L1 – 1.5 Average Rates of Change
MHF4U
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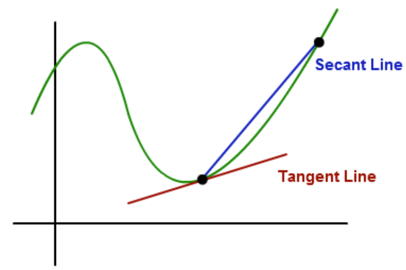
Part 1: Terminology

Rate of Change: *a measure of the change in one quantity (the dependent variable) with respect to a change in another quantity (the independent variable).*

Secant Line: *a line that passes through two points on the graph of a relation*

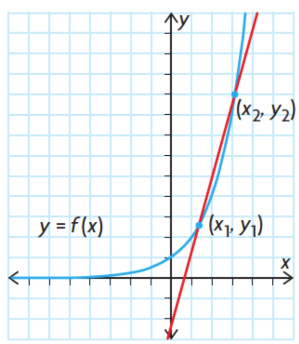
Tangent Line: *a line that touches the graph of a relation at only one point within a small interval*



An **average rate of change** is a change that takes place over an **interval**, while an **instantaneous rate of change** is a change that takes place in an **instant**. We will focus an average rates of change in this section.

An average rate of change corresponds to the slope of a **SECANT** between two points on a curve.

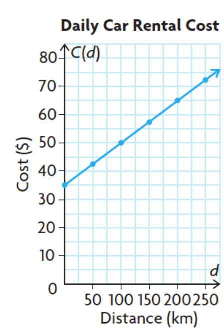
Average rate of change = slope of secant $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$



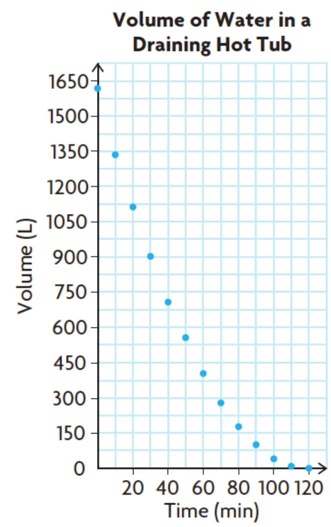
Part 2: Average Rates of Change from a Table or Graph

Note: All **linear** relationships have a constant rate of change. Average rate of change calculations over different intervals of the independent variable give the **SAME** result.

We will be focusing on **non-linear** relationships. Non-linear relationships do not have a constant rate of change. Average rate of change calculations over different intervals of the independent interval give **DIFFERENT** results.



Example 1: Andrew drains water from a hot tub. The tub holds 1600 L of water. It takes 2 hours for the water to drain completely. The volume V, in Liters, of water remaining in the tub at various times t, in minutes, is shown in the table and graph.



Time (min)	Volume (L)
0	1600
10	1344
20	1111
30	900
40	711
50	544
60	400
70	278
80	178
90	100
100	44
110	10
120	0

a) Calculate the average rate of change in volume during each of the following time intervals.

i) $30 \leq t \leq 90$

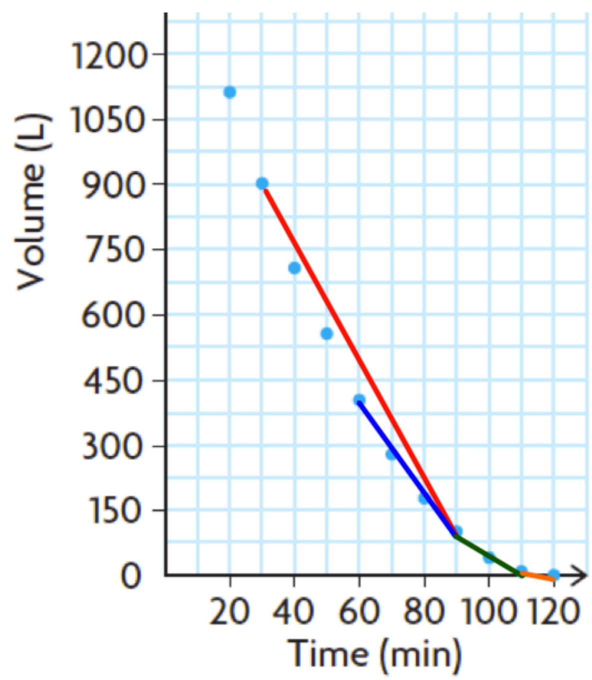
$$m = \frac{\Delta V}{\Delta t}$$

$$= \frac{V(90) - V(30)}{90 - 30}$$

$$= \frac{100 - 900}{60}$$

$$= -13.3 \text{ L/min}$$

The volume of water is decreasing by 13.3 L/min.



ii) $60 \leq t \leq 90$

$$m = \frac{\Delta V}{\Delta t}$$

$$= \frac{V(90) - V(60)}{90 - 60}$$

$$= \frac{100 - 400}{30}$$

$$= -10 \text{ L/min}$$

The volume of water is decreasing by 10 L/min.

iii) $90 \leq 110$

$m = \frac{\Delta V}{\Delta t}$

$= \frac{V(110) - V(90)}{110 - 90}$

$= \frac{10 - 100}{20}$

$= -4.5$ L/min

The volume of water is decreasing by 4.5 L/min.

iv) $110 \leq 120$

$m = \frac{\Delta V}{\Delta t}$

$= \frac{V(120) - V(110)}{120 - 110}$

$= \frac{0 - 10}{10}$

$= -1$ L/min

The volume of water is decreasing by 1 L/min.

b) Does the tub drain at a constant rate?

The tub does not drain at a constant rate. This can be seen from the graph which shows a non-linear relationship. The rate slows as the water empties because of the pressure decrease.

A **negative** rate of change indicates the quantity of the dependent variable is decreasing over the interval. The secant line has a negative slope.

A **positive** rate of change indicates the quantity of the dependent variable is increasing over the interval. The secant line has a positive slope.

Part 2: Average Rate of Change from an Equation

Example 2: A rock is tossed upward from a cliff that is 120 meters above the water. The height of the rock above the water is modelled by $h(t) = -5t^2 + 10t + 120$, where h is the height in meters and t is the time in seconds. Calculate the average rate of change in height during each time intervals.

a) $0 \leq t \leq 1$

$$m = \frac{\Delta h}{\Delta t}$$

$$= \frac{h(1) - h(0)}{1 - 0}$$

$$= \frac{125 - 120}{1}$$

$= 5$ m/s

b) $1 \leq t \leq 2$

$$m = \frac{\Delta h}{\Delta t}$$

$$= \frac{h(2) - h(1)}{2 - 1}$$

$$= \frac{120 - 125}{1}$$

$= -5$ m/s

c) $2 \leq t \leq 3$

$$m = \frac{\Delta h}{\Delta t}$$

$$= \frac{h(3) - h(2)}{3 - 2}$$

$$= \frac{105 - 120}{1}$$

$= -15$ m/s

http://danaernst.com/CalculusApplets/SecantTangent/