L3 – Velocity, Acceleration, and Second Derivatives Unit 1
MCV4U
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Part 1: Second Derivatives

The second derivative of a function is determined by differentiating the first derivative of the function.

Example 1: For the function $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 4$

a) Calculate $f'(x)$

$f'(x) = x^2 - 2x - 3$

b) When is $f'(x) = 0$?

$0 = x^2 - 2x - 3$

$0 = (x - 3)(x + 1)$

$x_1 = 3 \qquad x_2 = -1$

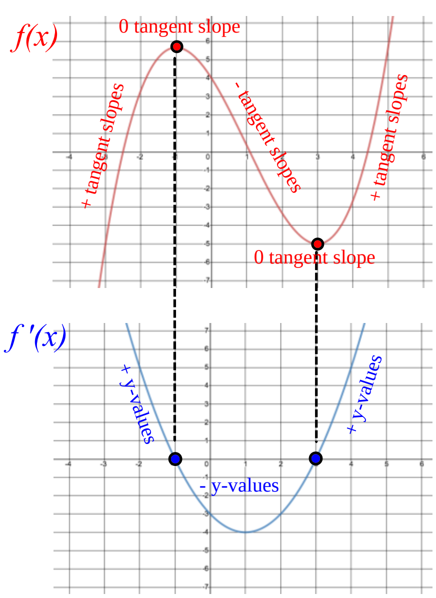
c) Calculate $f''(x)$

$f''(x) = 2x - 2$

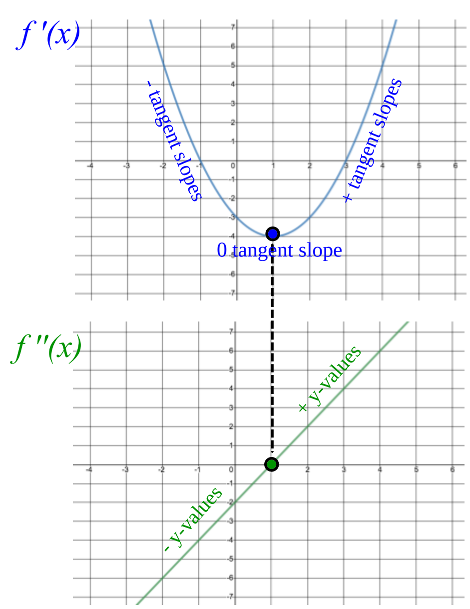
d) When is $f''(x) = 0$?

$0 = 2x - 2$

$x = 1$



Negative y-values from $-1 < x < 3$ on the graph of $f'(x)$ correspond to the negative tangent slopes on the graph of $f(x)$. The x-intercepts of -1 and 3 on the graph of $f'(x)$ correspond to the points on $f(x)$ that has a tangent slope of zero.



Negative y-values for $x < 1$ on the graph of $f''(x)$ correspond to the negative tangent slopes on the graph of $f'(x)$. The x-intercept of 1 on the graph of $f''(x)$ correspond to the point on $f'(x)$ that has a tangent slope of zero.

Part 2: Displacement, Velocity, and Acceleration

	Displacement (s)	**Velocity (v)**	**Acceleration (a)**
Definition	Distance an object has moved from the origin over a period of time (t)	Rate of change of displacement (s) with respect to time (t). Speed with direction.	Rate of change of velocity(v) with respect to time (t)
Relationship	$s(t)$	$v(t) = s'(t)$	$a(t) = v'(t) = s''(t)$
Possible Units	m	m/s	m/s^2

Important: *speed and velocity are often confused for one another. Speed is a scalar quantity. It describes the magnitude of motion but does not describe the direction. Velocity has both magnitude and direction. The sign indicates the direction the object is travelling relative to the origin.*

Example 2: A construction worker accidentally drops a hammer from a height of 90 meters. The height, s, in meters, of the hammer t seconds after it is dropped can be modelled by the function $s(t) = 90 - 4.9t^2$.

a) What is the velocity of the hammer at 1s vs. 4s?

$v(t) = s'(t) = -9.8t$

$v(1) = -9.8(1) = -9.8\ m/s$

$v(4) = -9.8(4) = 39.2\ m/s$

b) When does the hammer hit the ground? When is $s(t) = 0$?

$0 = 90 - 4.9t^2$

$4.9t^2 = 90$

$$t = \pm\sqrt{\frac{90}{4.9}}$$

$t \cong 4.3\ seconds$

c) What is the velocity of the hammer when it hits the ground?

$$v(4.3) = -9.8\left(\sqrt{\frac{90}{4.9}}\right) = -42\ m/s$$

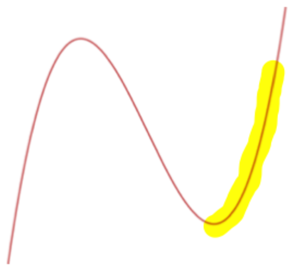
d) Determine the acceleration function.

$a(t) = v'(t) = s''(t) = -9.8$ (acceleration due to gravity)

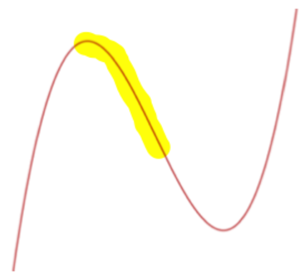
Speeding Up vs. Slowing Down

An object is **speeding up** if the graph of $s(t)$ has a positive slope that is increasing OR has a negative slope that is decreasing. In these scenarios, $v(t) \times a(t) > 0$.

Positive slope increasing
$v(t) \times a(t) = + \times + = +$

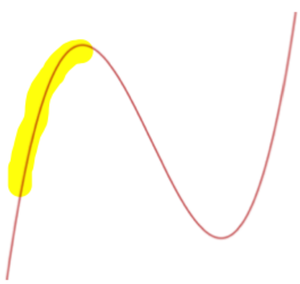


Negative slope decreasing
$v(t) \times a(t) = - \times - = +$

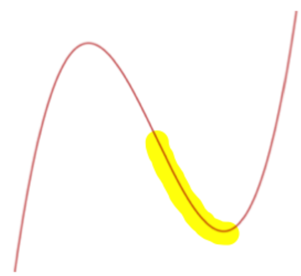


An object is **slowing down** if the graph of $s(t)$ has a positive slope that is decreasing OR has a negative slope that is increasing. In these scenarios, $v(t) \times a(t) < 0$.

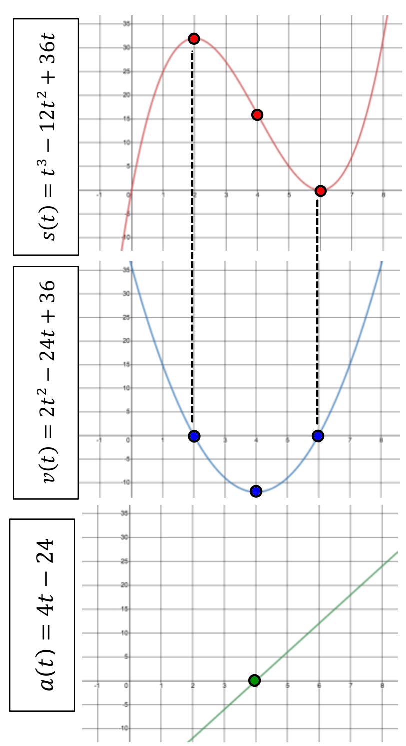
Positive slope decreasing
$v(t) \times a(t) = + \times - = -$



Negative slope increasing
$v(t) \times a(t) = - \times + = -$

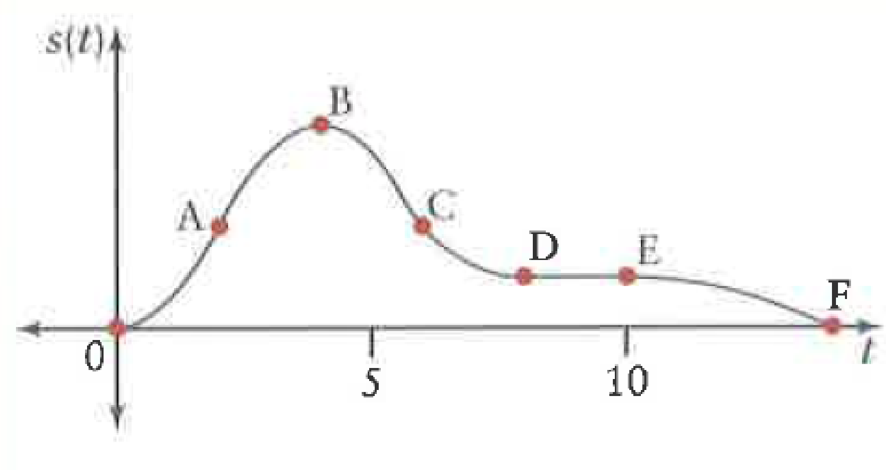


Example 3: The position of a particle moving along a straight line can be modelled by the function below where t is the time in seconds and s is the displacement in meters. Use the graphs of $s(t)$, $v(t)$, and $a(t)$ to determine when the particle is speeding up and slowing down.



Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	Slope of $s(t)$	Motion of particle
$(0,2)$	+	-	-	positive slope that is decreasing	Slowing down and moving forward
$(2,4)$	-	-	+	Negative slope that is decreasing	Speeding up and moving in reverse
$(4,6)$	-	+	-	Negative slope that is increasing	Slowing down and moving in reverse
$(6,8)$	+	+	+	Positive slope that is increasing	Speeding up and moving forward

Example 4: Given the graph of $s(t)$, figure out where $v(t)$ and $a(t)$ are + or – and use this information to state when the particle is speeding up and slowing down.



Interval	$\boldsymbol{v(t)}$	$\boldsymbol{a(t)}$	$v(t) \times \boldsymbol{a(t)}$	**Slope of $\boldsymbol{s(t)}$**	**Motion of particle**
$(0, A)$	+	+	+	Positive and increasing	Speeding up and moving forward
(A, B)	+	-	-	Positive and decreasing	Slowing down and moving forward
(B, C)	-	-	+	Negative and decreasing	Speeding up and moving in reverse
(C, D)	-	+	-	Negative and increasing	Slowing down and moving in reverse
(D, E)	0	0	0	0	Not moving
(E, F)	-	-	+	Negative and decreasing	Speeding up and moving in reverse