L2 – Maxima and Minima Unit 2
MCV4U
Jensen

Part 1: Review

Determine all local extrema for the function below using critical numbers and the first derivative test. State when the function is increasing and decreasing.

> ***Remember:*** *Local extrema occur when the sign of the derivative CHANGES. If the sign of the derivative does not change, you have neither local extrema.*

$f(x) = 2x^3 + 3x^2 - 36x + 5$

$f'(x) = 6x^2 + 6x - 36$

$0 = 6(x^2 + x - 6)$

$0 = 6(x + 3)(x - 2)$

$x = -3$ and $x = 2$ are critical numbers

Critical points:

$f(-3) = 2(-3)^3 + 3(-3)^2 - 36(-3) + 5 = 86$ $(-3, 86)$

$f(2) = 2(2)^3 + 3(2)^2 - 36(2) + 5 = -39$ $(2, -39)$

Interval boundaries: $-\infty$, -3, 2, ∞

Test value for x	-4	0	3
$f'(x)$	$+$	$-$	$+$
$f(x)$	Increasing	Decreasing	Increasing

Local max at $(-3, 86)$

Local min at $(2, -39)$

Interval of increasing: $(-\infty, -3) \cup (2, \infty)$

Interval of decreasing: $(-3, 2)$

Part 2: Local vs Absolute Extrema

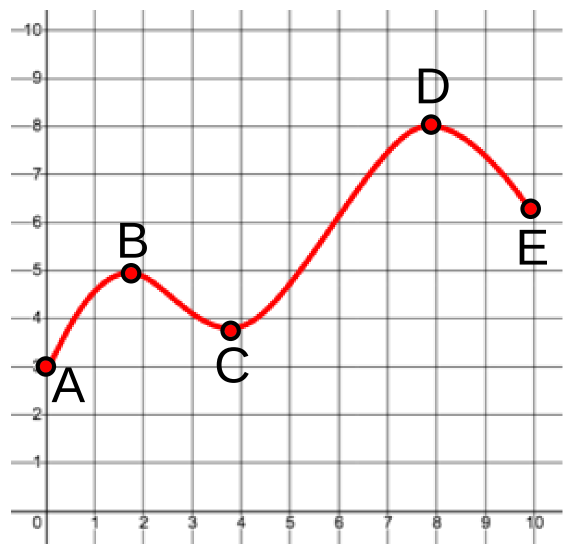
Local max or min values of a function are also called local extrema, or turning points.

Local max: If the y-coordinate of all points in the vicinity are less than the y-coordinate of the point. The sign of the derivative would change from positive before the point, to zero at the point, to negative after.

Local min: If the y-coordinate of all points in the vicinity are greater than the y-coordinate of the point. The sign of the derivative would change from negative before the point, to zero at the point, to positive after.

Absolute max/min: A function $f(x)$ has an ABSOLUTE max or min at point a if $f(a)$ is the biggest or smallest value of $f(x)$ for ALL x in the domain.

Example 1: Consider the graph of a function on the interval $[0, 10]$.



a) Identify the local maximum points.

B and D

b) Identify the local minimum points.

C

c) What do all the points identified in parts a) and b) have in common?

They each would have horizontal tangent lines.

d) Identify the absolute max and min points in the interval $[0,10]$

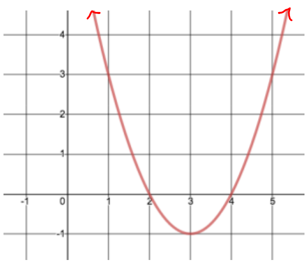
Absolute max is at D

Absolute min is at A

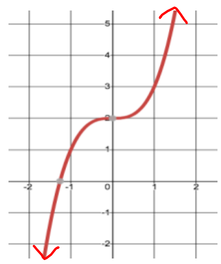
Reminder: A critical number of a function is a value of a **in the domain of the function** where either $f'(a) = 0$ or $f'(a)$ does not exist. If a is a critical number, $(a, f(a))$ is a critical point.

Scenarios for critical numbers:

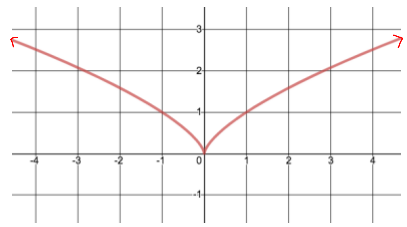
1) $f'(a) = 0$
Local extrema at $(a, f(a))$
$f(x) = x^2 - 6x + 8$



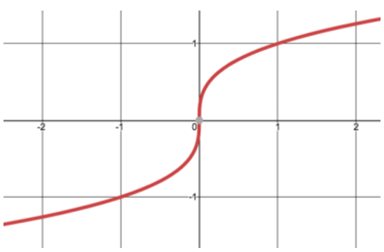
2) $f'(a) = 0$
No local extrema at $(a, f(a))$
$f(x) = x^3 + 2$



3) $f'(a)$ does not exist
local extrema at $(a, f(a))$ (cusp)
$f(x) = x^{\frac{2}{3}}$

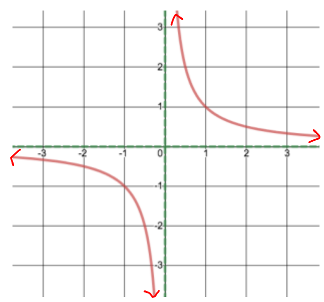


4) $f'(a)$ does not exist
No local extrema at $(a, f(a))$
$f(x) = x^{\frac{1}{3}}$



5) $\boldsymbol{f'(a)}$ **does not exist**
No local extrema
$f(a)$ does not exist either, therefore a is NOT a critical number

$f(x) = \frac{1}{x}$



To determine the absolute extrema values of a function on an interval, find the critical numbers, then substitute the critical numbers and also the x-coordinates of the endpoints of the interval into the function.

Example 2: Find the absolute max and min of the function $f(x) = x^3 - 12x - 3$ on the interval $-3 \leq x \leq 4$.

Test critical numbers AND endpoints of interval.

$f'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$0 = 3(x - 2)(x + 2)$

$x = 2$ and $x = -2$ are critical numbers

$f(-3) = (-3)^3 - 12(-3) - 3 = 6$

$f(-2) = 13$
$f(2) = -19$
$f(4) = 13$

Absolute min: $(2, -19)$

Absolute max: $(-2, 13)$ and $(4, 13)$

Example 3: The surface area of a cylindrical container is to be 100 cm^2. Its volume is given by the function $V(r) = 50r - \pi r^3$, where r is the radius of the cylinder in cm. Find the max volume of the cylinder if the radius cannot exceed 3 cm.

$V'(r) = 50 - 3\pi r^2$

Find any critical numbers:

$0 = 50 - 3\pi r^2$

$3\pi r^2 = 50$

$r = \sqrt{\frac{50}{3\pi}}$ is a critical number

Test endpoints of interval and critical number to find absolute max

$V(0) = 0$ cm^3

$V\left(\sqrt{\frac{50}{3\pi}}\right) = 76.8$ cm^3

$V(3) = 65.2$ cm^3

Therefore, the max volume of the cylinder is about 76.8 cm^3 when the radius is about 2.3 cm.