L3 – Newton Quotient
MHF4U
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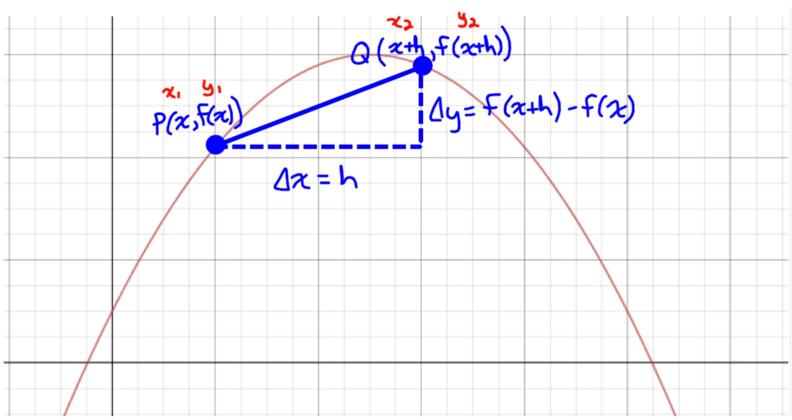
Part 1: Difference between Average and Instantaneous Rates of Change

Average Rate of Change	Instantaneous Rate of Change
Over an interval	At one exact point
Slope of secant	Slope of tangent
Calculated	Estimated
m	$\frac{dy}{dx}$

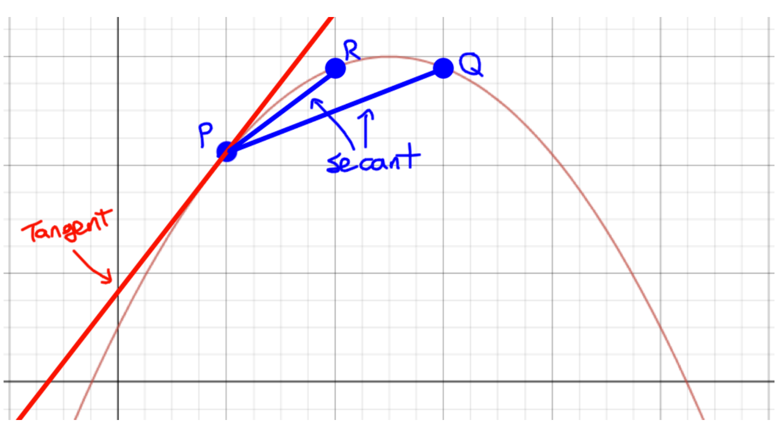
Part 2: Estimating Instantaneous Rate of Change Using Difference Quotient

Looking at the graph of $f(x) = -0.2(x-5)^2 + 6$, points $P(x, f(x))$ and $Q(x+h, f(x+h))$ are on the graph of the function. By connecting the points we have a secant line.

Finding the slope of the secant line will tell us the average rate of change between the two points.



The closer we move point Q towards point P, the closer the average rate of change will get to the instantaneous rate of change at P. In other words, the slope of the secant will get closer to the slope of the tangent line at P. Notice that the slope of PR is much closer to the tangent's slope than the slope of PQ is.



The smaller the difference in the x values (smaller the secant line), the closer we get to the actual instantaneous rate of change. What if we put the difference in the x values at 0? This would give us the instantaneous rate of change exactly, but how do we divide by 0?

We have to find the LIMIT of the secant slopes as the Δx approaches 0.

It is often useful to find the general equation for instantaneous rate of change of $f(x)$ that can be used to find the slope of the tangent at any point throughout the domain. This equation is called the 'derivative' and is written as $f'(x)$.

Newton Quotient:

The instantaneous rate of change at any x for $y = f(x)$ is

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This is called "Derivative from First Principles."

Example 1: Find an equation for the instantaneous rate of change of $f(x) = \frac{1}{4}x^2 - x + 1$. Then use the equation to find $f'(4)$.

Note: *To find a general formula for instantaneous rate of change, leave the x's in the equation.*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{4}(x+h)^2 - (x+h) + 1 - \left(\frac{1}{4}x^2 - x + 1\right)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{4}(x^2 + 2xh + h^2) - x - h + 1 - \frac{1}{4}x^2 + x - 1}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{\frac{1}{4}x^2} + \frac{1}{2}xh + \frac{1}{4}h^2 \cancel{- x} - h \cancel{+ 1} \cancel{- \frac{1}{4}x^2} \cancel{+ x} \cancel{- 1}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{2}xh + \frac{1}{4}h^2 - h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}\left(\frac{1}{2}x + \frac{1}{4}h - 1\right)}{\cancel{h}}$$

$$f'(x) = \frac{1}{2}x + \frac{1}{4}(0) - 1$$

$$f'(x) = \frac{1}{2}x - 1$$

$$f'(4) = \frac{1}{2}(4) - 1$$

$$f'(4) = 1$$

Example 2: Find a formula for the instantaneous rate of change for the function $f(x) = x^2 - 5x + 4$

$f(x+h) = (x+h)^2 - 5(x+h) + 4$

$= x^2 + 2xh + h^2 - 5x - 5h + 4$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h + 4 - (x^2 - 5x + 4)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h + 4 - x^2 + 5x - 4}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 - 5h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h(2x + h - 5)}{h}$$

$$f'(x) = \lim_{h \to 0} 2x + h - 5$$

$$f'(x) = 2x + 0 - 5$$

$$f'(x) = 2x - 5$$