Chapter 5 Exam Review – Probability Distributions

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Section 5.1 – Probability Distributions

1) What must be the value of $P(4)$ if this is a valid probability distribution? Why?

The probabilities of a valid probability distribution must add to 1. Therefore $P(4) = 0.35$

X	$P(X)$
0	0.1
1	0.2
2	0.05
3	0.2
4	**0.35**
5	0.1

2) Use the given frequency distribution to…

a) create a probability distribution for n, the number of dogs per household in a small town.

Dogs	*Households*
0	1500
1	430
2	175
3	52
4	16

n	$P(n)$
0	$\frac{1500}{2173}$ **OR 0.6902899218**
1	$\frac{430}{2173}$ **OR 0.1978831109**
2	$\frac{175}{2173}$ **OR 0.0805338242**
3	$\frac{52}{2173}$ **OR 0.0239300506**
4	$\frac{16}{2173}$ **OR 0.0073630925**

b) Determine the expected number of dogs in a home in the small town?

$E(n) = 0\left(\frac{1500}{2173}\right) + 1\left(\frac{430}{2173}\right) + 2\left(\frac{175}{2173}\right) + 3\left(\frac{52}{2173}\right) + 4\left(\frac{16}{2173}\right) = \frac{1000}{2173}$ **OR 0.4601932812**

Section 5.2 – Hypergeometric Probability Distributions

3) The door prizes at a dance are gift certificates from local merchants. There are four $10 certificates, five $20 certificates, and three $50 certificates. The prize envelopes are mixed together in a bag and are drawn at random.

a) Create a probability distribution for the number of $50 prizes drawn, n, on the first three draws.

# of $50 prizes drawn ($n$)	$P(n)$
0	$\frac{\binom{3}{0}\binom{9}{3}}{\binom{12}{3}} = \frac{84}{220} = \frac{21}{55}$ OR 0.3818181818
1	$\frac{\binom{3}{1}\binom{9}{2}}{\binom{12}{3}} = \frac{108}{220} = \frac{27}{55}$ OR 0.490909090909
2	$\frac{\binom{3}{2}\binom{9}{1}}{\binom{12}{3}} = \frac{27}{220}$ OR 0.122727272727
3	$\frac{\binom{3}{3}\binom{9}{0}}{\binom{12}{3}} = \frac{1}{220}$ OR 0.004545454545

b) What is the expected number of $50 certificates among the first three prizes drawn?

$E(n) = r\left(\frac{a}{n}\right) = 3\left(\frac{3}{12}\right) = \frac{9}{12} = 0.75$

Note: you could also calculate the expected value using $\sum n \cdot P(n)$

c) What is the probability that at least 1 $50 prize is drawn in the first three draws?

$P(n \geq 1) = 1 - P(0) = 1 - \frac{21}{55} = \frac{34}{55}$ OR 0.618181818

Section 5.3 – Binomial Distributions

4) A family plans on having four children. Assuming the probability of having a boy is equal to the probability of having a girl…

a) Create a probability distribution for the number of boys, X, the family will have

# of boys (X)	$\boldsymbol{P(X)}$
0	$binompdf(4,0.5,0) = 0.0625$
1	$binompdf(4,0.5,1) = 0.25$
2	$binompdf(4,0.5,2) = 0.375$
3	$binompdf(4,0.5,3) = 0.25$
4	$binompdf(4,0.5,4) = 0.0625$

b) Find the expected number of boys in a family with four children

$E(X) = np = 4(0.5) = 2$

Note: you could also calculate the expected value using $\sum X \cdot P(X)$

5) A basketball player has a shooting percentage of 0.450

a) Create a probability distribution table for the number of baskets made in a quarter where he takes 4 shots.

Number of Baskets Made (X)	**P(X)**
0	$binompdf(4{,}0.45{,}0) = 0.09150625$
1	$binompdf(4{,}0.45{,}1) = 0.299475$
2	$binompdf(4{,}0.45{,}2) = 0.3675375$
3	$binompdf(4{,}0.45{,}3) = 0.200475$
4	$binompdf(4{,}0.45{,}4) = 0.04100625$

b) What is the expected number of baskets made in the quarter?

$E(X) = np = 4(0.45) = 1.8$

Note: you could also calculate the expected value using $\sum X \cdot P(X)$

6) The Choco-Latie Candies company makes candy-coated chocolates, 40% of which are red. The production line mixes the candies randomly and packages ten per box.

a) Calculate the probability that exactly 5 of the candies in a box are red.

$P(X = 5) = binompdf(10{,}0.4{,}5) = 0.2006581248$

b) Calculate the probability that fewer than 5 in a box are red.

$P(X < 5) = P(X \leq 4) = binomcdf(10{,}0.4{,}4) = 0.6331032576$

c) Calculate the probability that at least 3 of the candies in a box are red.

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - binomcdf(10{,}0.4{,}2) = 0.8327102464$

7) A certain type of rocket has a failure rate of 1.5%

a) Calculate the probability of there being exactly 1 failure in 100 launches. (answer to 6 decimal places)

$P(X = 1) = binompdf(100, 0.015,1) = 0.335953$

b) Calculate the probability that there are more than 4 failures in 100 launches (answer to 6 decimal places)

$P(X > 4) = 1 - P(X \leq 4) = 1 - binomcdf(100,0.015,4) = 0.017693$

c) What is the expected number of failures in 100 launches of the rocket?

$E(X) = np = 100(0.015) = 1.5$

8) Suppose that 65% of the families in a town own computers. If eight families are surveyed at random…

a) What is the probability exactly 3 own a computer?

$P(X = 3) = binompdf(8,0.65,3) = 0.0807733916$

b) What is the probability that all 8 own a computer?

$P(X = 8) = binompdf(8,0.65,8) = 0.0318644813$

c) What is the probability that 6 or fewer families own a computer

$P(X \leq 6) = binomcdf(8,0.65,6) = 0.8308731378$

d) What is the expected number of families that will own a computer.

$E(X) = np = 8(0.65) = 5.2$

9) A recent survey of a gas-station's customers showed that 68% paid with credit cards, 29% used debit cards, and only 3% paid with cash. During her eight-hour shift as cashier at this gas station, Serena had a total of 223 customer. What is the probability that…

a) at least 142 customers used a credit card?

$P(X \geq 142) = 1 - P(X \leq 141) = 1 - binomcdf(223{,}0.68{,}141) = 0.9260539149$

b) fewer than 220 customers paid with credit or debit cards

$P(X < 220) = P(X \leq 219) = binomcdf(223{,}0.97{,}219) = 0.9040253448$

<u>Section 5.4 – Geometric Distributions</u>

10) From experience, you know that the probability that you will make a sale on any given telephone call is 0.23. Find the probability…

a) On any given day, your first sale won't be until your 5th call.

$P(Y = 5) = geometpdf(0.23{,}5) = 0.0808519943$

b) It takes less than 4 calls to make a sale

$P(Y < 4) = P(Y \leq 3) = geometcdf(0.23{,}3) = 0.543467$

c) It takes more than 10 calls to make a sale

$P(Y > 10) = 1 - P(Y \leq 10) = 1 - geometcdf(0.23{,}10) = 0.0732668047$

11) Basketball player Shaquille O'Neal makes a free throw shot about 54% of the time. Find the probability…

a) The first free throw he makes is his 3rd free throw attempt

$P(Y = 3) = geometpdf(0.54{,}3) = 0.114264$

b) It takes him more than 5 attempts to make his first free throw

$P(Y > 5) = 1 - P(Y \leq 5) = 1 - geometcdf(0.54{,}5) = 0.0205962976$

12) A cereal maker places a game piece in its cereal boxes. The probability of winning a prize in the game is 1 in 4. Find the probability that…

a) You win your first prize with your fourth purchase

$P(Y = 4) = geometpdf(0.25,4) = 0.10546875$

b) It takes you fewer than 3 purchases to win a prize

$P(Y < 3) = P(Y \leq 2) = geometcdf(0.25,2) = 0.4375$

Section 5.5 – Binomial Theorem

13) Find the binomial expansion of each expression in simplified form using the binomial theorem.

a) $(2x + 3)^4$

$= \binom{4}{0}(2x)^4(3)^0 + \binom{4}{1}(2x)^3(3)^1 + \binom{4}{2}(2x)^2(3)^2 + \binom{4}{3}(2x)^1(3)^3 + \binom{4}{4}(2x)^0(3)^4$

$= (1)(16)(x^4)(1) + (4)(8)(x^3)(3) + (6)(4)(x^2)(9) + (4)(2)(x)(27) + (1)(1)(81)$

$= 16x^4 + 96x^3 + 216x^2 + 216x + 81$

b) $(2x - 1)^4$

$= \binom{4}{0}(2x)^4(-1)^0 + \binom{4}{1}(2x)^3(-1)^1 + \binom{4}{2}(2x)^2(-1)^2 + \binom{4}{3}(2x)^1(-1)^3 + \binom{4}{4}(2x)^0(-1)^4$

$= (1)(16)(x^4)(1) + (4)(8)(x^3)(-1) + (6)(4)(x^2)(1) + (4)(2)(x)(-1) + (1)(1)(1)$

$= 16x^4 - 32x^3 + 24x^2 - 8x + 1$

c) $(3x - 2y)^5$

$= \binom{5}{0}(3x)^5(-2y)^0 + \binom{5}{1}(3x)^4(-2y)^1 + \binom{5}{2}(3x)^3(-2y)^2 + \binom{5}{3}(3x)^2(-2y)^3 + \binom{5}{4}(3x)^1(-2y)^4 + \binom{5}{5}(3x)^0(-2y)^5$

$= (1)(243)(x^5)(1) + (5)(81)(x^4)(-2)(y) + 10(27)(x^3)(4)(y^2) + 10(9)(x^2)(-8)(y^3) + 5(3)(x)(16)(y^4) + 1(1)(-32)(y^5)$

$= 243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$

14) In the expansion of $\left(x^3 + \frac{2}{x}\right)^8$ find:

a) The number of terms

$= 9$

b) The general term

$t_{r+1} = \binom{8}{r}(x^3)^{8-r}\left(\frac{2}{x}\right)^r$

$t_{r+1} = \binom{8}{r}(x^{24-3r})(2^r)(x^{-r})$

$t_{r+1} = \binom{8}{r}(2^r)(x^{24-4r})$

c) The third term

$t_{r+1} = \binom{8}{r}(2^r)(x^{24-4r})$

$t_{2+1} = \binom{8}{2}(2^2)\left(x^{24-4(2)}\right)$

$t_3 = 28(4)(x^{24-8})$

$t_3 = 112x^{16}$

d) The constant term

$t_{r+1} = \binom{8}{r}(2^r)(x^{24-4r})$

$t_{6+1} = \binom{8}{6}(2^6)\left(x^{24-4(6)}\right)$

$t_7 = 28(64)x^0$

$t_7 = 1792$

15) Marshall is walking from his house to school. His route from home always takes him 4 blocks west and 9 blocks south to school but he likes to vary the path he takes.

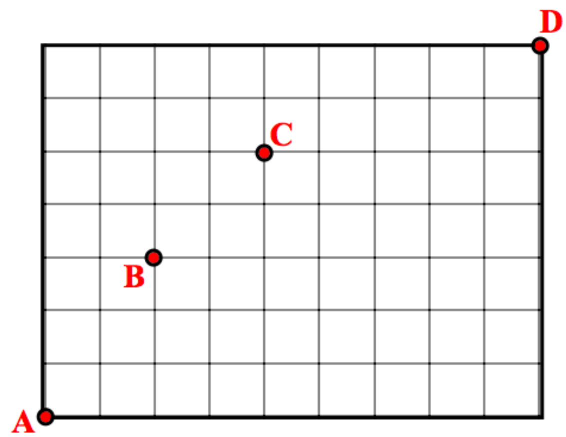
a) How many different routes can he take?

$n(routes) = \binom{13}{4} = 715$

b) If Marshall needs to stop for a coffee at the Tim Horton's that is 2 blocks west and three blocks south of his house, how many routes pass by this store?

$n(routes) = \binom{5}{2}\binom{8}{2} = 10(28) = 280$

16) The grid below shows the streets in your neighbourhood.



a) How many different routes are there to get from A to D?

$$n(routes) = \binom{16}{7} = 11440$$

b) How many different routes from A to D pass by the point B on the way?

$$n(routes) = \binom{5}{2}\binom{11}{4} = 10(330) = 3300$$

c) What is the probability that you pass C on your way from A to D?

$$P(pass\ C) = \frac{\binom{9}{5}\binom{7}{2}}{11440} = \frac{2646}{11440} = \frac{1323}{5720} \quad \text{OR} \quad 0.2312937063$$