L2 – 4.4 Compound Angle Formulas
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Compound angle: *an angle that is created by adding or subtracting two or more angles.*

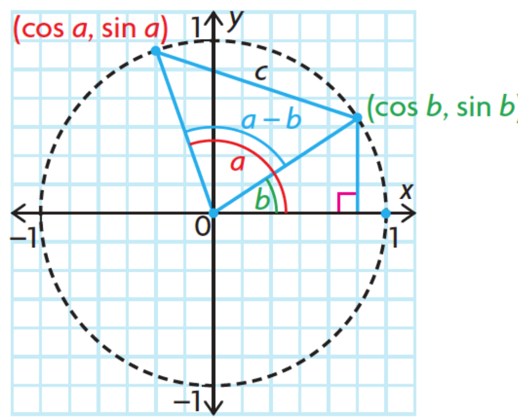
Part 1: Proof of $\cos(x - y)$

Normal algebra rules do not apply:

$$\cos(x - y) \neq \cos x - \cos y$$

So what does $\cos(x - y) =?$

Consider the diagram to the right…



By the cosine law:

$c^2 = 1^2 + 1^2 - 2(1)(1)\cos(a - b)$

$c^2 = 2 - 2\cos(a - b)$ THIS IS EQUATION 1

But notice that c has endpoints of $(\cos a, \sin a)$ and $(\cos b, \sin b)$

Using the distance formula $distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$c = \sqrt{(\cos a - \cos b)^2 + (\sin a - \sin b)^2}$

$c^2 = (\cos a - \cos b)^2 + (\sin a - \sin b)^2$

$c^2 = \cos^2 a - 2\cos a \cos b + \cos^2 b + \sin^2 a - 2\sin a \sin b + \sin^2 b$

$c^2 = 1 - 2\cos a \cos b - 2\sin a \sin b + 1$

$c^2 = 2 - 2\cos a \cos b - 2\sin a \sin b$ THIS IS EQUATION 2

Set equations 1 and 2 equal

$2 - 2\cos(a - b) = 2 - 2\cos a \cos b - 2\sin a \sin b$

$-2\cos(a - b) = -2\cos a \cos b - 2\sin a \sin b$

Part 2: Proofs of other compound angle formulas

Example 1: Prove $\cos(x+y) = \cos x \cos y - \sin x \sin y$

LS	RS

LS = RS

Example 2: Prove addition and subtraction formulas for sine using co-function identities and the subtraction formula for cosine.

Co-Function Identities

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$ $\quad\quad$ $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

a) Prove $\sin(x+y) = \sin x \cos y + \cos x \sin y$

LS	RS

LS = RS

b) Prove $\sin(x - y) = \sin x \cos y - \cos x \sin y$

LS	**RS**

LS = RS

Compound Angle Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

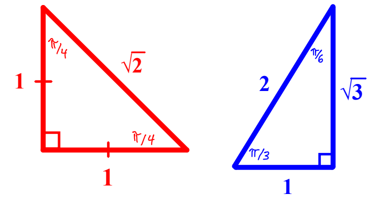
$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Part 3: Determine Exact Trig Ratios for Angles other than Special Angles



By expressing an angle as a sum or difference of angles in the special triangles, exact values of other angles can be determined.

Example 3: Use compound angle formulas to determine exact values for

a) $\sin\frac{\pi}{12}$

$$\sin\frac{\pi}{12} =$$

b) $\tan\left(-\frac{5\pi}{12}\right)$

$$\tan\left(-\frac{5\pi}{12}\right) =$$

Part 4: Use Compound Angle Formulas to Simplify Trig Expressions

Example 4: Simplify the following expression

$$\cos\frac{7\pi}{12}\cos\frac{5\pi}{12}+\sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$$

Part 5: Application

Example 5: Evaluate $\sin(a+b)$, where a and b are both angles in the second quadrant; given $\sin a=\frac{3}{5}$ and $\sin b=\frac{5}{13}$

Start by drawing both terminal arms in the second quadrant and solving for the third side.