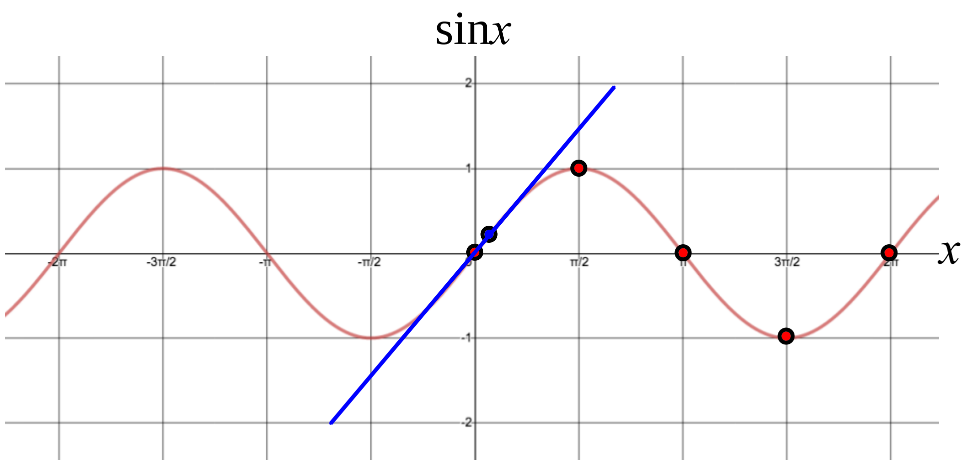
L1 – Derivatives of Sine and Cosine Unit 3

MCV4U

Jensen

Part 1: Investigation

Example 1: Find the derivative of $\sin x$



x	$\sin x$	$\frac{d}{dx}\sin x$
0	0	1
$\frac{\pi}{2}$	1	0
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1

a) Complete the $\sin x$ column in the table.

b) Estimate the instantaneous rate of change (tangent slope) of $\sin x$ at $x = 0$ using a secant line. Use the interval $\left[\frac{0\pi}{100}, \frac{1\pi}{100}\right]$.

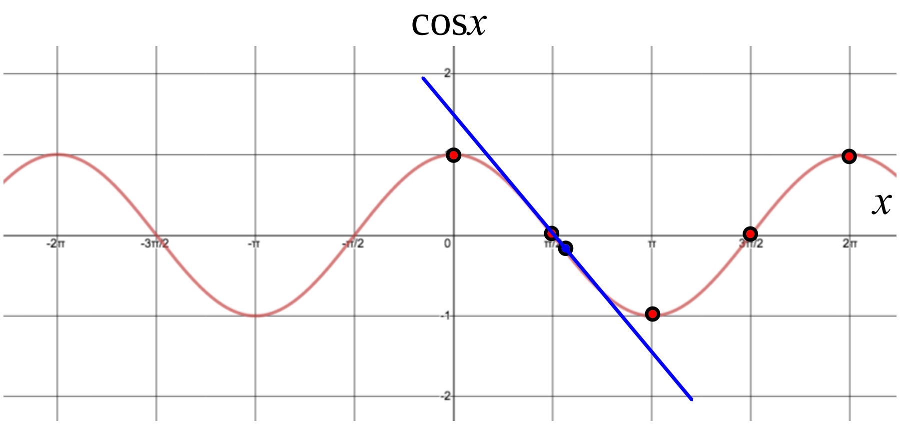
$$\left.\frac{d(\sin x)}{dx}\right|_{x=0} \cong \frac{y_2 - y_1}{x_2 - x_1} = \frac{\sin\left(\frac{\pi}{100}\right) - \sin(0)}{\frac{\pi}{100} - 0} = \frac{0.0314107591}{0.0314159265} = 0.9998355122 \cong 1$$

c) Complete the instantaneous rate of change column.

d) What do you notice about the values of $\frac{d}{dx}\sin x$? Plot the values and graph the derivative of $\sin x$. What is the derivative of $\sin x$?

The derivative of $\sin x$ is equivalent to $\cos x$

Example 2: Repeat the process to find the derivative of $\cos x$



x	$\cos x$	$\frac{d}{dx}\cos x$
0	1	0
$\frac{\pi}{2}$	0	-1
π	-1	0
$\frac{3\pi}{2}$	0	1
2π	1	0

a) Complete the $\cos x$ column in the table.

b) Estimate the instantaneous rate of change (tangent slope) of $\cos x$ at $x = \frac{\pi}{2}$ using a secant line. Use the interval $\left[\frac{50\pi}{100}, \frac{51\pi}{100}\right]$.

$$\left.\frac{d(\cos x)}{dx}\right|_{x=\frac{\pi}{2}} \cong \frac{y_2 - y_1}{x_2 - x_1} = \frac{\cos\left(\frac{51\pi}{100}\right) - \cos\left(\frac{50\pi}{100}\right)}{\frac{51\pi}{100} - \frac{50\pi}{100}} = \frac{-0.0314107591}{0.0314159265} = -0.9998355122 \cong -1$$

c) Complete the instantaneous rate of change column.

d) What do you notice about the values of $\frac{d}{dx}\cos x$? Plot the values and graph the derivative of $\cos x$. What is the derivative of $\cos x$?

The derivative of $\cos x$ is a vertically flipped $\sin x$ function. Therefore, the derivative of $\cos x$ is equivalent to $-\sin x$.

Example 3: Find the derivative of $\tan x$.

$y = \tan x$

$y = \frac{\sin x}{\cos x}$

$\frac{dy}{dx} = \frac{\cos x(\cos x) - (-\sin x)(\sin x)}{\cos^2 x}$

$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$\frac{dy}{dx} = \frac{1}{\cos^2 x}$

$\frac{dy}{dx} = \sec^2 x$

Derivatives of Trig Functions:

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\tan x = \sec^2 x$$

Part 2: Differentiating equations involving trig functions

The rules for differentiation apply to sinusoidal functions. A reminder of these rules is below:

Rule	Derivative
Power Rule If $f(x) = x^n$	$f'(x) = nx^{n-1}$
Constant Multiple Rule If $f(x) = c \cdot g(x)$ where c is a constant	$f'(x) = c \cdot g'(x)$
Sum Rule If $h(x) = f(x) + g(x)$	$h'(x) = f'(x) + g'(x)$
Difference Rule If $h(x) = f(x) - g(x)$	$h'(x) = f'(x) - g'(x)$
Product Rule If $h(x) = f(x)g(x)$	$h'(x) = f'(x)g(x) + f(x)g'(x)$
Quotient Rule If $h(x) = f(x) \div g(x)$	$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$
Power of a Function Rule If $h(x) = \left(f(x)\right)^n$	$h'(x) = n[f(x)]^{n-1} \times f'(x)$
Chain Rule If $h(x) = f(g(x))$	$h'(x) = f'[g(x)] \times g'(x)$

Example 4: Differentiate each of the following

a) $y = 2\sin x$

$$\frac{dy}{dx} = 2\frac{d}{dx}\sin x$$

$$\frac{dy}{dx} = 2\cos x$$

b) $f(x) = -3\cos x$

$$f'(x) = -3(-\sin x)$$

$$f'(x) = 3\sin x$$

c) $y = 4\tan x$

$$\frac{dy}{dx} = 4\frac{d}{dx}\tan x$$

$$\frac{dy}{dx} = 4\sec^2 x$$

Example 5: Differentiate with respect to x

a) $y = \sin x + \cos x$

$$\frac{dy}{dx} = \cos x + (-\sin x)$$

$$\frac{dy}{dx} = \cos x - \sin x$$

b) $y = 2\cos x - 4\sin x$

$$\frac{dy}{dx} = 2(-\sin x) - 4\cos x$$

$$\frac{dy}{dx} = -2\sin x - 4\cos x$$

Example 6: Find the slope of the tangent line to the graph of $f(x) = 3\sin x$ at the point where $x = \frac{\pi}{4}$

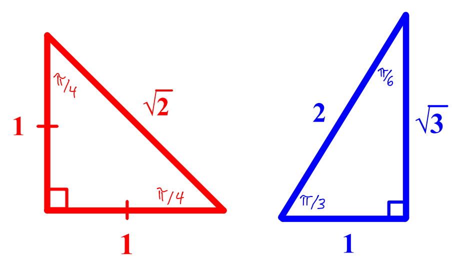
$$f'(x) = 3\cos x$$

$$f'\left(\frac{\pi}{4}\right) = 3\cos\left(\frac{\pi}{4}\right)$$

$$f'\left(\frac{\pi}{4}\right) = 3\left(\frac{1}{\sqrt{2}}\right)$$

$$f'\left(\frac{\pi}{4}\right) = \frac{3}{\sqrt{2}}$$

Remember special triangles:



Example 7: Find the equation of the tangent line to the curve $f(x) = -2\sin x$ at the point where $x = \frac{\pi}{6}$.

Slope of tangent line:

$$f'(x) = -2\cos x$$

$$f'\left(\frac{\pi}{6}\right) = -2\cos\left(\frac{\pi}{6}\right)$$

$$f'\left(\frac{\pi}{6}\right) = -2\left(\frac{\sqrt{3}}{2}\right)$$

$$f'\left(\frac{\pi}{6}\right) = -\sqrt{3}$$

$$m = -\sqrt{3}$$

Point on tangent line:

$$f\left(\frac{\pi}{6}\right) = -2\sin\left(\frac{\pi}{6}\right)$$

$$f\left(\frac{\pi}{6}\right) = -2\left(\frac{1}{2}\right)$$

$$f\left(\frac{\pi}{6}\right) = -1$$

$$\left(\frac{\pi}{6}, -1\right)$$

Equation of tangent line:

$$y = mx + b$$

$$-1 = -\sqrt{3}\left(\frac{\pi}{6}\right) + b$$

$$b = \frac{\sqrt{3}\pi}{6} - 1$$

$$y = -\sqrt{3}x + \frac{\sqrt{3}\pi}{6} - 1$$