L3 – Applications of the Dot Product Unit 5
MCV4U
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Warm-Up

Example 1: A desk is pushed with a force of 50 N at an angle of 45 degrees below the horizontal. If the desk is pushed 5 meters, how much work is done?

> ***Remember:*** *Mechanical work is the product of the magnitude of the displacement travelled by an object and the magnitude of the force applied in the direction of the motion.*

Part 1: Angle Between 2 Vectors

To determine the angle between two vectors, you can rearrange the dot product formula, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, to isolate $\cos\theta$:

Example 2: Determine the angle between each pair of vectors.

a) $\vec{g} = [5, 1]$ and $\vec{h} = [-3, 8]$

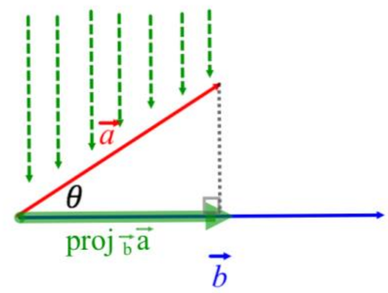
b) $\vec{a} = [-3, 6]$ and $\vec{b} = [4, 2]$

Part 2: Vector Projections

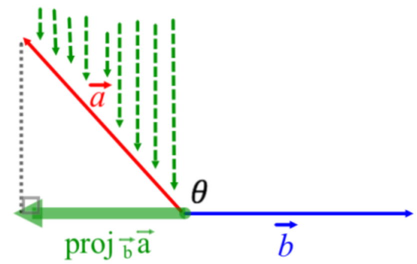
You can think of a vector projection like a shadow. The vertical arrows in the diagrams represent light from above.

Think of the projection of $\vec{a}$ on $\vec{b}$ as the shadow that $\vec{a}$ casts on $\vec{b}$.

If the angle between $\vec{a}$ and $\vec{b}$ is less than $90°$, then the projection of $\vec{a}$ on $\vec{b}$, or $proj_{\vec{b}}\,\vec{a}$, is the vector component of $\vec{a}$ in the direction of $\vec{b}$.

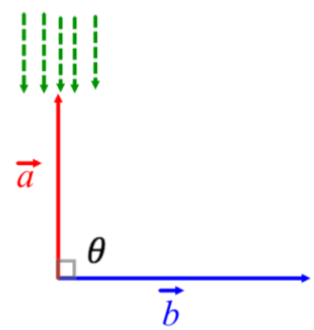


If the angle between $\vec{a}$ and $\vec{b}$ is between $90°$ and $180°$, the direction of $proj_{\vec{b}}\,\vec{a}$ is in the opposite direction of $\vec{b}$.



If $\vec{a}$ is perpendicular to $\vec{b}$, then $\vec{a}$ casts 'no shadow' on to $\vec{b}$. So if $\theta = 90°$, $proj_{\vec{b}}\,\vec{a} = 0$.

Note: *This is why the dot product $\vec{a} \cdot \vec{b}$ would be zero for perpendicular vectors.*



Formulas for Vector Projection:

Geometric Formulas:

Cartesian Formulas:

OR

Formulas for Magnitude of Vector Projection:

If $0° < \theta < 90°$

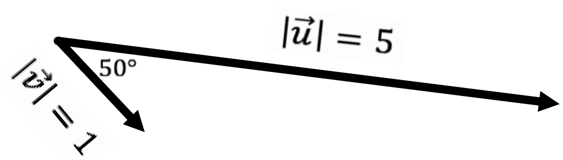
OR

If $90° < \theta < 180°$

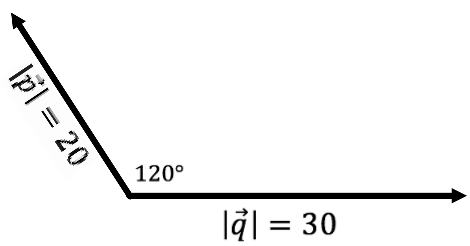
Note: $\frac{\vec{b}}{|\vec{b}|}$ is a unit vector in the direction of $\vec{b}$. Sometimes the symbol $\hat{b}$ is used to denote a unit vector in the direction of $\vec{b}$.

Example 3: Determine the following projections of one vector on another.

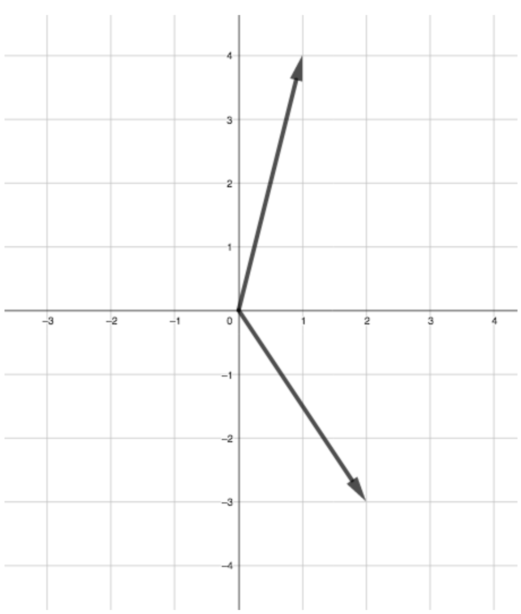
a) Determine the projection of $\vec{u}$ on $\vec{v}$



b) Determine $proj_{\vec{q}}\,\vec{p}$

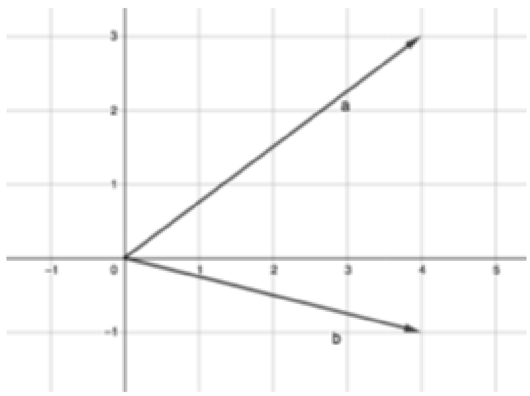


c) Determine the projection of $\vec{d} = [2, -3]$ on $\vec{c} = [1,4]$



d) Find the magnitude of the projection of $\vec{a} = [4,3]$ on $\vec{b} = [4,-1]$

e) Find the projection of $\vec{a} = [4,3]$ on $\vec{b} = [4,-1]$



Part 3: Dot Product with Sales

Example 4: A shoe store sold 350 pairs of Nike shoes and 275 pairs of Adidas shoes in a year. Nike shoes sell for $175 and Adidas shoes sell for $250.

a) Write a Cartesian vector, $\vec{s}$, to represent the numbers of pairs of shoes sold.

b) Write a Cartesian vector, $\vec{p}$, to represent the prices of the shoes.

c) Find the dot product $\vec{s} \cdot \vec{p}$. What does this dot product represent?