## Part 1: Proof of Product Law of Logarithms

Let  $x = \log_b m$  and  $y = \log_b n$ 

Written in exponential form:

 $b^x = m$  and  $b^y = n$ 

 $mn = b^x b^y$ 

 $mn = b^{x+y}$ 

 $\log_b(mn) = x + y$ 

 $\log_b(mn) = \log_b m + \log_b n$ 

## Part 2: Summary of Log Rules

Power Law of Logarithms	$\log_b x^n = n \log_b x  \text{for } b > 0, b \neq 1, x > 0$		
Product Law of Logarithms	$\log_b(mn) = \log_b m + \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$		
Quotient Law of Logarithms	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n  \text{for } b > 0, b \neq 1, m > 0, n > 0$		
Change of Base Formula	$\log_b m = \frac{\log m}{\log b}$ , $m > 0, b > 0, b \neq 1$		
Exponential to Logarithmic	$y = b^x \rightarrow x = \log_b y$		
Logarithmic to Exponential	$y = \log_b x \rightarrow x = b^y$		
Other useful tips	$\log_a(a^b) = b \qquad \qquad \log_a a = \log_{10} a \qquad \qquad \log_b b = 1$		

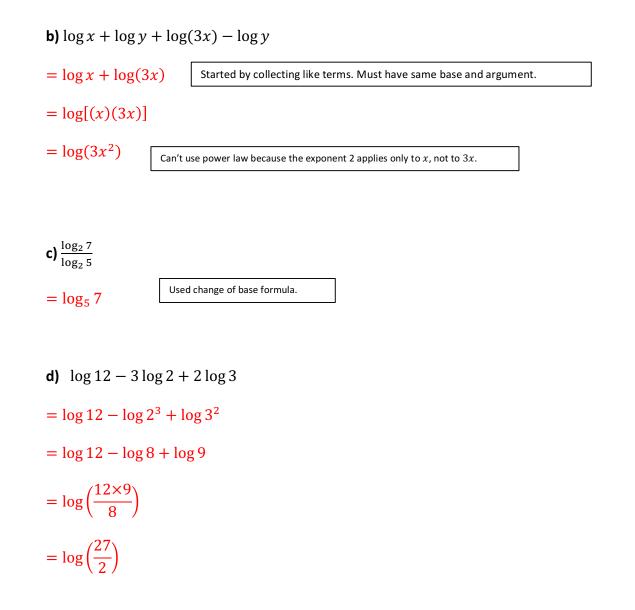
## Part 3: Practice Using Log Rules

Example 1: Write as a single logarithm

**a)**  $\log_5 6 + \log_5 8 - \log_5 16$ 

 $=\log_5\left(\frac{6\times 8}{16}\right)$ 

 $= \log_5 3$ 



## Example 2: Write as a single logarithm and then evaluate

<b>a)</b> $\log_8 4 + \log_8 16$	<b>b)</b> $\log_3 405 - \log_3 5$	<b>c)</b> $2\log 5 + \frac{1}{2}\log 16$
$= \log_8(4 \times 16)$	$=\log_3\left(\frac{405}{5}\right)$	$= \log 5^2 + \log \sqrt{16}$
$=\log_8 64$	$= \log_3 81$	$= \log 25 + \log 4$
$=\frac{\log 64}{\log 8}$	$=\frac{\log 81}{\log 3}$	$= \log(25 \times 4)$
= 2	log 3	= log 100
	= 4	= 2

a) 
$$\log_3(xy)$$
b)  $\log 20$ c)  $\log(ab^2c)$  $= \log_3 x + \log_3 y$  $= \log 4 + \log 5$  $= \log a + \log b^2 + \log c$  $= \log a + 2 \log b + \log c$ 

**Example 4:** Simplify the following algebraic expressions

a) 
$$\log\left(\frac{\sqrt{x}}{x^2}\right)$$
  

$$= \log\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right)$$

$$= \log x^{\frac{3}{2}} + \log x^2 - \log x^{\frac{1}{2}}$$

$$= \log \left(\frac{2x-2}{x^2-1}\right)$$

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$$= \log \left(\frac{2(x-1)}{(x-1)(x+1)}\right)$$

$$= \log x^{-\frac{3}{2}}$$

$$= \frac{3}{2}\log x + \frac{4}{2}\log x - \frac{1}{2}\log x$$

$$= \log \frac{2}{x+1}$$

$$= 3\log x$$