L2 – Vector Addition Unit 4

MCV4U

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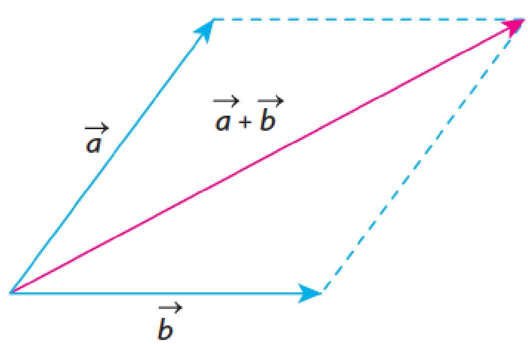
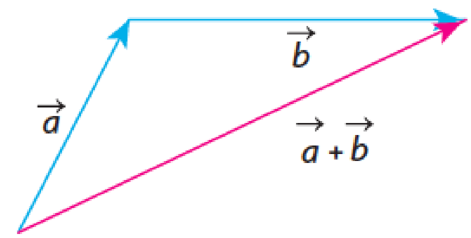
Part 1: Adding Vectors

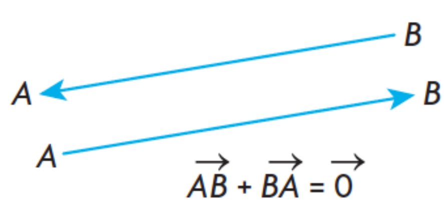
When you add two or more vectors, you are finding a single vector, called the **RESULTANT**, that has the same effect as the original vectors applied one after the other.

Two methods:

Parallelogram	Tip to Tail (triangle)
To determine the sum of any two vectors $\vec{a}$ and $\vec{b}$, arranged tail-to-tail, complete the parallelogram formed by the two vectors. Their sum is the vector that is the diagonal of the constructed parallelogram.	The sum of vectors $\vec{a}$ and $\vec{b}$ can also be found by translating the tail of vector $\vec{b}$ to the head of vector $\vec{a}$. The resultant is the vector from the tail of $\vec{a}$ to the head of $\vec{b}$.
	

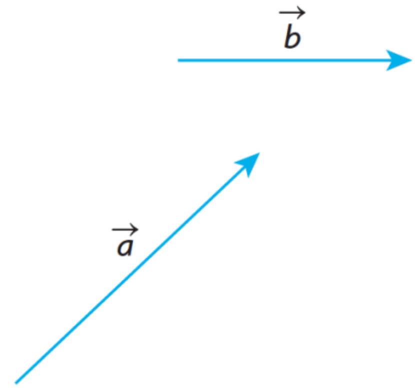
What if we add opposite vectors?

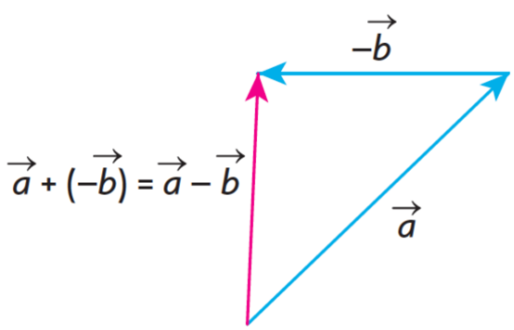
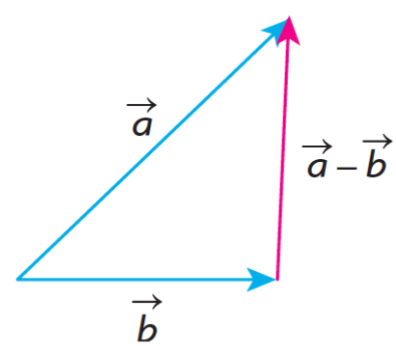
When two opposite vectors are added, the resultant is the zero vector. This means that the combined effect of a vector and its opposite is the zero vector.



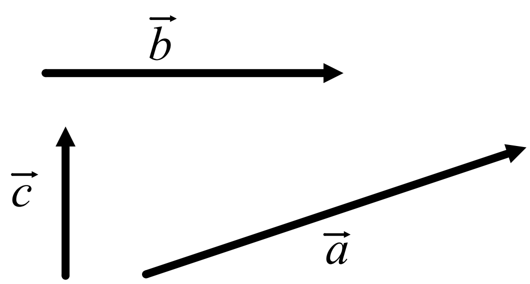
Part 2: Difference of 2 Vectors

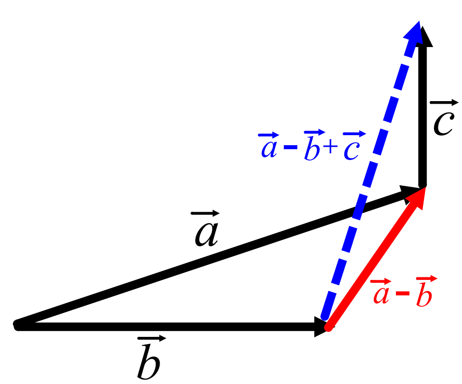
If you want to determine the difference between two vectors, $\vec{a} - \vec{b}$, there are two options:



Adding the Opposite	Tail to Tail
The difference between $\vec{a}$ and $\vec{b}$ is found by adding the opposite of vector $\vec{b}$ to $\vec{a}$ using the triangle law of addition.	Another way to think about $\vec{a} - \vec{b}$ is to arrange the vectors tail to tail. In this case, $\vec{a} - \vec{b}$ is the vector that must be added to $\vec{b}$ to get $\vec{a}$
	

Example 1: Suppose you are given the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ as shown below. Using these three vectors, sketch $\vec{a} - \vec{b} + \vec{c}$





Example 2: In the rectangular box shown below, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{b}$, and $\overrightarrow{OD} = \vec{c}$. Express each of the following vectors in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$.

a) $\overrightarrow{BC} = -\vec{a}$

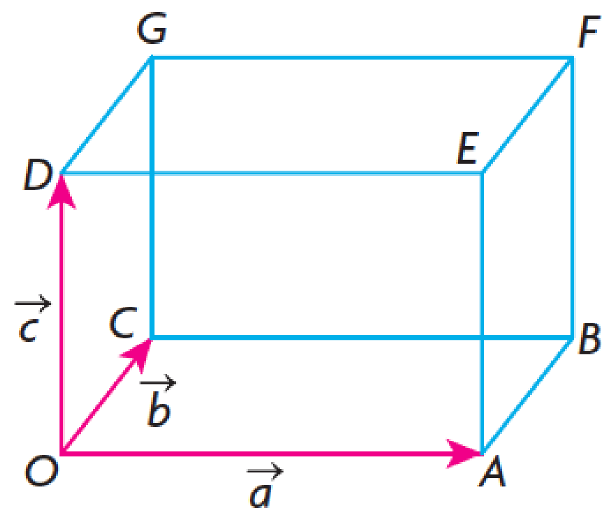
b) $\overrightarrow{GF} = \vec{a}$

c) $\overrightarrow{OB} = \vec{a} + \vec{b}$

d) $\overrightarrow{AC} = \vec{b} - \vec{a}$

e) $\overrightarrow{BG} = \vec{c} + (-\vec{a}) = \vec{c} - \vec{a}$

f) $\overrightarrow{OF} = \vec{a} + \vec{b} + \vec{c}$



Part 3: Properties of Vector Addition

Commutative Property	$\vec{u} + \vec{v} = \vec{v} + \vec{u}$
Associative Property	$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
Identity Property	$\vec{v} + \vec{0} = \vec{v} = \vec{0} + \vec{v}$

Example 3: Simplify each of the following

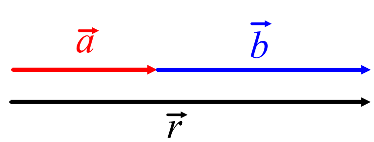
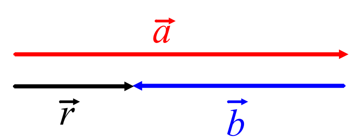
a) $(\vec{u} + \vec{v}) - \vec{u}$

$= (\vec{v} + \vec{u}) + (-\vec{u})$
$= \vec{v} + [\vec{u} + (-\vec{u})]$
$= \vec{v} + \vec{0}$
$= \vec{v}$

b) $[(\vec{p} + \vec{q}) - \vec{p}] - \vec{q}$

$= [\vec{q} + (\vec{p} - \vec{p})] - \vec{q}$
$= (\vec{q} + \vec{0}) - \vec{q}$
$= \vec{q} - \vec{q}$
$= \vec{0}$

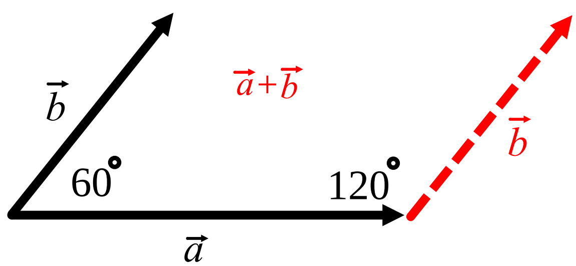
Part 4: Solving Problems involving Vector Addition and Subtraction

If you have two vectors acting in the same direction, the overall magnitude is equal to the sum of the two individual magnitudes.  $\lvert\vec{r}\rvert = \lvert\vec{a}\rvert + \lvert\vec{b}\rvert$	If you have two vectors acting in opposite directions, the overall magnitude is equal to the difference of the two individual magnitudes.  $\lvert\vec{r}\rvert = \lvert\vec{a}\rvert - \lvert\vec{b}\rvert$

However, not all forces act in the same or opposite direction. Therefore, we will need some trigonometry to determine the magnitude of resultant vectors.

Rule	When to Use It	
Pythagorean Theorem $a^2 + b^2 = c^2$	Right Triangle Know: 2 sides Want: 3rd side	
$S\frac{O}{H}C\frac{A}{H}T\frac{O}{a}$	Right Triangle Know: 2 sides Want: Angle (use inverse ratio)	Right Triangle Know: 1 side, 1 angle Want: Side
Sine Law $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Oblique Triangle (no right angle) Know: 2 sides and opposite angle Want: Angle	Oblique Triangle (no right angle) Know: 1 side and all angles Want: Side
Cosine Law $a^2 = b^2 + c^2 - 2bc(\cos A)$ $\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$	Oblique Triangle Know: 2 sides and contained angle Want: 3rd side (use top formula)	Oblique Triangle Know: All 3 sides Want: Angle (use bottom formula)

Example 4: Given vectors $\vec{a}$ and $\vec{b}$ such that the angle between the two vectors is $60°$, $|\vec{a}| = 3$, and $|\vec{b}| = 2$, determine $|\vec{a} + \vec{b}|$.



Note: *"angle between vectors" means the angle between the vectors when placed tail to tail.*

Translate them tip to tail to determine the resultant vector.

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(120)$$

$$|\vec{a} + \vec{b}|^2 = (3)^2 + (2)^2 - 2(3)(2)\cos(120)$$

$$|\vec{a} + \vec{b}|^2 = 13 - 12\left(-\frac{1}{2}\right)$$

$$|\vec{a} + \vec{b}|^2 = 19$$

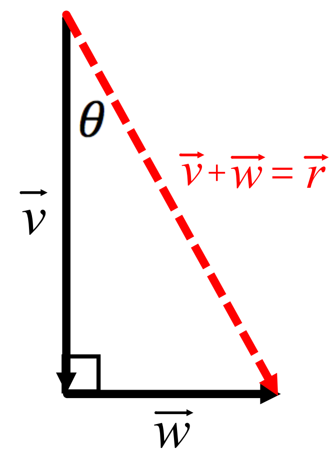
$$|\vec{a} + \vec{b}| = \sqrt{19}$$

Example 5: An airplane heads due south at a speed of 300 km/h and meets a wind from the west at 100 km/h. What is the resultant velocity of the airplane (relative to the ground)?

Let $\vec{v}$ represent the velocity of the airplane without the wind.

Let $\vec{w}$ represent the velocity of the wind.

Let $\vec{r}$ represent the resultant velocity of airplane with wind taken in to account relative to a fixed point on the ground.



Magnitude of resultant (speed):

$|\vec{r}|^2 = |\vec{v}|^2 + |\vec{w}|^2$

$|\vec{r}|^2 = (300)^2 + (100)^2$

$|\vec{r}|^2 = 100\ 000$

$|\vec{r}| \cong 316.23$ km/h

Direction:

$\tan\theta = \frac{100}{300}$

$\theta = \tan^{-1}\left(\frac{1}{3}\right)$

$\theta \cong 18.4°$

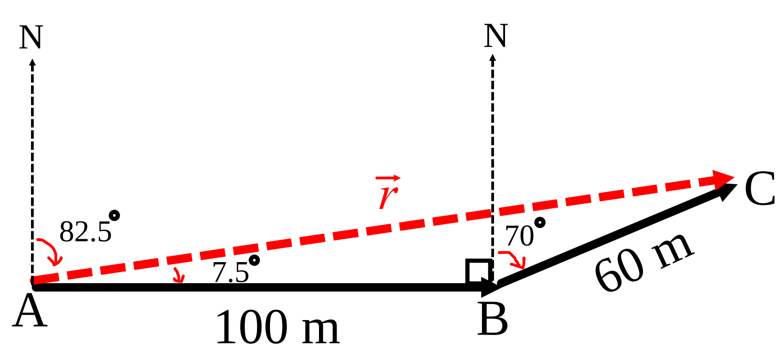
Therefore, the plane is heading $S18.4°E$ at a speed of 316.23 km/h.

Example 6: In an orienteering race, you walk 100 m due east and then walk $N70°E$ for 60 m. How far are you from your starting position, and at what bearing?

$|\vec{r}|^2 = |\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 - 2|\overrightarrow{AB}||\overrightarrow{BC}|\cos(160)$

$|\vec{r}|^2 = (100)^2 + (60)^2 - 2(100)(60)\cos(160)$

$|\vec{r}| \cong 157.7$ m



$\frac{157.7}{\sin 160} = \frac{60}{\sin A}$

$\sin A = \frac{60\sin 160}{157.7}$

$\angle A = \sin^{-1}(\frac{60\sin 160}{157.7})$

$\angle A \cong 7.5°$

You have travelled about 157.7 m at a quadrant bearing of $N82.5°E$.