L2 – MORE Derivatives of Sine and Cosine Unit 3
MCV4U
Jensen

Reminder of rules:

Rule	Derivative
Power Rule If $f(x) = x^n$	$f'(x) = nx^{n-1}$
Constant Multiple Rule If $f(x) = c \cdot g(x)$ where c is a constant	$f'(x) = c \cdot g'(x)$
Sum Rule If $h(x) = f(x) + g(x)$	$h'(x) = f'(x) + g'(x)$
Difference Rule If $h(x) = f(x) - g(x)$	$h'(x) = f'(x) - g'(x)$
Product Rule If $h(x) = f(x)g(x)$	$h'(x) = f'(x)g(x) + f(x)g'(x)$
Quotient Rule If $h(x) = f(x) \div g(x)$	$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$
Power of a Function Rule If $h(x) = (f(x))^n$	$h'(x) = n[f(x)]^{n-1} \times f'(x)$
Chain Rule If $h(x) = f(g(x))$	$h'(x) = f'[g(x)] \times g'(x)$

Derivatives of Trig Functions:

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

Derivatives of Composite Trig Functions:

$$\frac{d}{dx}\sin f(x) = \cos f(x) \times f'(x)$$

$$\frac{d}{dx}\cos f(x) = -\sin f(x) \times f'(x)$$

$$\frac{d}{dx}\tan f(x) = \sec^2 f(x) \times f'(x)$$

Example 1: Determine the derivative with respect to x

a) $y = \sin(2x)$

$$\frac{dy}{dx} = \cos(2x)\,(2)$$

$$\frac{dy}{dx} = 2\cos(2x)$$

b) $y = \sin^2 x$

$$\frac{dy}{dx} = 2\sin x(\cos x)$$

c) $y = \sin(x^2)$

$$\frac{dy}{dx} = \cos(x^2)\,(2x)$$

$$\frac{dy}{dx} = 2x\cos(x^2)$$

d) $x^2 \sin x$

$$\frac{dy}{dx} = (2x)\sin x + \cos x(x^2)$$

$$\frac{dy}{dx} = x(2\sin x + x\cos x)$$

Example 2: Find the derivative with respect to x for each function.

a) $y = \cos(3x)$

$$\frac{dy}{dx} = -\sin(3x)\,(3)$$

$$\frac{dy}{dx} = -3\sin(3x)$$

b) $f(x) = 2\sin(\pi x)$

$$f'(x) = 2\cos(\pi x)(\pi)$$

$$f'(x) = 2\pi\cos(\pi x)$$

c) $g(x) = \tan(x^2 + 3x)$

$$g'(x) = \sec^2(x^2 + 3x)\,(2x + 3)$$

$$g'(x) = (2x + 3)\sec^2(x^2 + 3x)$$

Example 3: Differentiate with respect to x.

a) $y = \cos^3 x$

$$\frac{dy}{dx} = 3\cos^2 x(-\sin x)$$

$$\frac{dy}{dx} = -3\cos^2 x\sin x$$

b) $f(x) = 2\sin^3 x - 4\cos^2 x$

$$f'(x) = 6\sin^2 x(\cos x) - 8\cos x\,(-\sin x)$$

$$f'(x) = 6\sin^2 x(\cos x) + 8\cos x\,(\sin x)$$

$$f'(x) = 2\sin x\cos x\,(3\sin x + 4)$$

$$f'(x) = \sin(2x)(2\sin x + 4)$$

Notice the double angle identity $\sin(2x) = 2\sin x\cos x$ was used to simplify.

Example 4: Find each derivative with respect to t.

a) $y = t^3\cos t$

$$\frac{dy}{dx} = 3t^2\cos t + (-\sin t)t^3$$

$$\frac{dy}{dx} = 3t^2\cos t - \sin t\,(t^3)$$

$$\frac{dy}{dx} = t^2(3\cos t - t\sin t)$$

b) $h(t) = \sin(4t)\cos^2 t$

$$h'(t) = 4\cos(4t)\cos^2 t + 2\cos t\,(-\sin t)\sin(4t)$$

$$h'(t) = 2\cos t\,[2\cos t\cos(4t) - \sin t\sin(4t)]$$

Example 5: Find the derivative of $y = x\tan(2x-1)$

$$\frac{dy}{dx} = 1\tan(2x-1) + \sec^2(2x-1)(2)(x)$$

$$\frac{dy}{dx} = \tan(2x-1) + 2x\sec^2(2x-1)$$