L1 – 4.3 Co-function Identities
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Part 1: Remembering How to Prove Trig Identities

Fundamental Trigonometric Identities		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
$csc\,\theta = \frac{1}{sin\,\theta}$ $sec\,\theta = \frac{1}{cos\,\theta}$ $cot\,\theta = \frac{1}{tan\,\theta}$	$\frac{sin\,\theta}{cos\,\theta} = tan\,\theta$ $\frac{\cos\theta}{\sin\theta} = \cot\theta$	$\sin^2\theta + \cos^2\theta = 1$

Tips and Tricks		
Reciprocal Identities	**Quotient Identities**	**Pythagorean Identities**
Square both sides $csc^2\,\theta = \frac{1}{sin^2\,\theta}$ $sec^2\,\theta = \frac{1}{cos^2\,\theta}$ $cot^2\,\theta = \frac{1}{tan^2\,\theta}$	Square both sides $\frac{sin^2\,\theta}{cos^2\,\theta} = tan^2\,\theta$ $\frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$	Rearrange the identity $sin^2\theta = 1 - cos^2\theta$ $cos^2\theta = 1 - sin^2\theta$ Divide by either $\sin^2\theta$ or $\sin^2\theta$ $1 + cot^2\theta = \csc^2\theta$ $tan^2\theta + 1 = sec^2\theta$

General tips for proving identities:

- **i)** Separate into LS and RS. Terms may NOT cross between sides.
- **ii)** Try to change everything to $sin\,\theta$ or $cos\,\theta$
- **iii)** If you have two fractions being added or subtracted, find a common denominator and combine the fractions.
- **iv)** Use difference of squares → $1 - sin^2\,\theta = (1 - sin\,\theta)(1 + sin\,\theta)$
- **v)** Use the power rule → $sin^6\,\theta = (sin^2\,\theta)^3$

Example 1: Prove each of the following identities

a) $\tan^2 x + 1 = \sec^2 x$

LS	RS
$= \tan^2 x + 1$	$= \sec^2 x$
QI: $= \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x}$ (common denom)	R.I.: $= \frac{1}{\cos^2 x}$
$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$	
P.I.: $= \frac{1}{\cos^2 x}$	

LS=RS

b) $\cos^2 x = (1 - \sin x)(1 + \sin x)$

LS	RS
$= \cos^2 x$	$= (1 - \sin x)(1 + \sin x)$
	DOS: $= 1 - \sin^2 x$
	PI: $= \cos^2 x$

LS=RS

c) $\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$

LS	RS
$= \frac{\sin^2 x}{1 - \cos x}$	$= 1 + \cos x$
PI: $= \frac{1 - \cos^2 x}{1 - \cos x}$	
DOS: $= \frac{(1 - \cos x)(1 + \cos x)}{1 - \cos x}$	
$= 1 + \cos x$	

LS=RS

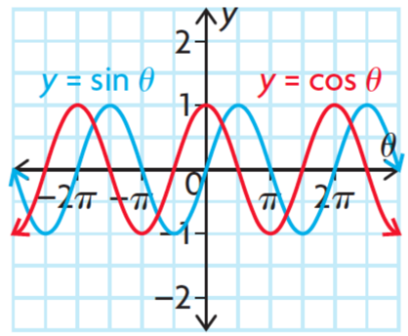
Part 2: Transformation Identities

Because of their periodic nature, there are many equivalent trigonometric expressions .

Horizontal translations of $\frac{\pi}{2}$ that involve both a sine function and a cosine function can be used to obtain two equivalent functions with the same graph.

Translating the cosine function $\frac{\pi}{2}$ to the **right**, $f(x) = \cos\left(x - \frac{\pi}{2}\right)$ results in the graph of the sine function, $f(x) = \sin x$.

Similarly, translating the sine function $\frac{\pi}{2}$ to the **left**, $f(x) = \sin\left(x + \frac{\pi}{2}\right)$ results in the graph of the cosine function, $f(x) = \cos x$.

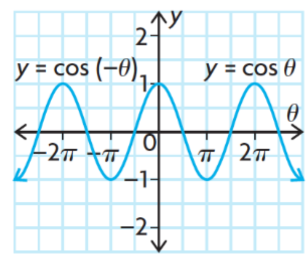


Transformation Identities

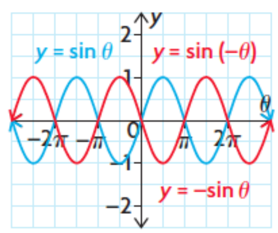
$$\cos\left(x - \frac{\pi}{2}\right) = \sin x \qquad \sin\left(x + \frac{\pi}{2}\right) = \cos x$$

Part 3: Even/Odd Function Identities

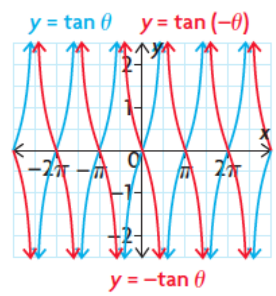
Remember that $\mathbf{cos}\ \boldsymbol{x}$ is an **even** function. Reflecting its graph across the y-axis results in two equivalent functions with the same graph.



$\sin x$ and $\tan x$ are both **odd** functions. They have rotational symmetry about the origin.



sin (−θ) = −sin θ



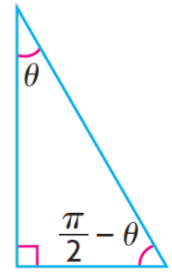
tan (−θ) = −tan θ

Even/Odd Identities

$$\cos(-x) = \cos x \qquad \sin(-x) = -\sin x \qquad \tan(-x) = -\tan x$$

Part 4: Co-function Identities

The co-function identities describe trigonometric relationships between complementary angles in a right triangle.



Co-Function Identities	
$\cos\left(\frac{\pi}{2} - x\right) = \sin x$	$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

We could identify other equivalent trigonometric expressions by comparing principle angles drawn in standard position in quadrants II, III, and IV with their related acute (reference) angle in quadrant I.

Principle in Quadrant II	Principle in Quadrant III	Principle in Quadrant IV
$\sin(\pi - x) = \sin x$	$\sin(\pi + x) = -\sin x$	$\sin(2\pi - x) = -\sin x$

Example 2: Prove both co-function identities using transformation identities

a) $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

LS	RS
$= \cos\left(\frac{\pi}{2} - x\right)$	$= \sin x$
$= \cos\left(-x + \frac{\pi}{2}\right)$ (factor −1)	
$= \cos\left[-\left(x - \frac{\pi}{2}\right)\right]$ (even)	
$= \cos\left(x - \frac{\pi}{2}\right)$ (T.I.)	
$= \sin x$	

LS=RS

b) $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

LS	RS
$= \sin\left(\frac{\pi}{2} - x\right)$	$= \cos x$ (even)
	$= \cos(-x)$ (TI)
	$= \sin\left(-x + \frac{\pi}{2}\right)$
	$= \sin\left(\frac{\pi}{2} - x\right)$

LS=RS

Part 5: Apply the Identities

Example 3: Given that $\sin\frac{\pi}{5} \cong 0.5878$, use equivalent trigonometric expressions to evaluate the following:

a) $\cos\frac{3\pi}{10}$

$= \sin\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$

$= \sin\left(\frac{5\pi}{10} - \frac{3\pi}{10}\right)$

$= \sin\left(\frac{2\pi}{10}\right)$

$= \sin\left(\frac{\pi}{5}\right)$

$\cong 0.5878$

b) $\cos\frac{7\pi}{10}$

$= \sin\left(\frac{\pi}{2} - \frac{7\pi}{10}\right)$

$= \sin\left(\frac{5\pi}{10} - \frac{7\pi}{10}\right)$

$= \sin\left(-\frac{2\pi}{10}\right)$

$= -\sin\left(\frac{\pi}{5}\right)$

$\cong -0.5878$