L3 – 4.5 Double Angle Formulas
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Part 1: Proofs of Double Angle Formulas

Example 1: Prove $\sin(2x) = 2\sin x \cos x$

LS	RS
$= \sin(2x)$ $= \sin(x + x)$ $= \sin x \cos x + \cos x \sin x$ $= 2\sin x \cos x$	$= 2\sin x \cos x$

LS = RS

Example 2: Prove $\cos(2x) = \cos^2 x - \sin^2 x$

LS	RS
$= \cos(2x)$ $= \cos(x + x)$ $= \cos x \cos x - \sin x \sin x$ $= \cos^2 x - \sin^2 x$	$= \cos^2 x - \sin^2 x$

LS = RS

Note: *There are alternate versions of $\cos 2x$ where either $\cos^2 x$ OR $\sin^2 x$ are changed using the Pythagorean Identity.*

Double Angle Formulas

$$\sin(2x) = 2\sin x\cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos(2x) = 2\cos^2 x - 1$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$$

Part 2: Use Double Angle Formulas to Simplify Expressions

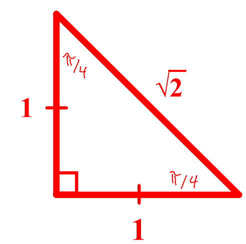
Example 1: Simplify each of the following expressions and then evaluate

a) $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= \sin\left[2\left(\frac{\pi}{8}\right)\right]$

$= \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$

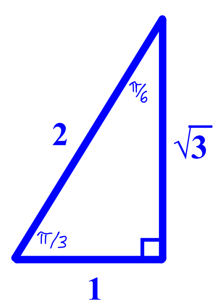


b) $\frac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}}$

$= \tan\left[2\left(\frac{\pi}{6}\right)\right]$

$= \tan\frac{\pi}{3}$

$= \sqrt{3}$



Part 3: Determine the Value of Trig Ratios for a Double Angle

If you know one of the primary trig ratios for any angle, then you can determine the other two. You can then determine the primary trig ratios for this angle doubled.

Example 2: If $\cos\theta = -\frac{2}{3}$ and $\frac{\pi}{2} \leq \theta \leq \pi$, determine the value of $\cos(2\theta)$ and $\sin(2\theta)$

We can solve for $\cos(2\theta)$ without finding the sine ratio if we use the following version of the double angle formula:

$$\cos(2\theta) = 2\cos^2\theta - 1$$

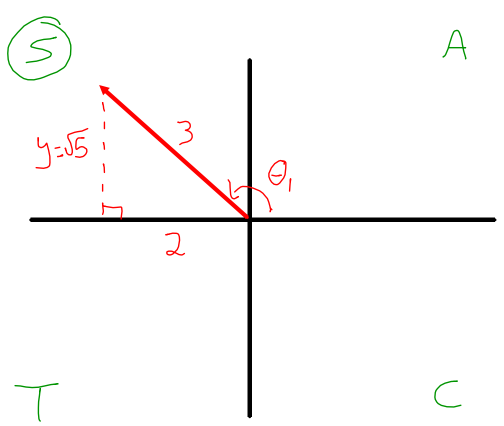
$$\cos(2\theta) = 2\left(-\frac{2}{3}\right)^2 - 1$$

$$\cos(2\theta) = 2\left(\frac{4}{9}\right) - 1$$

$$\cos(2\theta) = \frac{8}{9} - \frac{9}{9}$$

$$\cos(2\theta) = -\frac{1}{9}$$

Now to find $\sin(2\theta)$:



$2^2 + y^2 = 3^2$

$y^2 = 5$

$y = \sqrt{5}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

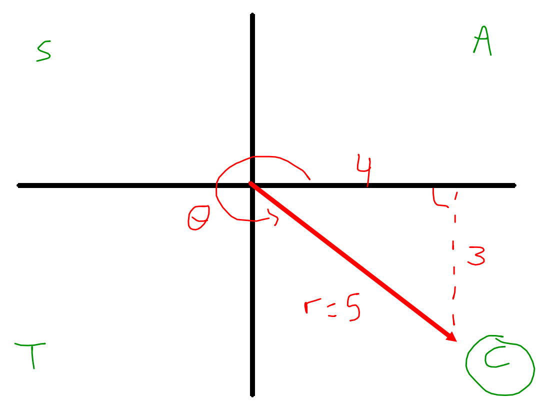
$= 2\left(\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$

$= \frac{-4\sqrt{5}}{9}$

$\cos(2\theta) = -\frac{1}{9}$ and $\sin(2\theta) = -\frac{4\sqrt{5}}{9}$

Example 3: If $\tan\theta = -\frac{3}{4}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, determine the value of $\cos(2\theta)$.

We are given that the terminal arm of the angle lies in quadrant **4**:



$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25}$$
$$= \frac{7}{25}$$

$$3^2 + 4^2 = r^2$$
$$25 = r^2$$
$$r = 5$$

$$\boxed{\cos(2\theta) = \frac{7}{25}}$$