L4 – 7.1/7.2 – Solving Exponential Equations
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Part 1: Changing the Base of Powers

Exponential functions can be written in many different ways. It is often useful to express an exponential expression using a different base than the one that is given.

Example 1: Express each of the following in terms of a power with a base of 2.

a) 8 **b)** 4^3 **c)** $\sqrt{16}\times\left(\sqrt[5]{32}\right)^3$ **d)** 12

Part d) shows that any positive number can be expressed as a power of any other positive number.

Example 2: Solve each equation by getting a common base

Remember: if $x^a = x^b$, then $a = b$

a) $4^{x+5} = 64^x$ **b)** $4^{2x} = 8^{x-3}$

Part 2: Solving Exponential Equations

When you have powers in your equation with different bases and it is difficult to write with the same base, it may be easier to solve by taking the _______________ of both sides and applying the ______________ of logarithms to remove the variable from the __________________.

Example 3: Solve each equation

a) $4^{2x-1} = 3^{x+2}$

Take log of both sides
Use power law of logarithms
Use distributive property to expand
Move variable terms to one side
Common factor
Isolate the variable

b) $2^{x+1} = 3^{x-1}$

Part 3: Applying the Quadratic Formula

Sometimes there is no obvious method of solving an exponential equation. If you notice two powers with the same base and an exponent of x, there may be a hidden quadratic.

Example 4: Solve the following equation

Multiply both sides by 2^x

Distribute

Rearrange in to standard form $ax^2 + bx + c = 0$

Solve using quadratic formula

Don’t forget to simplify the radical expression

Now substitute 2^x back in for k and solve

Case 1

Case 2

Part 4: Application Question

Remember:

Equation: $\boldsymbol{y = a(b)^x}$

$a =$ initial amount

$b =$ growth $(b > 1)$ or decay $(0 < b < 1)$ factor

$y =$ future amount

$x =$ number of times a has increased or decreased

To calculate x, use the equation: $x = \frac{total\ time}{time\ it\ takes\ for\ one\ growth\ or\ decay\ period}$

Example 5: A bacteria culture doubles every 15 minutes. How long will it take for a culture of 20 bacteria to grow to a population of 163 840?

Example 6: One minute after a 100-mg sample of Polonium-218 is placed into a nuclear chamber, only 80-mg remains. What is the half-life of polonium-218?