L1 – Derivatives of Sine and Cosine Unit 3

MCV4U

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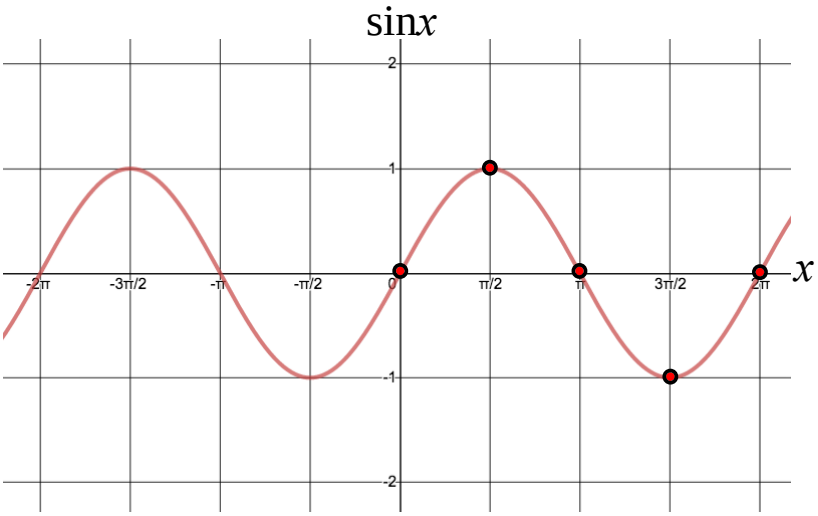
Part 1: Investigation

Example 1: Find the derivative of $\sin x$



x	$\sin x$	$\frac{d}{dx}\sin x$
0		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

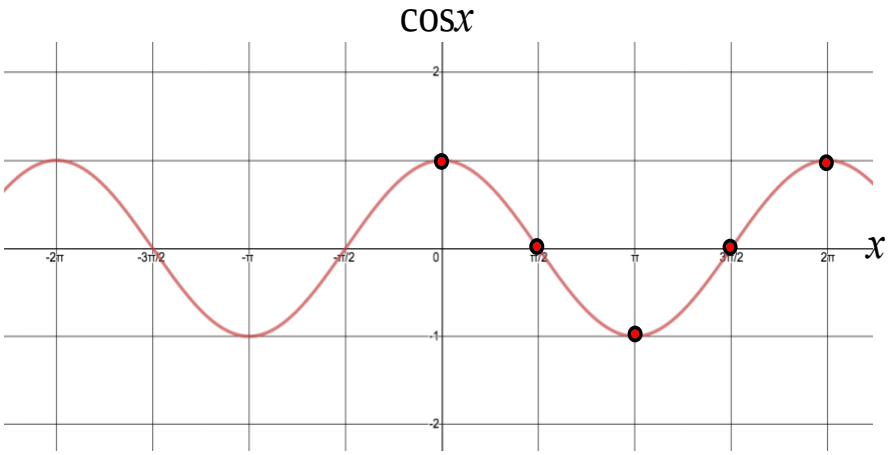
a) Complete the $\sin x$ column in the table.

b) Estimate the instantaneous rate of change (tangent slope) of $\sin x$ at $x = 0$ using a secant line. Use the interval $\left[\frac{0\pi}{100}, \frac{1\pi}{100}\right]$.

c) Complete the instantaneous rate of change column.

d) What do you notice about the values of $\frac{d}{dx}\sin x$? Plot the values and graph the derivative of $\sin x$. What is the derivative of $\sin x$?

Example 2: Repeat the process to find the derivative of $\cos x$



x	$\cos x$	$\frac{d}{dx}\cos x$
0		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

a) Complete the $\cos x$ column in the table.

b) Estimate the instantaneous rate of change (tangent slope) of $\cos x$ at $x = \frac{\pi}{2}$ using a secant line. Use the interval $\left[\frac{50\pi}{100}, \frac{51\pi}{100}\right]$.

c) Complete the instantaneous rate of change column.

d) What do you notice about the values of $\frac{d}{dx}\cos x$? Plot the values and graph the derivative of $\cos x$. What is the derivative of $\cos x$?

Example 3: Find the derivative of $\tan x$.

Derivatives of Trig Functions:

$\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\tan x = \sec^2 x$

Part 2: Differentiating equations involving trig functions

The rules for differentiation apply to sinusoidal functions. A reminder of these rules is below:

Rule	Derivative
Power Rule If $f(x) = x^n$	$f'(x) = nx^{n-1}$
Constant Multiple Rule If $f(x) = c \cdot g(x)$ where c is a constant	$f'(x) = c \cdot g'(x)$
Sum Rule If $h(x) = f(x) + g(x)$	$h'(x) = f'(x) + g'(x)$
Difference Rule If $h(x) = f(x) - g(x)$	$h'(x) = f'(x) - g'(x)$
Product Rule If $h(x) = f(x)g(x)$	$h'(x) = f'(x)g(x) + f(x)g'(x)$
Quotient Rule If $h(x) = f(x) \div g(x)$	$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$
Power of a Function Rule If $h(x) = \big(f(x)\big)^n$	$h'(x) = n[f(x)]^{n-1} \times f'(x)$
Chain Rule If $h(x) = f(g(x))$	$h'(x) = f'[g(x)] \times g'(x)$

Example 4: Differentiate each of the following

a) $y = 2\sin x$ **b)** $f(x) = -3\cos x$ **c)** $y = 4\tan x$

Example 5: Differentiate with respect to x

a) $y = \sin x + \cos x$

b) $y = 2\cos x - 4\sin x$

Example 6: Find the slope of the tangent line to the graph of $f(x) = 3\sin x$ at the point where $x = \frac{\pi}{4}$

Remember special triangles:

Example 7: Find the equation of the tangent line to the curve $f(x) = -2\sin x$ at the point where $x = \frac{\pi}{6}$.