}$$

$$P(heart \cup face\ card) = \frac{11}{26}$$

7) An electronics manufacturer is testing a new product to see whether it requires a surge protector. The tests show that a voltage spike has a 0.2% probability of damaging the product's power supply, a 0.6% probability of damaging downstream components, and a 0.1% probability of damaging both. Determine the probability that a voltage spike will damage the product.

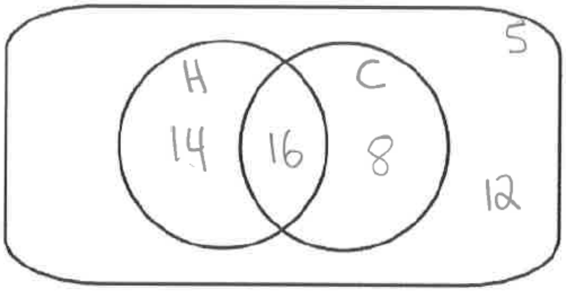
$$P(power\ supply \cup components) = P(power\ supply) + P(components) - P(power\ supply \cap components)$$

$$P(power\ supply \cup components) = 0.2\% + 0.6\% - 0.1\%$$

$$P(power\ supply \cup components) = 0.7\%$$

8) At the start of flu season, Dr. Anna Ahmeed examines 50 patients over two days. Of those 50 patients, 30 have a headache, 24 have a cold, and 12 have neither symptom. Some patients have both symptoms.

a) Draw a Venn diagram and determine the number of patients that have both symptoms.



$$n(H \cup C) = n(H) + n(C) - n(H \cap C)$$
$$38 = 30 + 24 - n(H \cap C)$$
$$n(H \cap C) = 54 - 38$$
$$n(H \cap C) = 16$$

b) Find the probability that a patient selected at random…

i) has just a headache

$$P(headache\ only) = \frac{14}{50} = \frac{7}{25}$$

ii) has a headache or a cold

$$P(H \cup C) = \frac{n(H \cup C)}{n(S)} = \frac{38}{50} = \frac{19}{25}$$

iii) does not have cold symptoms

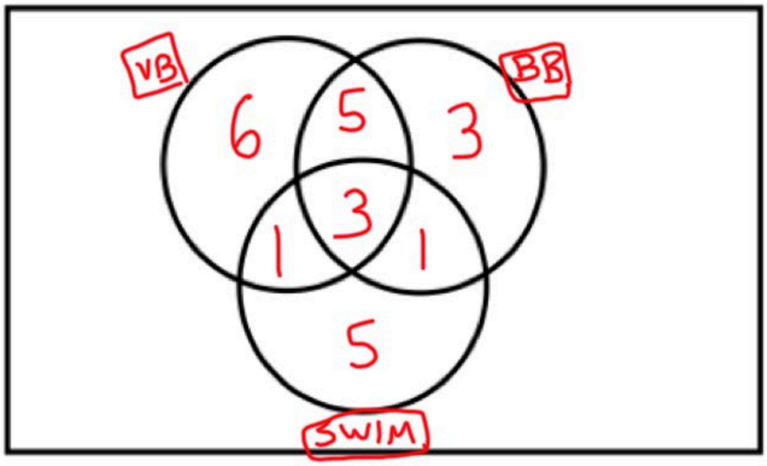
$$P(cold') = 1 - P(cold)$$

$$P(cold') = 1 - \frac{24}{50}$$

$$P(cold') = \frac{26}{50}$$

$$P(cold') = \frac{13}{25}$$

9) a) Use the table to complete the following Venn Diagram:



Team	# of Students
Volleyball	15
Basketball	12
Swimming	10
VB and BB	8
VB and Swim	4
BB and Swim	4
All three	3

b) What is the probability that a student plays volleyball?

$$P(volleyball) = \frac{15}{24} = \frac{5}{8}$$

c) What is the probability that a student plays basketball?

$$P(basketball) = \frac{12}{24} = \frac{1}{2}$$

d) What is the probability that a student plays basketball **and** volleyball?

$$P(baskeball \cap volleyball) = \frac{8}{24} = \frac{1}{3}$$

e) What is the probability that a student plays basketball **or** volleyball?

$$P(baskeball \cup volleyball) = \frac{19}{24}$$

Section 4.4 – Conditional Probability

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Refer to part 1 of 4.4 lesson for help with the following questions

1) Joel surveyed his class and summarized responses to the question, “Do you like school?”

	Liked	**Disliked**	**No Opinion**	**Total**
Males	12	5	2	19
Females	10	3	1	14
Total	22	8	3	33

Find each of the following probabilities:

a) $P(likes\ school\ |\ student\ is\ male)$

$$P(likes\ school\ |\ student\ is\ male) = \frac{n(likes\ school \cap male)}{n(male)} = \frac{12}{19}$$

b) $P(student\ is\ female\ |\ student\ dislikes\ school)$

$$P(student\ is\ female\ |\ student\ dislikes\ school) = \frac{n(female \cap dislikes\ school)}{n(dislikes\ school)} = \frac{3}{8}$$

2) A person is chosen at random from shoppers at a department store. If the person’s probability of having blonde hair and glasses is $\frac{2}{25}$ and the probability of wearing glasses is $\frac{9}{25}$, determine the probability that a person has blonde hair given that they wear glasses.

$$P(blonde\ hair\ |\ wears\ glasses) = \frac{P(blonde \cap glasses)}{P(glasses)} = \frac{\left(\frac{2}{25}\right)}{\left(\frac{9}{25}\right)} = \frac{2}{9}$$

3) From a medical study of 10 000 male patients, it was found that 2500 were smokers; 720 died from lung cancer and of these, 610 were smokers. Determine:

a) $P(dying\ from\ lung\ cancer\ |\ smoker)$

$$P(dying\ from\ lung\ cancer\ |\ smoker) = \frac{n(dying\ from\ lung\ cancer \cap smoker)}{n(smoker)} = \frac{610}{2500} = \frac{61}{250}$$

b) $P(dying\ from\ lung\ cancer\ |\ non - smoker)$

$$P(dying\ from\ lung\ cancer\ |\ non - smoker) = \frac{n(dying\ from\ lung\ cancer \cap non - smoker)}{n(non - smoker)} = \frac{110}{7500} = \frac{11}{750}$$

4) The table shows the results of a survey in which 146 families were asked if they own a computer and if they will be taking a summer vacation this year.

	Takes a Vacation	**Does not Take a Vacation**	**Total**
Owns a Computer	46	11	57
Does Not Own a Computer	55	34	89
Total	101	45	146

a) Find the probability a randomly selected family is taking a summer vacation this year given that they own a computer.

$$P(vacation|owns\ a\ computer) = \frac{n(vacation \cap computer)}{n(computer)} = \frac{46}{57}$$

b) Find the probability a randomly selected family is taking a summer vacation this year and owns a computer.

$$P(vacation \cap computer) = P(vacation) \times P(computer|vacation)$$

$$P(vacation \cap computer) = \frac{101}{146} \times \frac{46}{101} = \frac{46}{146} = \frac{23}{73}$$

Refer to part 2 of 4.4 lesson for help with the following questions

4) What is the probability of being dealt two clubs in a row from a well-shuffled deck of 52 playing cards without replacing the first card drawn?

$$P(1st\ club \cap 2nd\ club) = P(1st\ club) \times P(2nd\ club|1st\ club) = \frac{13}{52} \times \frac{12}{51} = \frac{156}{2652} = \frac{1}{17}$$

5) A bag contains three red marbles and five white marbles. What is the probability of drawing two red marbles at random if the first marble drawn is not replaced?

$$P(1st\ red \cap 2nd\ red) = P(1st\ red) \times P(2nd\ red|1st\ red) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$$

6) A road has two stop lights at two consecutive intersections. The probability of getting a green light at the first intersection is 0.6, and the probability of getting a green light at the second intersection, given that you got a green light at the first intersection, is 0.8. What is the probability of getting a green light at both intersections?

$$P(1st\ green \cap 2nd\ green) = P(1st\ green) \times P(2nd\ green|1st\ green) = 0.6 \times 0.8 = 0.48$$

You have a 48% chance of getting a green light at both intersections

7) Suppose the two joker cards are left in a standard deck of cards. One of the jokers is red and the other is black. A single card is drawn from the deck of 54 cards but not returned to the deck, and then a second card is drawn. Determine the probability of drawing:

a) one of the jokers on the first draw and an ace on the second draw

$$P(1st\ joker \cap 2nd\ ace) = P(1st\ joker) \times P(2nd\ ace|1st\ joker) = \frac{2}{54} \times \frac{4}{53} = \frac{8}{2862} = \frac{4}{1431}$$

b) a numbered card of any suit on the first draw and the red joker on the second draw

$$P(1st\ numbered\ card \cap 2nd\ red\ joker) = P(1st\ numbered\ card) \times P(2nd\ red\ joker|1st\ numbered\ card)$$

$$P(1st\ numbered\ card \cap 2nd\ red\ joker) = \frac{36}{54} \times \frac{1}{53}$$

$$P(1st\ numbered\ card \cap 2nd\ red\ joker) = \frac{36}{2862}$$

$$P(1st\ numbered\ card \cap 2nd\ red\ joker) = \frac{2}{159}$$

c) a queen on both draws

$$P(1st\ queen \cap 2nd\ queen) = P(1st\ queen) \times P(2nd\ queen|1st\ queen) = \frac{4}{54} \times \frac{3}{53} = \frac{12}{2862} = \frac{2}{477}$$

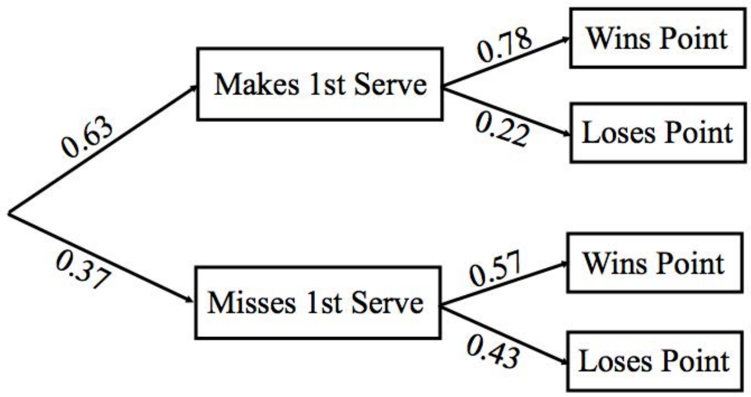
d) any black card on both draws

$$P(1st\ black \cap 2nd\ black) = P(1st\ black) \times P(2nd\ black|1st\ black) = \frac{27}{54} \times \frac{26}{53} = \frac{702}{2862} = \frac{13}{53}$$

Refer to part 3 of 4.4 lesson for help with the following questions

8) Tennis great Roger Federer made 63% of his first serves in 2011 season. When Federer made his first serve, he won 78% of the points. When Federer missed his first serve and had to serve again, he won only 57% of the points. Suppose we randomly choose a point on which Federer served.

a) Start by creating a tree diagram to model the situation.



b) What is the probability that Federer makes the first serve and wins the point?

$$P(makes\ 1st\ serve \cap wins\ point) = P(makes\ 1st\ serve) \times P(wins\ point | makes\ 1st\ serve)$$

$$P(makes\ 1st\ serve \cap wins\ point) = 0.63 \times 0.78$$

$$P(makes\ 1st\ serve \cap wins\ point) = 0.4914$$

There is a 49.14% chance Federer makes the first serve and wins the point.

c) What his the probability the he loses the point?

There are two ways he can lost he point; he can make his first serve and lose OR he can miss his first serve and lose.

$$P(loses) = P(wins\ 1st\ serve\ \cap loses) + P(misses\ 1st\ serve \cap loses)$$

$$P(loses) = 0.63 \times 0.22 + 0.37 \times 0.43$$

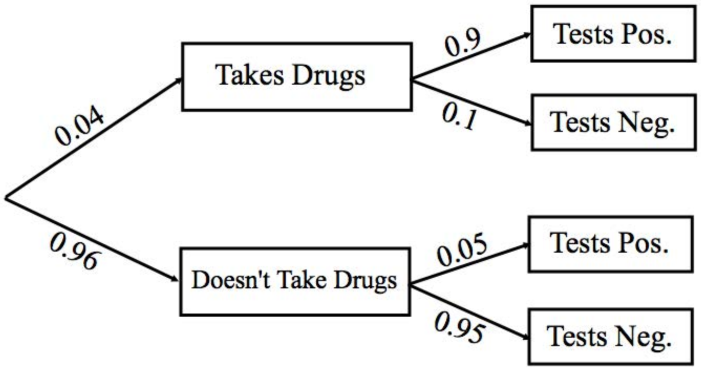
$$P(loses) = 0.1386 + 0.1591$$

$$P(loses) = 0.2977$$

There is a 29.77% chance Federer loses a point when he is serving.

9) Many employers require prospective employees to take a drug test. A positive result on this test indicates that the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose that 4% of prospective employees use drugs. Of the employees who use drugs, 90% would test positive. Of the employees who don't use drugs, 5% would test positive.

a) Start by creating a tree diagram to model the situation.



b) A randomly selected prospective employee tests positive for drugs. What is the probability that he actually took drugs?

$$P(took\ drugs | positive\ test) = \frac{P(took\ drugs \cap positive\ test)}{P(positive\ test)}$$

$$P(took\ drugs | positive\ test) = \frac{0.04 \times 0.9}{0.04 \times 0.9 + 0.96 \times 0.05} = \frac{0.036}{0.084} = 0.4286$$

There is about a 42.86% chance that he actually took drugs if he tested positive

Section 4.5 – Multiplication of Independent Events

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1) A truck driver has a choice of routes as he travels among four cities. He can choose from four routes between Toronto and Oakville, two between Oakville and Hamilton, and three between Hamilton and Guelph. Find the total number of routes possible for the complete Toronto-Oakville-Hamilton-Guelph trips.

$n(routes) = 4\times2\times3 = 24$

2) A test has four true/false questions. What is the probability that they will get all four correct by guessing?

$$P(all\ 4\ correct) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{16}$$

3) A test has three multiple choice questions, each question has four possible answers. What is the probability that you get all three questions correct by guessing?

$$P(all\ 3\ correct) = \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{64}$$

4) A standard deck of cards has had all the face cards (jacks, queens, and kings) removed so that only the ace through ten of each suit remain. A game is played in which a card is drawn from this deck and a six-sided die is rolled. For the purpose of this game, an ace is considered to have a value of 1.

a) Determine the total number of possible outcome for this game.

$n(outcomes) = n(card) \times n(die) = 40\times6 = 240$

b) Find the probability of each of these events:

i) an even card and an even roll of the die

$P(even\ card, even\ die) = P(even\ card)\times P(even\ die) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

ii) an even card and a roll of 3.

$P(even\ card, die\ 3) = P(even\ card)\times P(die\ 3) = \frac{1}{2}\times\frac{1}{6} = \frac{1}{12}$

iii) a card of 3 and a roll of the die of 3 or less

$P(card\ 3, die\ \leq 3) = P(card\ 3)\times P(die \leq 3) = \frac{4}{40}\times\frac{3}{6} = \frac{12}{240} = \frac{1}{20}$

5) Suppose the two joker cards are left in a standard deck of cards. One of the jokers is red and the other is black. A single card is drawn from the deck of 54 cards, returned, and then a second card is drawn. Determine the probability of drawing:

a) one of the jokers on the first draw and an ace on the second

$$P(joker, ace) = P(joker) \times P(ace) = \frac{2}{54} \times \frac{4}{54} = \frac{8}{2916} = \frac{2}{729}$$

b) the red joker on the second draw and a numbered card of any suit on the first

$$P(numbered\ card, red\ joker) = P(numbered\ card) \times P(red\ joker) = \frac{36}{54} \times \frac{1}{54} = \frac{36}{2916} = \frac{1}{81}$$

c) a queen on both draws

$$P(queen, queen) = P(queen) \times P(queen) = \frac{4}{54} \times \frac{4}{54} = \frac{16}{2916} = \frac{4}{729}$$

d) any black card on both draws

$$P(black, black) = P(black) \times P(black) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

e) any numbered card less than 10 on the first draw and a card with the same number on the second

$$P(< 10, same\ \#) = P(< 10) \times P(same\ \#) = \frac{32}{54} \times \frac{4}{54} = \frac{128}{2916} = \frac{32}{729}$$

6) A paper bag contains a mixture of 3 types of candy. There are ten chocolate bars, seven fruit bars, and three packages of toffee. Suppose a game is played in which a candy is randomly taken from the bag, replaced, and then a second candy is drawn from the bag. If you are allowed to keep the second candy only if it was the same type as the one that was drawn the first time, calculate the probability of each of the following:

a) you will be able to keep a chocolate bar

$$P(chocolate, chocolate) = P(chocolate) \times P(chocolate) = \frac{10}{20} \times \frac{10}{20} = \frac{100}{400} = \frac{1}{4}$$

b) you well be able to keep any candy

$$P(keep\ any) = P(chocolate, chocolate) + P(fruit, fruit) + P(toffee, toffee)$$

$$P(keep\ any) = \left(\frac{10}{20}\right)\left(\frac{10}{20}\right) + \left(\frac{7}{20}\right)\left(\frac{7}{20}\right) + \left(\frac{3}{20}\right)\left(\frac{3}{20}\right)$$

$$P(keep\ any) = \frac{100}{400} + \frac{49}{400} + \frac{9}{400}$$

$$P(keep\ any) = \frac{158}{400}$$

$$P(keep\ any) = \frac{79}{200}$$

c) you won't be able to keep any candy

$P(keep\ any') = 1 - P(keep\ any)$

$P(keep\ any') = 1 - \frac{79}{200}$

$P(keep\ any') = \frac{121}{200}$

7) A coin is tossed and a standard six-sided die is rolled.

a) How many different outcomes are possible?

$n(outcomes) = 2 \times 6 = 12$

b) What is the probability of flipping tails and rolling a number greater than 4?

$P(tails, > 4) = P(tails) \times P(> 4) = \frac{1}{2} \times \frac{2}{6} = \frac{2}{12} = \frac{1}{6}$

8) The probability that a salmon swims successfully through a dam is 0.85.

a) Find the probability that three salmon swim successfully through the dam.

$P(success, success, success) = 0.85 \times 0.85 \times 0.85 = 0.614$

b) Find the probability that none of the three salmon is successful.

$P(fail, fail, fail) = 0.15 \times 0.15 \times 0.15 = 0.003$

c) Find the probability that at least one of the three salmon is successful in swimming through the dam.

$P(atleast\ 1) = 1 - P(none) = 1 - 0.003 = 0.997$

9) There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B is accurate 89% of the time. If a patient is given both tests, find the probability that

a) both tests give the correct result

$P(A\ correct, B\ correct) = 0.95 \times 0.89 = 0.8455$

b) neither test gives the correct result

$P(A\ wrong, B\ wrong) = 0.05 \times 0.11 = 0.0055$

c) at least one of the tests gives the correct result

$P(at\ least\ one\ correct) = 1 - P(A\ wrong, B\ wrong) = 1 - 0.0055 = 0.9945$

Section 4.6 – Permutations

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1) Evaluate each of the following

a) $5!$

$$= 5\times4\times3\times2\times1$$
$$= 120$$

b) ${}_5P_3$

$$= \frac{5!}{(5-3)!}$$

$$= \frac{5!}{2!}$$

$$= \frac{5\times4\times3\times\cancel{2}\times\cancel{1}}{\cancel{2}\times\cancel{1}}$$

$$= 5\times4\times3$$

$$= 60$$

c) $\frac{5!}{4!}$

$$= \frac{5\times\cancel{4!}}{\cancel{4!}}$$

$$= 5$$

d) $P(10, 3)$

$$= \frac{10!}{(10-3)!}$$

$$= \frac{10!}{7!}$$

$$= \frac{10\times9\times8\times\cancel{7!}}{\cancel{7!}}$$

$$= 10\times9\times8$$

$$= 720$$

2) Simplify each of the following

a) $\frac{n!}{(n-1)!}$

$$= \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}}$$

$$= n$$

b) $\frac{(3n)!}{(3n-1)!}$

$$= \frac{(3n)\cancel{(3n-1)!}}{\cancel{(3n-1)!}}$$

$$= 3n$$

3) Express the following using factorials

a) $5\times4\times3\times2\times1$

$$= 5!$$

b) $8\times7\times6$

$$= \frac{8!}{5!}$$

c) $\frac{30\times29\times28}{3\times2\times1}$

$$= \frac{30!}{27!\,3!}$$

4) Ten students are to line up for a photograph.

a) In how many ways can the 10 students standing in a line be arranged?

$n(arrangements) = 10! = 3\,628\,800$

b) In how many ways can the 10 students standing in a line be arranged if Jill must be first?

$n(arrangements\ with\ Jill\ first) = 9! = 362\,880$

c) In how many ways can 10 students standing in a line be arranged if Jill must be first and Meera last?

$n(arrangements\ with\ Jill\ first\ and\ Meera\ last) = 8! = 40\,320$

5) The senior choir has rehearsed five songs for an upcoming assembly. In how many different orders can the choir perform the songs?

$n(orders\ of\ songs) = 5! = 120$

6) In how many ways is it possible to elect a president, a vice-president, and a secretary for a club consisting of 15 members?

$= P(15, 3)$

$$= \frac{15!}{(15-3)!}$$

$$= \frac{15!}{12!}$$

$= 15\times14\times13$

$= 2\ 730$

7) In how many ways can the letters of the word MONDAY be arranged?

$= 6!$

$= 720$

8) In how many different ways can the letters of the word MISSISSAUGA be arranged?

$$= \frac{11!}{(4!\,2!\,2!)}$$

$= 415\ 800$

9) Forty-three race cars started the 2004 Daytona 500. How many ways can the cars finish first, second, and third?

$= P(43, 3)$

$$= \frac{43!}{(43-3)!}$$

$$= \frac{43!}{40!}$$

$= 43\times42\times41$

$= 74\ 046$

10) There are 12 people entered in a swimming race. Assuming that there are no ties, in how many different ways can these people finish first, second, and third?

$= P(12, 3)$

$$= \frac{12!}{(12-3)!}$$

$$= \frac{12!}{9!}$$

$= 12 \times 11 \times 10$

$= 1\,320$

11) A landscaper wants to plant four oak trees, eight maple trees, and six poplar trees along the border of a lawn. The trees are to be spaced evenly apart. In how many distinguishable ways can they be planted?

$$= \frac{18!}{(4!\,8!\,6!)}$$

$= 9\,189\,180$

12) There are ten questions on a test.

a) In how many ways can these questions be arranged?

$= 10!$

$= 3\,628\,800$

b) In how many ways can these questions be arranged if the easiest question and the hardest question are side-by-side.

Treat the easiest question and the hardest question as a single question unit making nine questions that are to be arranged. Also, don't forget that the two questions can be arranged in 2! Ways within their unit.

$n(arrangements) = 9! \times 2! = 725\,760$

13) In how many ways can the 12 members of a volleyball team line up, if the captain and assistant captain must remain together?

Treat he captain and assistant captain as a single unit making there be 11 members to be arranged. Don't forget that the captain and assistant can be arranged in 2! ways within their unit.

$n(arrangements) = 11! \times 2! = 79\,833\,600$

14) A combination lock opens when the right combination of three numbers from 00 to 99 is entered. The same number may be used more than once.

a) What is the probability of getting the correct combination by chance?

$$P(correct) = \frac{n(correct)}{n(combinations)} = \frac{1}{100\times100\times100} = \frac{1}{1\,000\,000}$$

b) What is the probability of getting the right combination if you already know the first digit?

$$P(correct) = \frac{n(correct)}{n(combinations)} = \frac{1}{100\times100} = \frac{1}{10\,000}$$

c) How many possible combinations would there be if numbers could NOT be re-used?

$$n(combinations) = P(100, 3) = 100\times99\times98 = 970\,200$$

15) You are taking a chemistry test and are asked to list the first 10 elements of the periodic table in order as they appear in the table. You know the first 10 elements but not the order. What is the probability of you guessing the correct answer?

$$P(correct) = \frac{n(correct)}{n(orders)} = \frac{1}{10!} = \frac{1}{3\,628\,800}$$

16) Plates issued by the Motor Vehicle License Office now use four letters followed by three numbers. How many such plates are possible?

$$n(plates) = 26\times26\times26\times26\times10\times10\times10 = 456\,976\,000$$

17) Solve for n: $\frac{(n-1)!}{(n-3)!} = 20$

$$\frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 20$$

$$n^2 - 3n + 2 - 20 = 0$$

$$n^2 - 3n - 18 = 0$$

$$(n-6)(n+3) = 0$$

$$n = 6 \qquad \cancel{n = -3}$$

Section 4.7 Worksheet – Combinations

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1) Evaluate each of the following

a) C(8, 3)

$$=\frac{8!}{(8-3)!\,3!}=\frac{8!}{5!\,3!}=56$$

b) $_7C_4$

$$=\frac{7!}{(7-4)!\,4!}=\frac{7!}{3!\,4!}=35$$

c) $\binom{12}{11}$

$$=\frac{12!}{(12-11)!\,11!}=\frac{12!}{1!\,11!}=12$$

d) C(10, 3)

$$=\frac{10!}{(10-3)!\,3!}=\frac{10!}{7!\,3!}=120$$

2) In how many ways can a team of six female volleyball players be chosen to start the game from a roster of 12 players?

$$n(starting\ lineups)=\binom{12}{6}=924$$

3) In the card game Crazy Eights, how many different eight-card hands can be dealt from a standard 52-card deck?

$$n(hands)=\binom{52}{8}=752\ 538\ 150$$

4) From a group of 40 people, a jury of 12 people is selected. In how many different ways can a jury of 12 people be selected?

$$n(juries)=\binom{40}{12}=5\ 586\ 853\ 480$$

5) There are 15 qualified applicants for 5 trainee positions in a fast-food management program. How many different groups of trainees can be selected?

$$n(groups)=\binom{15}{5}=3\ 003$$

6) A pizza shop offers nine toppings. No topping is used more than once. In how many different ways can a three-topping pizza be formed?

$$n(3\ topping\ pizzas) = \binom{9}{3} = 84$$

7) Ursula runs a small landscaping business. She has on hand 8 kinds of rose bushes, 10 kinds of small shrubs, 5 kinds of evergreen seedlings, and 7 kinds of flower lilies. In how many ways can Ursula fill an order if a customer wants 8 different varieties consisting of 3 roses, 3 shrubs, and 2 lilies?

$$n(3\ roses, 3\ shrubs, 2\ lilies) = \binom{8}{3}\times\binom{10}{3}\times\binom{7}{2} = 56\times120\times21 = 141\ 120$$

8) From a group of five men and four women, determine how many committees of five people can be formed with

a) no restrictions

$$n(committees) = \binom{9}{5} = 126$$

b) exactly three women

$$n(3\ women, 2\ men) = \binom{4}{3}\times\binom{5}{2} = 4\times10 = 40$$

c) exactly four men

$$n(1\ woman, 4\ men) = \binom{4}{1}\times\binom{5}{4} = 4\times5 = 20$$

d) no women

$$n(5\ men) = \binom{5}{5} = 1$$

e) at least two men

$$n(\geq 2\ men) = 126 - n(1\ man, 4\ women)$$
$$= 126 - \binom{5}{1}\binom{4}{4}$$
$$= 126 - 5$$
$$= 121$$

f) at least three women

$$n(\geq 3\ women) = n(3\ women, 2\ men) + n(4\ women, 1\ man)$$
$$= \binom{4}{3}\binom{5}{2} + \binom{4}{4}\binom{5}{1}$$
$$= 40 + 5$$
$$= 45$$

9) One professor grades homework by randomly choosing 5 out of 12 homework problems to grade.

a) How many different groups of 5 problems can be chosen from the 12 problems?

$$n(groups\ of\ problems) = \binom{12}{5} = 792$$

b) Jerry did only 5 problems of one assignment. What is the probability that the problems he did comprised the group that was selected to be graded?

$$P(right\ group) = \frac{1}{792}$$

c) Silvia did 7 problems. How many different groups of 5 did she complete? What is the probability that one of the groups of 5 she completed comprised the group selected to be graded?

$$P(right\ group) = \frac{\binom{7}{5}}{\binom{12}{5}} = \frac{21}{792} = \frac{7}{264}$$

10) The qualified applicant pool for six management trainee positions consists of seven women and five men.

a) How many different groups of applicants can be selected for the positions?

$$n(groups) = \binom{12}{6} = 924$$

b) How many different groups of trainees would consist entirely of women?

$$n(groups\ with\ only\ women) = \binom{7}{6} = 7$$

c) If the positions are selected at random, what is the probability that the trainee class will consist entirely of women?

$$P(only\ women) = \frac{7}{924} = \frac{1}{132}$$

11) Find the probability of being dealt five diamonds from a standard deck of playing cards.

$$P(5\ diamonds) = \frac{n(5\ diamonds)}{n(5\ card\ hands)} = \frac{\binom{13}{5}}{\binom{52}{5}} = \frac{1\ 287}{2\ 598\ 960}$$

12) Three cards are selected at random from a standard deck of 52 playing cards. Determine the probability that all three cards are

a) hearts

$$n(3\ hearts) = \frac{\binom{13}{3}}{\binom{52}{3}} = \frac{286}{22\ 100} = \frac{11}{850}$$

b) black

$$n(3\ black) = \frac{\binom{26}{3}}{\binom{52}{3}} = \frac{2\ 600}{22\ 100} = \frac{2}{17}$$

c) aces

$$n(3\ aces) = \frac{\binom{4}{3}}{\binom{52}{3}} = \frac{4}{22\ 100} = \frac{1}{5\ 525}$$

d) face cards

$$n(3\ face\ cards) = \frac{\binom{12}{3}}{\binom{52}{3}} = \frac{220}{22\ 100} = \frac{11}{1\ 105}$$

13) A paper bag contains a mixture of three types of candy. There are ten gum balls, seven candy bars, and three packages of toffee. Suppose a game is played in which a candy is randomly taken from the bag and then a second candy is drawn from the bag, without replacement. You are allowed to keep both candies, if, and only if, the second is the same type as the first.

a) Calculate the probability that you will be able to keep a gum ball on the first try.

$$P(win\ gum) = \frac{\binom{10}{2}}{\binom{20}{2}} = \frac{45}{190} = \frac{9}{38} \qquad \text{OR} \qquad P(win\ gum) = \frac{10}{20} \times \frac{9}{19} = \frac{90}{380} = \frac{9}{38}$$

b) Calculate the probability that you will be able to keep any candy on the first try.

$$P(win\ any\ candy) = \frac{\binom{10}{2}}{\binom{20}{2}} + \frac{\binom{7}{2}}{\binom{20}{2}} + \frac{\binom{3}{2}}{\binom{20}{2}} = \frac{45}{190} + \frac{21}{190} + \frac{3}{190} = \frac{69}{190}$$

c) Calculate the probability that you will not be able to keep any candy on the first try.

$$P(lose) = 1 - P(win\ any) = 1 - \frac{69}{190} = \frac{121}{190}$$

14) Melik has five quarters and six dimes in his pocket. He pulls out one coin.

a) What are the odds of the coin being a quarter?

5:6

b) What are the odds of the coin being a dime?

6:5

15) Suppose the probability of rain tomorrow is 80%. What are the odds of rain tomorrow?

80:20 = 4:1

16) The coach says that the probability of winning the next game is 40%. What are the odds the team will win?

40:60 = 2:3