In advanced functions, you should have been introduced to the idea that the instantaneous rate of change is represented by the slope of the $\qquad$ at a point on the curve. You also learned that you can determine this value by taking the derivative of the function using the Newton Quotient.

## Newton Quotient Example:

a) Find the equation of the derivative of $f(x)=3 x^{2}+2 x+4$

Newton's Quotient:

$$
f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}
$$

b) Calculate $f^{\prime}(5)$. What does it represent?


Mathematicians have derived a set of rules for calculating derivatives that make this process more efficient.

| Rule | Derivative | Example |
| :---: | :---: | :---: |
| If $f(x)=c$ where $c$ is a constant | $f^{\prime}(x)=0$ |  |
| Power Rule |  |  |
| If $f(x)=x^{n}$ | $f^{\prime}(x)=n x^{n-1}$ |  |
| Constant Multiple Rule <br> If $f(x)=c \cdot g(x)$ where $c$ is a <br> constant | $f^{\prime}(x)=c \cdot g^{\prime}(x)$ |  |
| Sum Rule | $h^{\prime}(x)=f^{\prime}(x)+g^{\prime}(x)$ |  |
| If $h(x)=f(x)+g(x)$ | $h^{\prime}(x)=f^{\prime}(x)-g^{\prime}(x)$ |  |
| Difference Rule |  |  |

## Proof of Power Rule:

Use Binomial Theorem:

$$
t_{r+1}=\binom{n}{r} a^{n-r} b^{r}
$$

Example 1: Determine the equation of the derivative of each of the following functions:
a) $f(x)=3 x^{5}$
b) $f(x)=71$
c) $f(x)=\sqrt{x}$
d) $y=\sqrt[3]{x}$
e) $y=\frac{1}{x}$
f) $y=-\frac{1}{x^{5}}$

Example 2: Differentiate each function
a) $y=5 x^{6}-4 x^{3}+6$
b) $f(x)=-3 x^{5}+8 \sqrt{x}-9.3$
c) $g(x)=(2 x-3)(x+1)$
d) $h(x)=\frac{-8 x^{6}+8 x^{2}}{4 x^{5}}$

Example 3: Determine an equation for the tangent to the curve $f(x)=4 x^{3}+3 x^{2}-5$ at $x=-1$.

Point on the tangent line:
Slope of tangent line:
Remember: The equation of the derivative tells you the slope of the tangent to the original function.

Equation of tangent line:

Example 4: Determine the point(s) on the graph of $y=x^{2}(x+3)$ where the slope of the tangent is 24.


