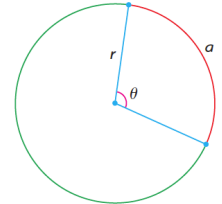## Part 1: What is a Radian?

Angles are commonly measured in **degrees**. However, in mathematics and physics, there are many applications in which expressing an angle as a pure number, without units, is more convenient than using degrees.

When measuring in **radians**, the size of an angle is expressed in terms of the length of an arc, a, that subtends the angle, θ, where $\theta = \frac{a}{r}$.

1 Radian is defined as the size of an angle that is subtended by an arc with a length **equal to the radius** of the circle.
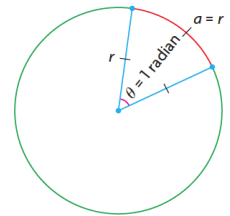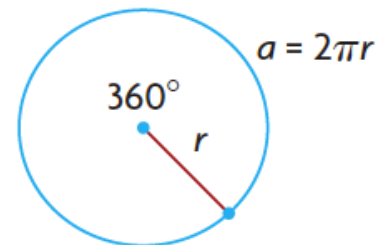
Therefore, when the arc length and radius are equal, $\theta = \frac{a}{r} = \frac{r}{r} = 1$ radian

How many radians are in a full circle? Or in other words, how many times can an arc length equal to the radius fit around the circumference of a circle?

Remember: $C = 2\pi r$

So if we use the full circumference of the circle for the arc length,
$\theta = \frac{a}{r} = \frac{2\pi r}{r} = 2\pi$ radians

Part 2: Switching Between Degrees and Radians

The key relationship you need to know in order to switch between degrees and radians is:

$$360° = 2\pi \text{ radians}$$

1 degree = $\frac{\pi}{180}$ radians To switch from degrees to radians, multiply by $$\frac{\pi}{180}$$	1 radian = $\frac{180}{\pi}$ degrees To switch from radians to degrees, multiply by $$\frac{180}{\pi}$$

Example 1: Start by converting the following common degree measures to radians

a) 180°

$$= \cancel{180°} \times \frac{\pi}{\cancel{180}}$$

$$= \pi \text{ radians}$$

b) 90°

$$= 90° \times \frac{\pi}{180}$$

$$= \frac{\pi}{2} \text{ radians}$$

c) 60°

$$= 60° \times \frac{\pi}{180}$$

$$= \frac{\pi}{3} \text{ radians}$$

d) 45°

$$= 45° \times \frac{\pi}{180}$$

$$= \frac{\pi}{4} \text{ radians}$$

e) 30°
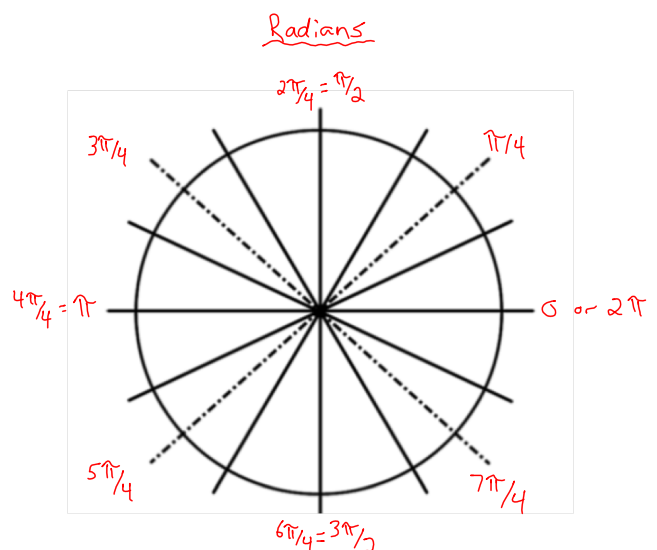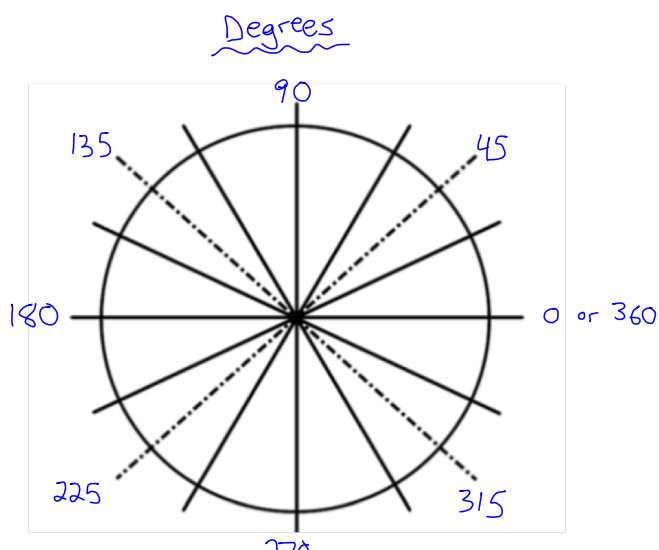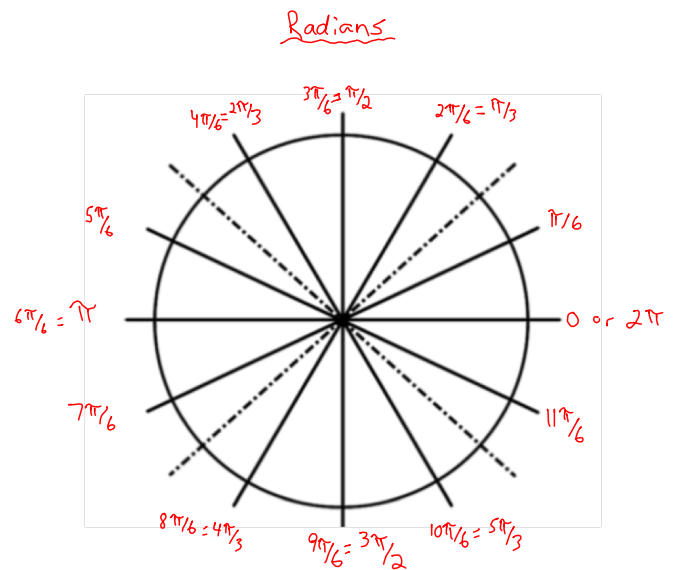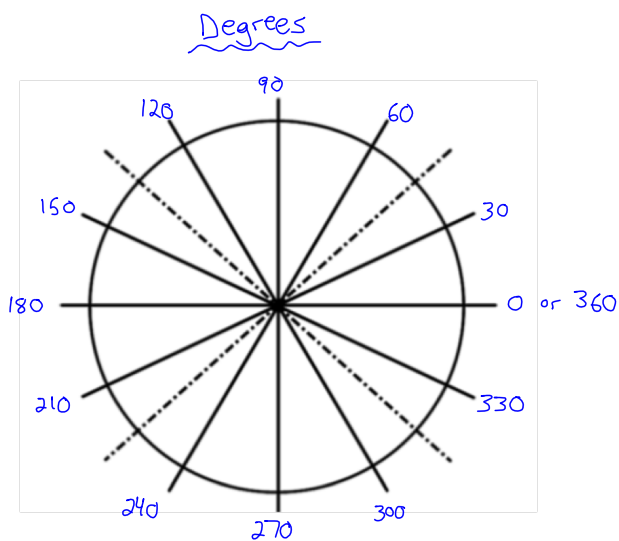
$$= 30° \times \frac{\pi}{180}$$

$$= \frac{\pi}{6} \text{ radians}$$

f) 1°

$$= 1° \times \frac{\pi}{180}$$

$$= \frac{\pi}{180} \text{ radians}$$

Degrees

Radians

Degrees

Radians

Example 2: Convert each of the following degree measures to radian measures

a) $225°$

$= 225° \times \dfrac{\pi}{180}$

$= \dfrac{5\pi}{4}$ radians

b) $80°$

$= 80° \times \dfrac{\pi}{180}$

$= \dfrac{4\pi}{9}$ radians

c) $450°$

$= 450° \times \dfrac{\pi}{180}$

$= \dfrac{5\pi}{2}$ radians

Example 3: Convert each of the following radian measures to degree measures

a) $\dfrac{2\pi}{3}$ radians

$= \dfrac{2\pi}{3} \times \dfrac{180}{\pi}$

$= 120°$

b) $\dfrac{9\pi}{4}$ radians

$= \dfrac{9\pi}{4} \times \dfrac{180}{\pi}$

$= 405°$

c) 1 radian

$= 1 \times \dfrac{180}{\pi}$

$= 57.296°$

Part 3: Application

Example 4: Suzette chooses a camel to ride on a carousel. The camel is located 9 m from the center of the carousel. It the carousel turns through an angle of $\dfrac{5\pi}{6}$, determine the length of the arc travelled by the camel, to the nearest tenth of a meter.
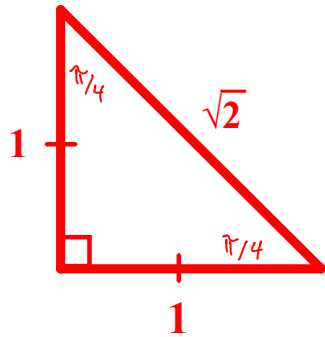
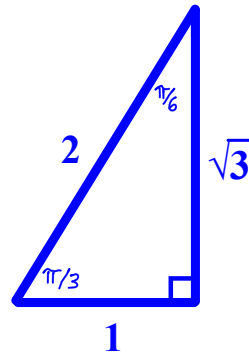$\theta = \dfrac{a}{r}$

$a = \theta r$

$a = \dfrac{5\pi}{6}(9)$

$a \cong 23.6$ m

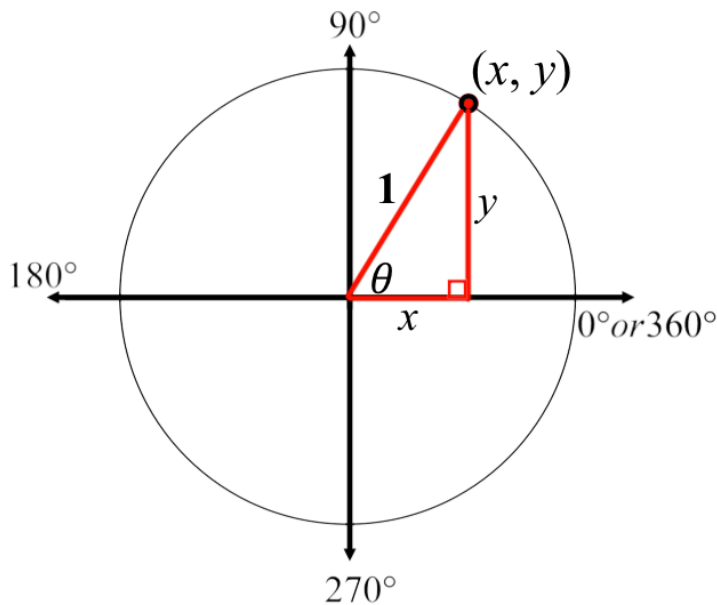## Part 4: Special Triangles Using Radian Measures

Draw the 45-45-90 triangle

Draw the 30-60-90 triangle

Also, you will need to remember the UNIT CIRLCE which is a circle that has a radius of 1. Use the unit circle to write expressions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ in terms of x, y, and r.

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x}$$

On the unit circle, the sine and cosine functions take a simple form:

$\sin\theta = y$

$\cos\theta = x$

The value of $\sin\theta$ is the **y-coordinate** of each point on the unit circle

The value of $\cos\theta$ is the **x-coordinate** of each point on the unit circle

Therefore, we can use the points where the terminal arm intersects the unit circle to get the sine and cosine ratios just by looking at the y and x co-ordinates of the points.

Example 5: Fill out chart of ratios using special triangles and the unit circle

	$0, 2\pi$	$\dfrac{\pi}{6}$	$\dfrac{\pi}{4}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$	π	$\dfrac{3\pi}{2}$
$\sin x$	0	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{3}}{2}$	1	0	-1
$\cos x$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$	0	-1	0
$\tan x$	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined	0	undefined