Chapter 5 Review – Probability Distributions
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Section 5.1 – Probability Distributions

1) The manager of "Fiona's Fine Foods" conducted a study to analyze the length of time her employees spent engaged in a grocery sale per customer. The results are shown in the table below, where time has been rounded to the nearest minute.

Time (min)	1	2	3	4	5	6	7	8	9	10
Frequency	4	7	11	22	38	26	19	15	8	3

a) Create a probability distribution for these data.

Time per customer (X)	$P(x)$
1	$\frac{4}{153}$ or 0.0261437908
2	$\frac{7}{153}$ or 0.045751634
3	$\frac{11}{153}$ or 0.0718954248
4	$\frac{22}{153}$ or 0.1437908497
5	$\frac{38}{153}$ or 0.2483660131
6	$\frac{26}{153}$ or 0.1699346405
7	$\frac{19}{153}$ or 0.1241830065
8	$\frac{15}{153} = \frac{5}{51}$ or 0.0980392157
9	$\frac{8}{153}$ or 0.0522875817
10	$\frac{3}{153} = \frac{1}{51}$ or 0.0196078431

b) Determine the expected length of a typical sale.

$$E(X) = 1\left(\frac{4}{153}\right) + 2\left(\frac{7}{153}\right) + 3\left(\frac{11}{153}\right) + 4\left(\frac{22}{153}\right) + 5\left(\frac{38}{153}\right) + 6\left(\frac{26}{153}\right) + 7\left(\frac{19}{153}\right) + 8\left(\frac{15}{153}\right) + 9\left(\frac{8}{153}\right) + 10\left(\frac{3}{153}\right)$$

$$E(X) = \frac{840}{153}$$

$$E(X) \cong 5.49$$

2) A school is having a fundraising lottery. A total of 2000 tickets are available at a price of $25 each for these prizes:

Prize	$10 000	$500	$25
Number	1	5	494

a) Create a probability distribution for the amount of money you could win if you play this lottery.

Payout (X)	$P(X)$
$10 000	$\frac{1}{2000}$
$500	$\frac{5}{2000} = \frac{1}{400}$
$25	$\frac{494}{2000} = \frac{247}{1000}$
$0	$\frac{1500}{2000} = \frac{3}{4}$

b) Determine the expected value of the lottery

$$E(X) = 10000\left(\frac{1}{2000}\right) + 500\left(\frac{5}{2000}\right) + 25\left(\frac{494}{2000}\right) + 0\left(\frac{1500}{2000}\right)$$

$$E(X) = \frac{24\,850}{2\,000}$$

$$E(X) = 12.425$$

c) What is your expected profit or loss if you buy one ticket?

$$Expected\ profit = expected\ value - cost\ of\ ticket = 12.425 - 25 = -12.575$$

You would expect to lost about $12.58

3) The table below shows the probability distribution of the number of languages spoken by a randomly selected high school student.

Languages	1	2	3	4	5
Probability	0.630	0.295	0.065	0.008	0.002

a) Is this a valid probability distribution? Give evidence.

$0.63 + 0.295 + 0.065 + 0.008 + 0.002 = 1$

The probabilities add to 1, therefore this is a valid probability distribution.

b) Calculate the expected number of languages spoken.

$E(languages) = 1(0.63) + 2(0.295) + 3(0.065) + 4(0.008) + 5(0.002) = 1.457$

Section 5.2 – Hypergeometric Probability Distributions

4) A basket of 12 apples has 5 bad apples in it. Four apples are drawn from the basket.

a) Find the probability distribution for *n*, the number of bad apples drawn from the basket.

# of bad apples (n)	$P(n)$
0	$\frac{\binom{5}{0}\binom{7}{4}}{\binom{12}{4}} = \frac{35}{495} = \frac{7}{99}$ or 0.0707070707
1	$\frac{\binom{5}{1}\binom{7}{3}}{\binom{12}{4}} = \frac{175}{495} = \frac{35}{99}$ or 0.3535353535
2	$\frac{\binom{5}{2}\binom{7}{2}}{\binom{12}{4}} = \frac{210}{495} = \frac{14}{33}$ or 0.4242424242
3	$\frac{\binom{5}{3}\binom{7}{1}}{\binom{12}{4}} = \frac{70}{495} = \frac{14}{99}$ or 0.1414141414
4	$\frac{\binom{5}{4}\binom{7}{0}}{\binom{12}{4}} = \frac{5}{495} = \frac{1}{99}$ or 0.0101010101

b) Compute the expected number of bad apples drawn.

$$E(n) = 0\left(\frac{35}{495}\right) + 1\left(\frac{175}{495}\right) + 2\left(\frac{210}{495}\right) + 3\left(\frac{70}{495}\right) + 4\left(\frac{5}{495}\right)$$

$$E(n) = \frac{825}{495}$$

$$E(n) = 1.666667$$

OR

$$E(n) = 4\left(\frac{5}{12}\right) = \frac{20}{12} = 1.666667$$

5) 5 cards are randomly dealt from a standard deck of cards.

a) Show the probability distribution for x, the number of cards dealt that are face cards.

# face cards (x)	$P(x)$
0	$\frac{\binom{12}{0}\binom{40}{5}}{\binom{52}{5}} = \frac{658\,008}{2\,598\,960}$ OR 0.2531812725
1	$\frac{\binom{12}{1}\binom{40}{4}}{\binom{52}{5}} = \frac{1\,096\,680}{2\,598\,960}$ OR 0.42319687875
2	$\frac{\binom{12}{2}\binom{40}{3}}{\binom{52}{5}} = \frac{652\,080}{2\,598\,960}$ OR 0.2509003601
3	$\frac{\binom{12}{3}\binom{40}{2}}{\binom{52}{5}} = \frac{171\,600}{2\,598\,960}$ OR 0.0660264106
4	$\frac{\binom{12}{4}\binom{40}{1}}{\binom{52}{5}} = \frac{19\,800}{2\,598\,960}$ OR 0.007618432
5	$\frac{\binom{12}{5}\binom{40}{0}}{\binom{52}{5}} = \frac{792}{2\,598\,960}$ OR 0.000304737

b) Calculate the expected number of face cards in a 4 card hand.

$$E(X) = 0\left(\frac{658\,008}{2\,598\,960}\right) + 1\left(\frac{1\,096\,680}{2\,598\,960}\right) + 2\left(\frac{652\,080}{2\,598\,960}\right) + 3\left(\frac{171\,600}{2\,598\,960}\right) + 4\left(\frac{19\,800}{2\,598\,960}\right) + 5\left(\frac{792}{2\,598\,960}\right)$$

$$E(X) = \frac{2\,998\,800}{2\,598\,960}$$

$$E(X) = 1.153846154$$

OR

$$E(X) = 5\left(\frac{12}{52}\right) = \frac{60}{52} = 1.153846154$$

c) What is the probability that you are dealt at least one face card?

$$P(X \geq 1) = 1 - P(0) = 1 - \frac{658\,008}{2\,598\,960} = \frac{1\,940\,952}{2\,598\,960} = 0.7468187275$$

6) In a 100-meter track race there are 16 competitors. 5 of the 16 competitors are from King's. The 16 competitors will be divided evenly in to four different heats by random draw.

a) Show the probability distribution for x, the number of students from King's who are running in the first heat.

# King's Runners (x)	$P(x)$
0	$\frac{\binom{5}{0}\binom{11}{4}}{\binom{16}{4}} = \frac{330}{1820}$ OR 0.1813186813
1	$\frac{\binom{5}{1}\binom{11}{3}}{\binom{16}{4}} = \frac{825}{1820}$ OR 0.4532967033
2	$\frac{\binom{5}{2}\binom{11}{2}}{\binom{16}{4}} = \frac{550}{1820}$ OR 0.3021978022
3	$\frac{\binom{5}{3}\binom{11}{1}}{\binom{16}{4}} = \frac{110}{1820}$ OR 0.0604395604
4	$\frac{\binom{5}{4}\binom{11}{0}}{\binom{16}{4}} = \frac{5}{1820}$ OR 0.0027472527

b) Calculate the expected number of King's runners in the first heat.

$$E(X) = 0\left(\frac{330}{1820}\right) + 1\left(\frac{825}{1820}\right) + 2\left(\frac{550}{1820}\right) + 3\left(\frac{110}{1820}\right) + 4\left(\frac{5}{1820}\right)$$

$$E(X) = \frac{2275}{1820}$$

$$E(X) = 1.25$$

OR

$$E(X) = 4\left(\frac{5}{16}\right) = \frac{20}{16} = 1.25$$

c) What is the probability that there will be more than 1 King's student in the first heat?

$$P(X > 1) = P(2) + P(3) + P(4) = \frac{550}{1820} + \frac{110}{1820} + \frac{5}{1820} = \frac{665}{1820} = 0.3653846154$$

Section 5.3 – Binomial Distributions

7) If a batter has a batting average of 0.320,

a) Create a probability distribution table for the number of hits per game if you assuming they get three chances to hit/bat.

Number of Hits (X)	P(X)
0	$\binom{3}{0}(0.32)^0(0.68)^3$ OR $binompdf(3, 0.32, 0) = 0.314432$
1	$\binom{3}{1}(0.32)^1(0.68)^2$ OR $binompdf(3, 0.32, 1) = 0.443904$
2	$\binom{3}{2}(0.32)^2(0.68)^1$ OR $binompdf(3, 0.32, 2) = 0.208896$
3	$\binom{3}{3}(0.32)^3(0.68)^0$ OR $binompdf(3, 0.32, 3) = 0.032768$

b) What is the expected number of hits in a game?

$$E(X) = 0(0.314432) + 1(0.443904) + 2(0.208896) + 3(0.032768) = 0.96$$

$$E(X) = 3(0.32) = 0.96$$

8) Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours?

$$P(X = 2) = binompdf\left(5, \frac{1}{6}, 2\right) = 0.1607510288$$

9) The Smiths hope to have four children. Assume that the probability of a boy being born is the same as the probability for a girl to be born.

a) Determine the probability that all four of the children will be boys.

$$P(X = 4) = binompdf(4, 0.5, 4) = 0.0625$$

b) Determine the probability that at least two of the children will be girls.

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - binomcdf(4, 0.5, 1) = 1 - 0.3125 = 0.6875$$

10) There are 10 questions on a multiple-choice quiz. Each question has four possible answers. Suppose Pitu knows the answer to six of the questions and guesses the answers to the other four. What is the probability of these events?

a) Pitu gets at least eight questions right

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - binomcdf(4, 0.25, 1) = 1 - 0.73828125 = 0.26171875$

b) Pitu gets all 10 questions right.

$P(X = 4) = binompdf(4, 0.25, 4) = 0.00390625$

11) Cal's Coffee prints prize coupons under the rim of 20% of its paper cups. If you buy ten cups of coffee…

a) What is the probability that you would win at least seven prizes?

$P(X \geq 7) = 1 - P(X \leq 6) = 1 - binomcdf(10, 0.2, 6) = 1 - 0.9991356416 = 0.0008643584$

b) What is your expected number of prizes?

$E(X) = np = 10(0.2) = 2$

12) A factory produces computer chips with a 0.9% defect rate. In a batch of 100 computer chips, what is the probability that…

a) exactly 1 is defective

$P(X = 1) = binompdf(100, 0.009, 1) = 0.3677344383$

b) at least 3 are defective

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - binomcdf(100, 0.009, 2) = 1 - 0.9379644283 = 0.0620355717$

c) less than 5 are defective

$P(X < 5) = P(X \leq 4) = binomcdf(100, 0.009, 4) = 0.9978089603$

Section 5.4 – Geometric Distributions

13) In the board game Monopoly, one way to get out of jail is to roll doubles. Suppose that this was the only way a player could get out of jail. The random variable of interest is $Y =$ number of attempts it takes to roll doubles one time.

a) Find the probability that it takes 3 turns to roll doubles.

$$P(Y = 3) = geometpdf\left(\frac{1}{6}, 3\right) = 0.1157407407$$

b) Find the probability that it takes more than 3 turns to roll doubles.

$$P(Y > 3) = 1 - P(Y \leq 3) = 1 - geometcdf\left(\frac{1}{6}, 3\right) = 1 - 0.4212962963 = 0.5787037037$$

c) How many times would you expect to have to roll the dice to get doubles once?

$$E(Y) = \frac{1}{p} = \frac{1}{\left(\frac{1}{6}\right)} = 6$$

14) Babe Ruth had a career batting average of 0.342, quite impressive for a home run hitter. The random variable of interest is the number of at bats it takes for him to get a hit. Calculate the probability that…

a) It takes him 5 at bats

$$P(Y = 5) = geometpdf(0.342, 5) = 0.0641105763$$

b) It takes him less than 4 at bats

$$P(Y < 4) = P(Y \leq 3) = geometcdf(0.342, 3) = 0.715109688$$

15) A factory making printed-circuit boards has a defect rate of 2.4% on one of its production lines. An inspector tests randomly selected circuit boards from this production line.

a) What is the probability that the first defective circuit board will be the sixth one tested?

$$P(Y = 6) = geometpdf(0.024, 6) = 0.0212549619$$

b) What is the probability that the first defective circuit board will be among the first six tested?

$$P(Y \leq 6) = geometcdf(0.024, 6) = 0.1356315509$$

c) What is the expected number of circuit boards that would have to be tested until the inspector finds a defective one?

$$E(Y) = \frac{1}{p} = \frac{1}{0.024} = 41.67$$

Section 5.5 – Binomial Theorem

16) Use Pascal's Identity to write an expression that is equivalent to each of the following.

a) $\binom{10}{3} + \binom{10}{4} = \binom{11}{4}$ **b)** $\binom{17}{13} + \binom{17}{12} + \binom{18}{14} = \binom{19}{4}$ **c)** $\binom{30}{r+5} + \binom{30}{r+6} = \binom{31}{r+6}$

17) Find the binomial expansion of each expression in simplified form.

a) $(1 + x)^3$

$= \binom{3}{0}(1)^3(x)^0 + \binom{3}{1}(1)^2(x)^1 + \binom{3}{2}(1)^1(x)^2 + \binom{3}{3}(1)^0(x)^3$

$= (1)(1)(1) + 3(1)(x) + 3(1)(x^2) + 1(1)(x^3)$

$= 1 + 3x + 3x^2 + x^3$

b) $(2 + y)^6$

$= \binom{6}{0}(2)^6(y)^0 + \binom{6}{1}(2)^5(y)^1 + \binom{6}{2}(2)^4(y)^2 + \binom{6}{3}(2)^3(y)^3 + \binom{6}{4}(2)^2(y)^4 + \binom{6}{5}(2)^1(y)^5 + \binom{6}{6}(2)^0(y)^6$

$= (1)(64)(1) + 6(32)(y) + 15(16)(y^2) + 20(8)(y^3) + 15(4)(y^4) + 6(2)(y^5) + 1(1)(y^6)$

$= 64 + 192y + 240y^2 + 160y^3 + 60y^4 + 12y^5 + y^6$

c) $(1 - 2x)^5$

$= \binom{5}{0}(1)^5(-2x)^0 + \binom{5}{1}(1)^4(-2x)^1 + \binom{5}{2}(1)^3(-2x)^2 + \binom{5}{3}(1)^2(-2x)^3 + \binom{5}{4}(1)^1(-2x)^4 + \binom{5}{5}(1)^0(-2x)^5$

$= (1)(1)(1) + 5(1)(-2)(x) + 10(1)(4)(x^2) + 10(1)(-8)(x^3) + 5(1)(16)(x^4) + 1(1)(-32)(x^5)$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

d) $(2x^2 - x)^4$

$= \binom{4}{0}(2x^2)^4(-1x)^0 + \binom{4}{1}(2x^2)^3(-1x)^1 + \binom{4}{2}(2x^2)^2(-1x)^2 + \binom{4}{3}(2x^2)^1(-1x)^3 + \binom{4}{4}(2x^2)^0(-1x)^4$

$= 1(16)(x^8)(1) + 4(8)(x^6)(-1)(x) + 6(4)(x^4)(1)(x^2) + 4(2)(x^2)(-1)(x^3) + 1(1)(1)(x^4)$

$= 16x^8 - 32x^7 + 24x^6 - 8x^5 + x^4$

18) Expand completely

a) $\left(1 - \frac{1}{x^2}\right)^4$

$= \binom{4}{0}(1)^4\left(\frac{-1}{x^2}\right)^0 + \binom{4}{1}(1)^3\left(\frac{-1}{x^2}\right)^1 + \binom{4}{2}(1)^2\left(\frac{-1}{x^2}\right)^2 + \binom{4}{3}(1)^1\left(\frac{-1}{x^2}\right)^3 + \binom{4}{4}(1)^0\left(\frac{-1}{x^2}\right)^4$

$= 1(1)(1) + 4(1)\left(\frac{-1}{x^2}\right) + 6(1)\left(\frac{1}{x^4}\right) + 4(1)\left(\frac{-1}{x^6}\right) + 1(1)\left(\frac{1}{x^8}\right)$

$= 1 - \frac{4}{x^2} + \frac{6}{x^4} - \frac{4}{x^6} + \frac{1}{x^8}$

b) $\left(x - \frac{1}{x}\right)^2$

$= \binom{2}{0}(x)^2\left(\frac{-1}{x}\right)^0 + \binom{2}{1}(x)^1\left(\frac{-1}{x}\right)^1 + \binom{2}{2}(x)^0\left(\frac{-1}{x}\right)^2$

$= (1)(x^2)(1) + 2(x)\left(\frac{-1}{x}\right) + 1(1)\left(\frac{1}{x^2}\right)$

$= x^2 - 2 + \frac{1}{x^2}$

c) $\left(\frac{t}{3} + 6v^2\right)^3$

$= \binom{3}{0}\left(\frac{t}{3}\right)^3(6v^2)^0 + \binom{3}{1}\left(\frac{t}{3}\right)^2(6v^2)^1 + \binom{3}{2}\left(\frac{t}{3}\right)^1(6v^2)^2 + \binom{3}{3}\left(\frac{t}{3}\right)^0(6v^2)^3$

$= 1\left(\frac{t^3}{27}\right)(1) + 3\left(\frac{t^2}{9}\right)(6)(v^2) + 3\left(\frac{t}{3}\right)(36)(v^4) + 1(1)(216)(v^6)$

$= \frac{t^3}{27} + 2t^2v^2 + 36tv^4 + 216v^6$

19) State the fourth and tenth term in the expansion of $(a - b)^{50}$

$t_{r+1} = \binom{n}{r}(a)^{n-r}(b)^r$

$t_{3+1} = \binom{50}{3}(a)^{50-3}(-1b)^3$

$t_4 = -19600a^{47}b^3$

$t_{9+1} = \binom{50}{9}(a)^{50-9}(-1b)^9$

$t_{10} = -2\ 505\ 433\ 700a^{41}b^9$

20) In the expansion of $\left(x^4 - \frac{2}{x}\right)^{10}$ find:

a) The number of terms

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b) The 5th term

General term:

$t_{r+1} = \binom{10}{r}(x^4)^{10-r}\left(\frac{-2}{x}\right)^r$

$t_{r+1} = \binom{10}{r}(-2)^r(x^{40-4r})(x^{-r})$

$t_{r+1} = \binom{10}{r}(-2)^r(x^{40-5r})$

5th term:

$t_{4+1} = \binom{10}{4}(-2)^4(x^{40-5(4)})$

$t_5 = 3360x^{20}$

c) The constant term

$t_{8+1} = \binom{10}{8}(-2)^8(x^{40-5(8)})$

$t_9 = 11\ 520$

d) The term containing x^{30}

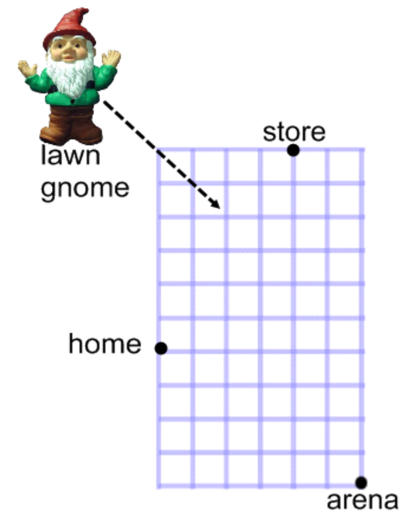
$t_{2+1} = \binom{10}{2}(-2)^2(x^{40-5(2)})$

$t_3 = 180x^{30}$

21) To walk from school to the local hamburger shack Inez must go eight blocks east and five blocks south of the school. If she walks east and south only, how many different routes can she follow from the school to the hamburger shack?

$n(routes) = \binom{13}{8} = 1287$

22) From home you are heading to the store to buy skate laces and sharpen your skates. You are then heading to the arena for practice, and after practice heading home. If you can only ever move towards your destination…



a) How many possible routes are there from your house to the store?

$\binom{10}{6} = 210$

b) How many possible routes are there from the store to the arena?

$\binom{12}{2} = 66$

c) How many possible routes are there from the arena back to your house?

$\binom{10}{6} = 210$

d) How many total possible routes do you have to choose from to complete all three trips?

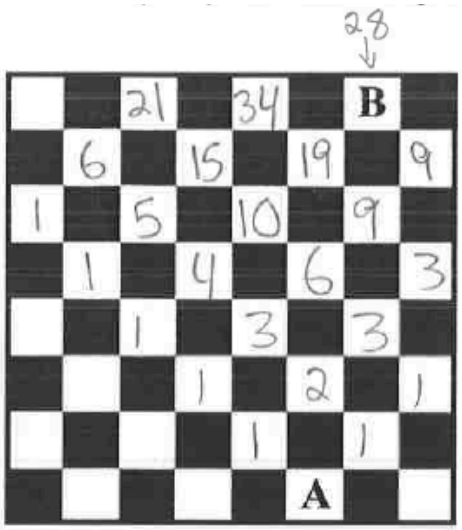
$n(routes) = (210)(66)(210) = 2\,910\,600$

e) What is the probability that you pass the garden gnome of the way to the store?

$P(pass\ gnome) = \frac{n(routes\ pass\ gnome)}{n(routes)} = \frac{\binom{6}{2}\binom{4}{2}}{\binom{10}{4}} = \frac{90}{210} = \frac{3}{7}$ or 0.429

23) How many ways are there to get from A to B moving only diagonally forward?

a)



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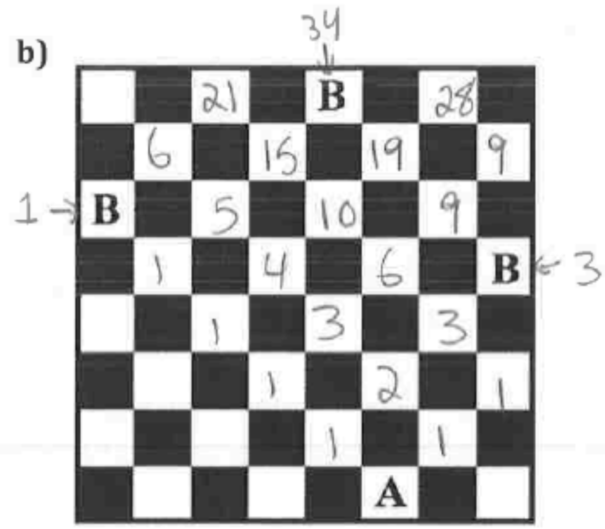
b)



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