L3 – 5.1/5.2 Graphing Trig Functions
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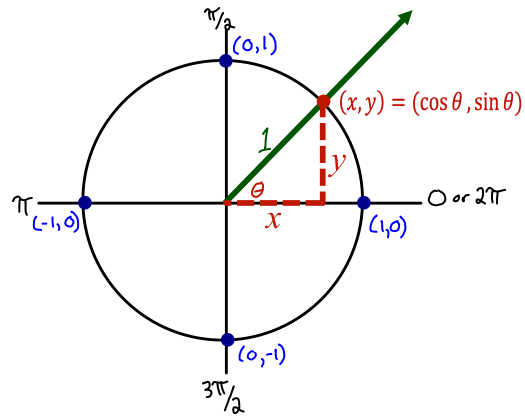
Part 1: Remember the Unit Circle https://www.geogebra.org/m/tKkYHMXC

The unit circle is a circle is a circle that is centered at the origin and has a radius of **1 unit**. On the unit circle, the sine and cosine functions take a simple form:



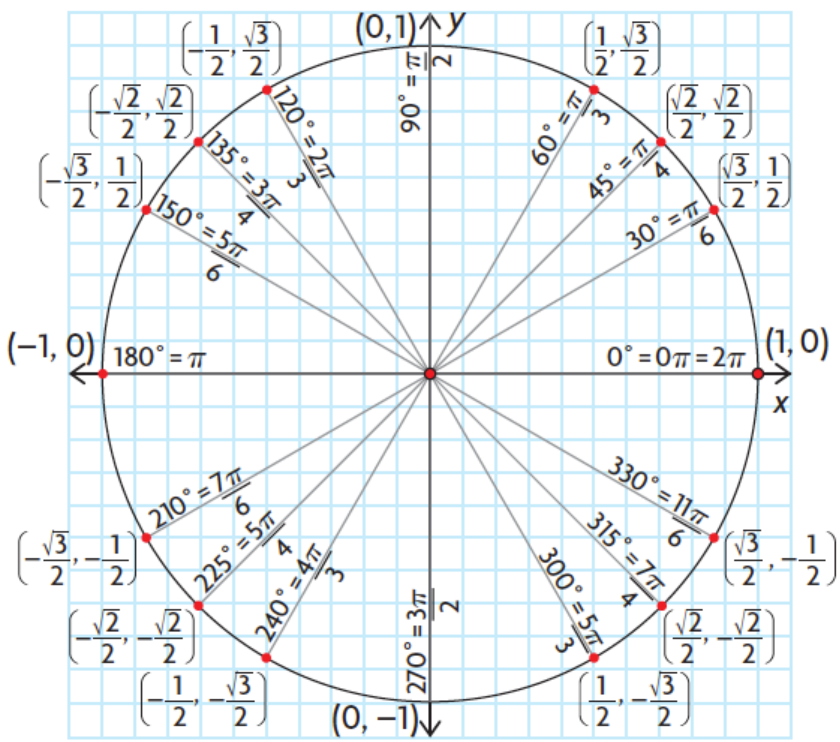
$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

The value of $\sin\theta$ is the ***y*-coordinate** of each point on the unit circle

The value of $\cos\theta$ is the ***x*-coordinate** of each point on the unit circle

$(x, y) = (\cos\theta, \sin\theta)$

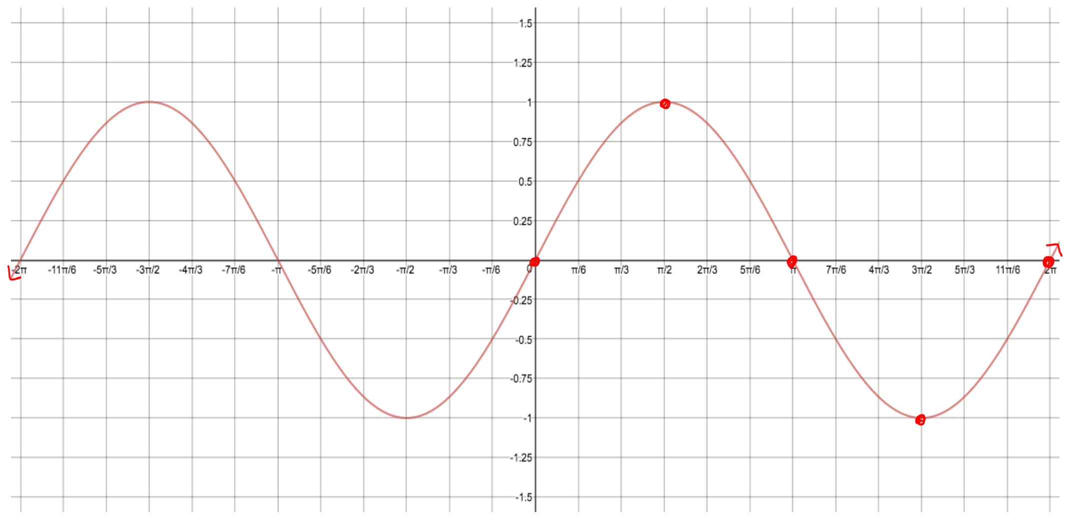


Part 2: Graphing Sine and Cosine

To graph sine and cosine, we will be using a Cartesian plane that has angles for x values.

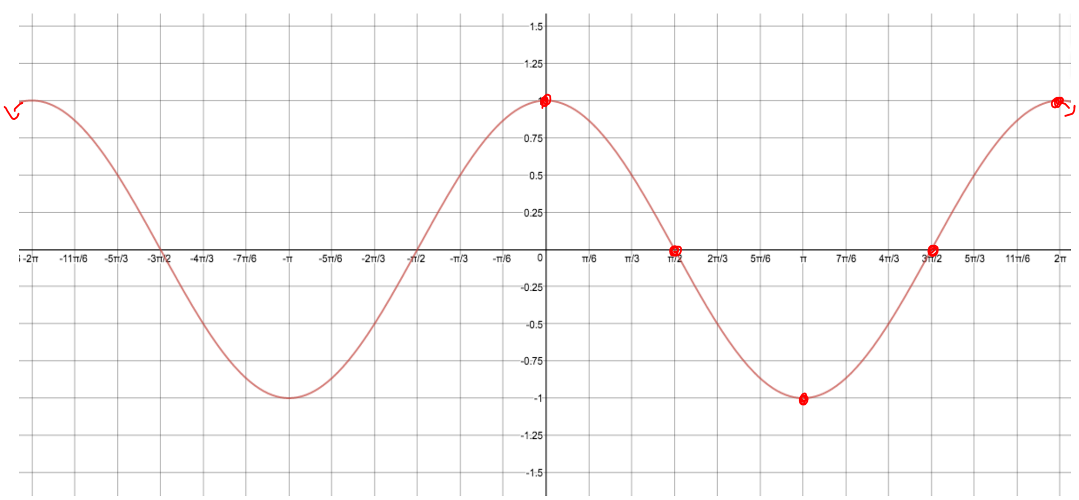
Example 1: Complete the following table of values for the function $f(x) = \sin(x)$. Use special triangles, the unit circle, or a calculator to find values for the function at $30° = \frac{\pi}{6}$ radian intervals.

x	$\sin x$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \sim 0.87$
$\frac{3\pi}{6} = \frac{\pi}{2}$	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2} \sim 0.87$
$\frac{5\pi}{6}$	$\frac{1}{2}$
$\frac{6\pi}{6} = \pi$	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2} \sim -0.87$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2} \sim -0.87$
$\frac{11\pi}{6}$	$-\frac{1}{2}$
$\frac{12\pi}{6} = 2\pi$	0



Example 2: Complete the following table of values for the function $f(x) = \cos(x)$. Use special triangles, the unit circle, or a calculator to find values for the function at $30° = \frac{\pi}{6}$ radian intervals.

x	$\cos x$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2} \sim 0.87$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{1}{2}$
$\frac{3\pi}{6} = \frac{\pi}{2}$	0
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\frac{1}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2} \sim -0.87$
$\frac{6\pi}{6} = \pi$	-1
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2} \sim -0.87$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{1}{2}$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	0
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$\frac{1}{2}$
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2} \sim 0.87$
$\frac{12\pi}{6} = 2\pi$	1



Properties of both Sine and Cosine Functions

Domain: $\{X \in \mathbb{R}\}$

Range: $\{Y \in \mathbb{R} | -1 \leq y \leq 1\}$

Period: 2π radians

Amplitude: $\frac{max-min}{2} = \frac{1-(-1)}{2} = 1$

PERIOD: *the horizontal length of one cycle on a graph.*

AMPLITUDE: *half the distance between the maximum and minimum values of a periodic function.*

Part 3: Graphing the Tangent Function

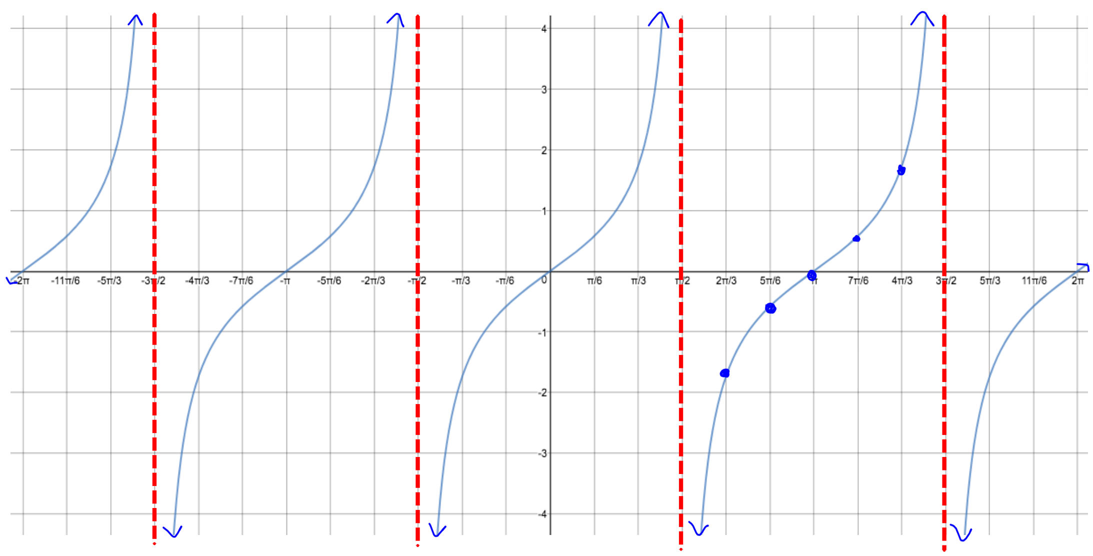
Recall: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Note: Since $\cos\theta$ is in the denominator, any time $\cos\theta = 0$, $\tan\theta$ will be undefined which will lead to a vertical asymptote.

Since $\sin\theta$ is in the numerator, any time $\sin\theta = 0$, $\tan\theta$ will equal 0 which will be an x-intercept.

Example 3: Complete the following table of values for the function $f(x) = \tan(x)$. Use the quotient identity to find y-values.

x	$\tan x$
0	$\frac{0}{1} = 0$
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}} \sim 0.58$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\sqrt{3} \sim 1.73$
$\frac{3\pi}{6} = \frac{\pi}{2}$	$\frac{1}{0} =$ und.
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\sqrt{3} \sim -1.73$
$\frac{5\pi}{6}$	$-\frac{1}{\sqrt{3}} \sim -0.58$
$\frac{6\pi}{6} = \pi$	$\frac{0}{-1} = 0$
$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}} \sim 0.58$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$\sqrt{3} \sim 1.73$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	$\frac{-1}{0} =$ und.
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\sqrt{3} \sim -1.73$
$\frac{11\pi}{6}$	$-\frac{1}{\sqrt{3}} \sim -0.58$
$\frac{12\pi}{6} = 2\pi$	$\frac{0}{1} = 0$



Properties of the Tangent Function

Domain: $\{X \in \mathbb{R} \mid x \neq \frac{\pi+2k\pi}{2}\}$ where $k \in Z$

Range: $\{Y \in \mathbb{R}\}$

Period: π radians

Amplitude: none (no max or min)

Part 4: Graphing Reciprocal Trig Functions

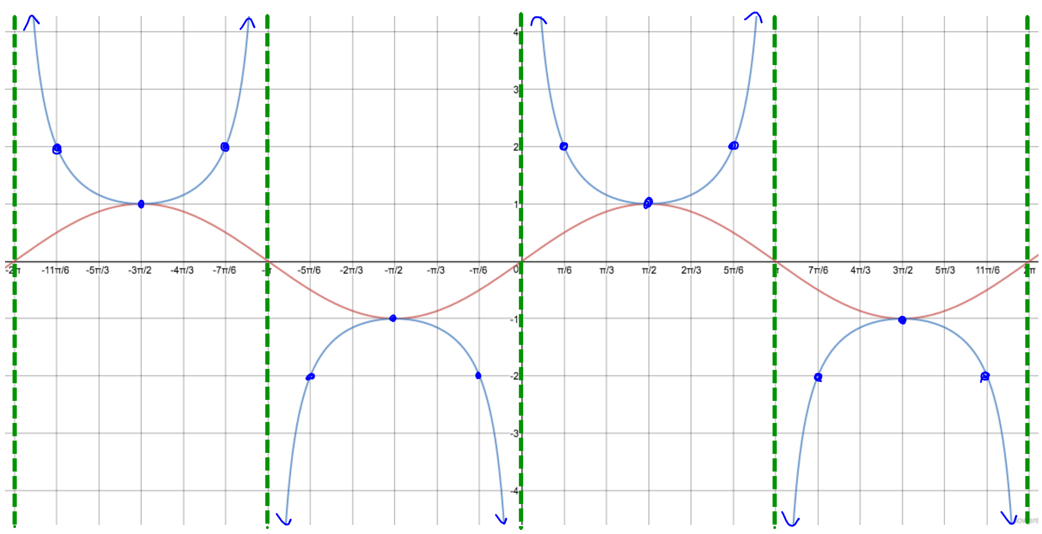
Reciprocal Identities		
$csc\,\theta = \frac{1}{sin\,\theta}$	$sec\,\theta = \frac{1}{cos\,\theta}$	$cot\,\theta = \frac{1}{tan\,\theta}$

The graph of a reciprocal trig function is related to the graph of its corresponding primary trig function in the following ways:

- Reciprocal has a vertical asymptote at each zero of its primary trig function
- Reciprocal has a zero at each vertical asymptote of its primary trig function
- Has the same positive/negative intervals but intervals of increasing/decreasing are reversed
- y-values of 1 and -1 do not change and therefore this is where the reciprocal and primary intersect
- Local min points of the primary become local max of the reciprocal and vice versa.

Example 4: Complete the following table of values for the function $f(x) = \csc(x)$. Use the reciprocal identity to find y-values.

x	$\csc x$
0	und.
$\frac{\pi}{6}$	2
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{3\pi}{6} = \frac{\pi}{2}$	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{5\pi}{6}$	2
$\frac{6\pi}{6} = \pi$	UNDEFINED
$\frac{7\pi}{6}$	-2
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{11\pi}{6}$	-2
$\frac{12\pi}{6} = 2\pi$	und.



Properties of the Cosecant Function

Domain: $\{X \in \mathbb{R} \mid x \neq k\pi\}$ where $k \in Z$

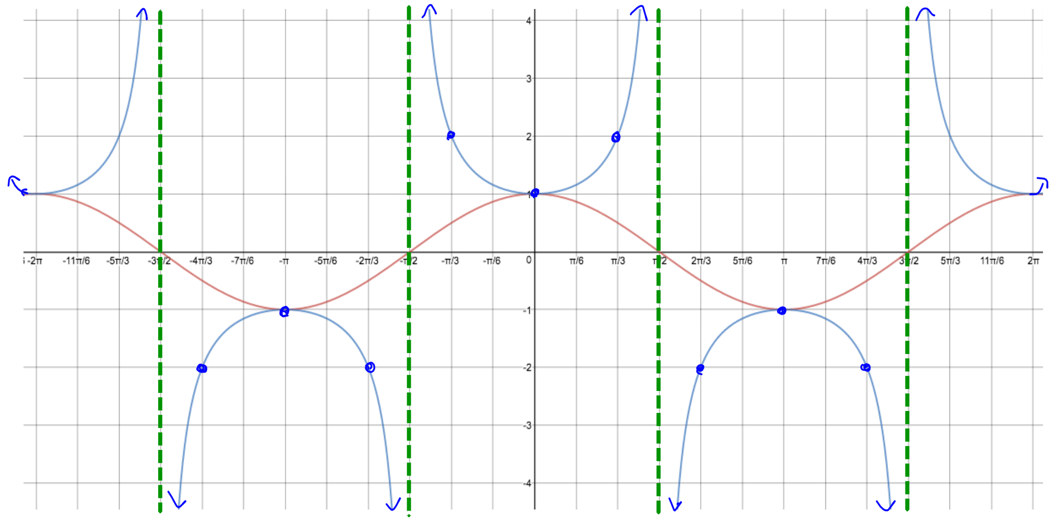
Range: $\{Y \in \mathbb{R} | y \leq -1 \text{ or } y \geq 1\}$

Period: 2π radians

Amplitude: none (no max or min)

Example 5: Complete the following table of values for the function $f(x) = sec\,(x)$. Use the reciprocal identity to find y-values.

x	$\sec x$
0	1
$\frac{\pi}{6}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{2\pi}{6} = \frac{\pi}{3}$	2
$\frac{3\pi}{6} = \frac{\pi}{2}$	und.
$\frac{4\pi}{6} = \frac{2\pi}{3}$	-2
$\frac{5\pi}{6}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{6\pi}{6} = \pi$	-1
$\frac{7\pi}{6}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	-2
$\frac{9\pi}{6} = \frac{3\pi}{2}$	und.
$\frac{10\pi}{6} = \frac{5\pi}{3}$	2
$\frac{11\pi}{6}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{12\pi}{6} = 2\pi$	1



Properties of the Secant Function

Domain: $\{X \in \mathbb{R} \mid x \neq \frac{\pi+2k\pi}{2}\}$ where $k \in Z$

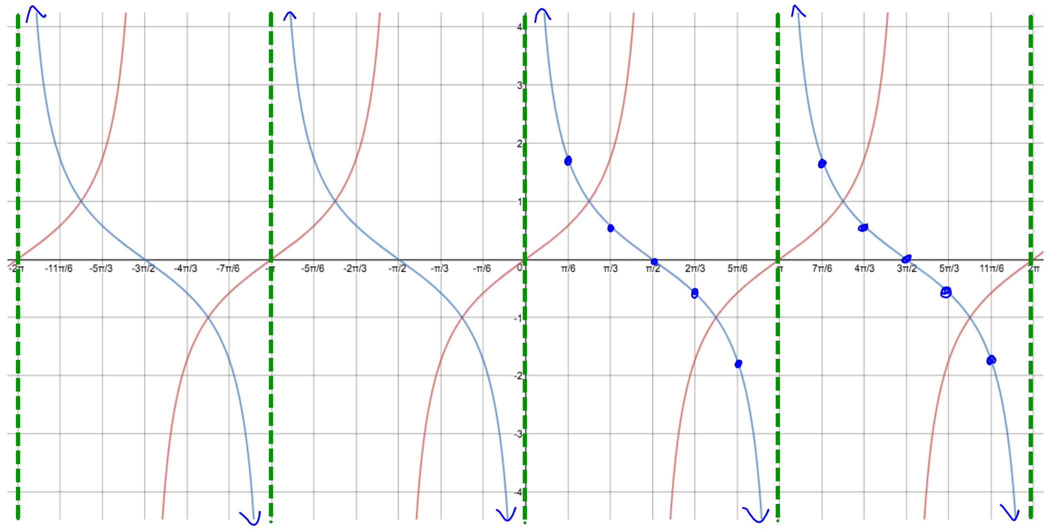
Range: $\{Y \in \mathbb{R} | y \leq -1 \; or \; y \geq 1\}$

Period: 2π radians

Amplitude: none (no max or min)

Example 6: Complete the following table of values for the function $f(x) = \cot(x)$. Use the reciprocal identity to find y-values.

x	$\cot x$
0	und.
$\frac{\pi}{6}$	$\sqrt{3} \sim 1.73$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{1}{\sqrt{3}} \sim 0.58$
$\frac{3\pi}{6} = \frac{\pi}{2}$	0
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\frac{1}{\sqrt{3}} \sim -0.58$
$\frac{5\pi}{6}$	$-\sqrt{3} \sim -1.73$
$\frac{6\pi}{6} = \pi$	und.
$\frac{7\pi}{6}$	$\sqrt{3} \sim 1.73$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$\frac{1}{\sqrt{3}} \sim 0.58$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	0
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{1}{\sqrt{3}} \sim -0.58$
$\frac{11\pi}{6}$	$-\sqrt{3} \sim -1.73$
$\frac{12\pi}{6} = 2\pi$	und.



Properties of the Cotangent Function

Domain: $\{X \in \mathbb{R} \mid x \neq k\pi\}$ where $k \in Z$

Range: $\{Y \in \mathbb{R}\}$

Period: π radians

Amplitude: none (no max or min)