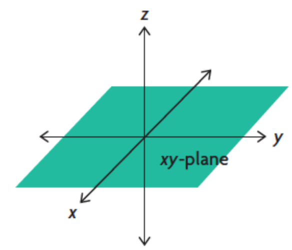
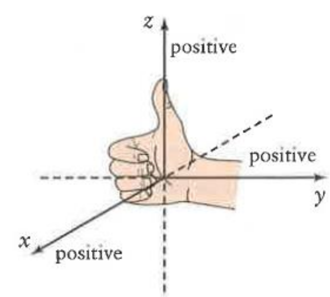
Part 1: Plotting Points in 3-Dimensions



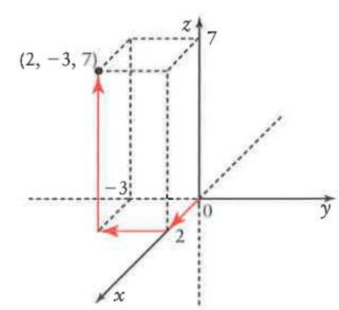
https://www.geogebra.org/3d?lang=en

In placing points in 3-Dimensions (R^3), we choose three axes called x-, y-, and z-axis. Each axis is perpendicular. Each point is written using ordered triples (x, y, z).



There are several ways to choose the orientation of the positive axes, but we will use what is called the right-handed system. If we imagine ourselves looking down the positive z-axis onto the xy plane so that, when the positive x-axis is rotated $90°$ counterclockwise it becomes coincident with the positive y-axis, then this is called the right-handed system.

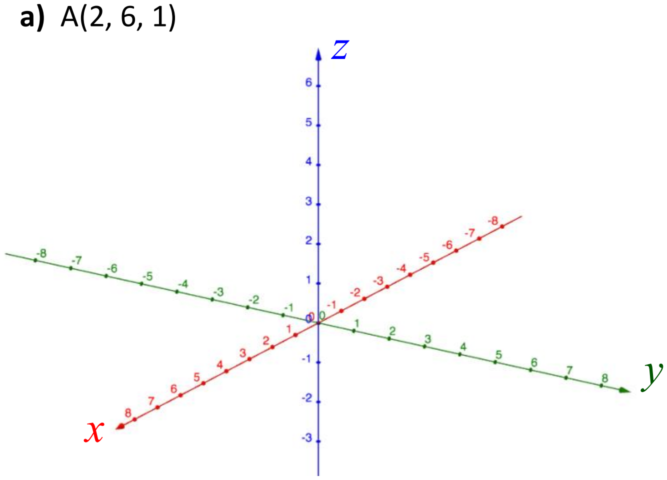
If you curl the fingers of your right hand from the positive x-axis to the positive y-axis your thumb will point along the positive z-axis.



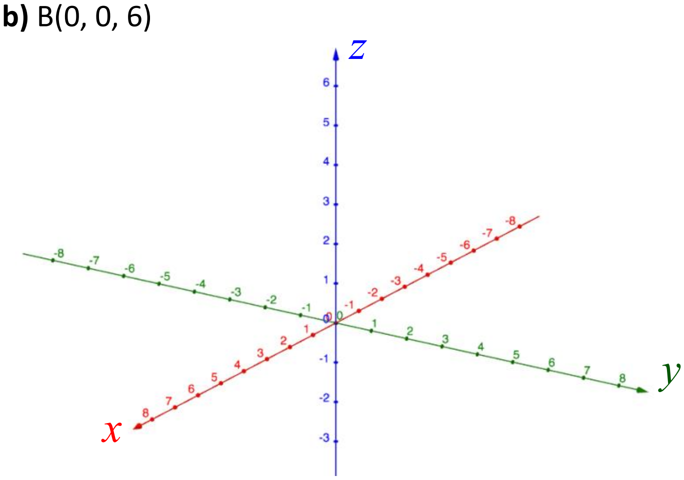
To plot the point of (2, -3, 7), start at the origin. Move two units along the positive x-axis, then 3 units parallel to the negative y-axis, and then 7 units parallel to the positive z-axis.

Example 1: Plot the following points in R^3

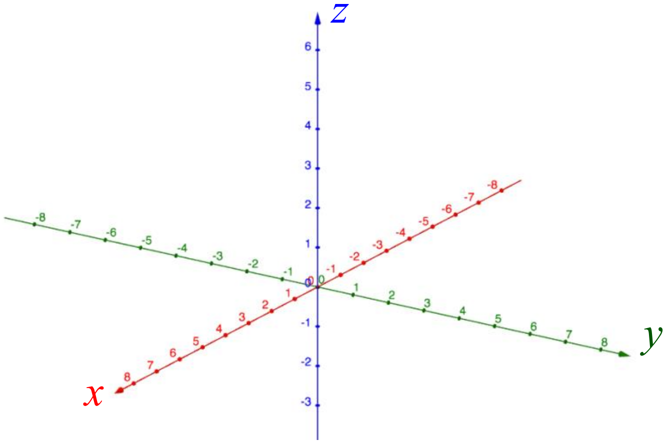
a) A(2, 6, 1)



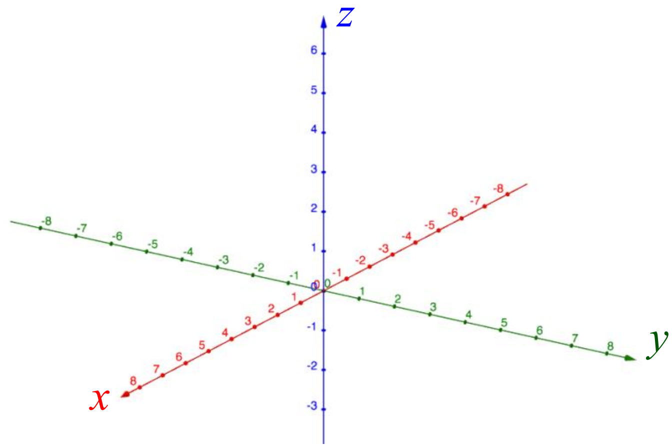
b) B(0, 0, 6)



c) C(2, 3, 0)

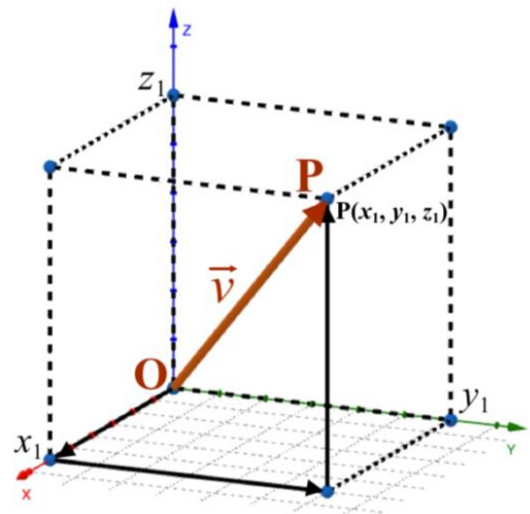


d) D(-1, -3, 4)



__**Part 2: 3-D Cartesian Vectors**__

Let $\vec{v}$ represent a vector in space. If $\vec{v}$ is translated so that its tail is at the origin, O, then its tip will be at some point $P(x_1, y_1, z_1)$. Then $\vec{v}$ is the position vector of the point P, and $\vec{v} = \overrightarrow{OP} = [x_1, y_1, z_1]$.



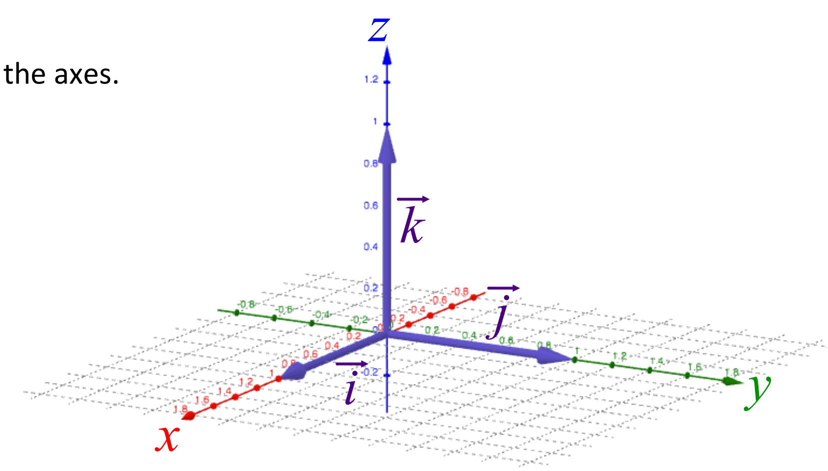
Unit Vectors in R^3

Unit vectors all have a magnitude of 1 and along the axes.
In 3-Dimensions, there are 3 unit vectors:

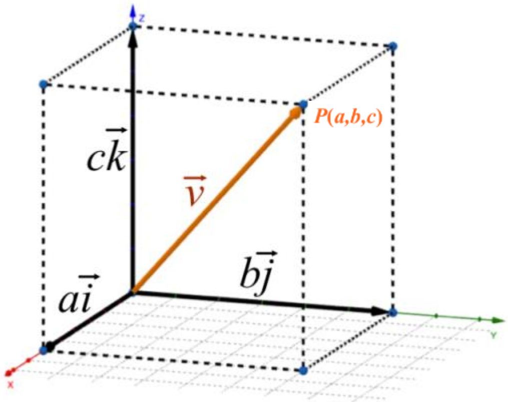
x-axis is $\vec{\imath} = [1,0,0]$

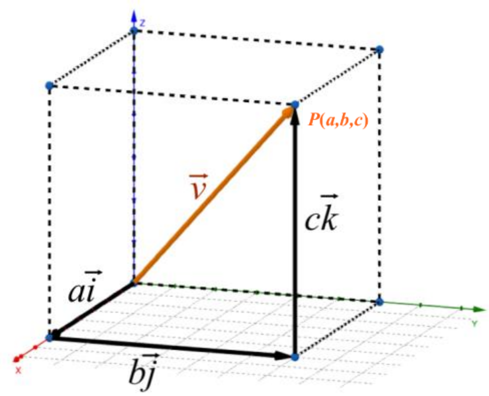
y-axis is $\vec{\jmath} = [0,1,0]$

z-axis is $\vec{k} = [0,0,1]$



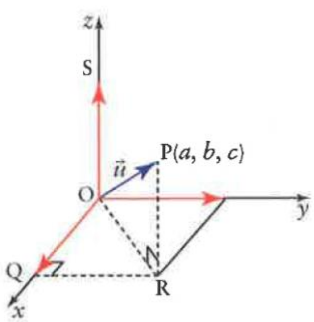
3-D vectors can be written as the sum of multiples of $\vec{\imath}, \vec{\jmath}$, and $\vec{k}$.





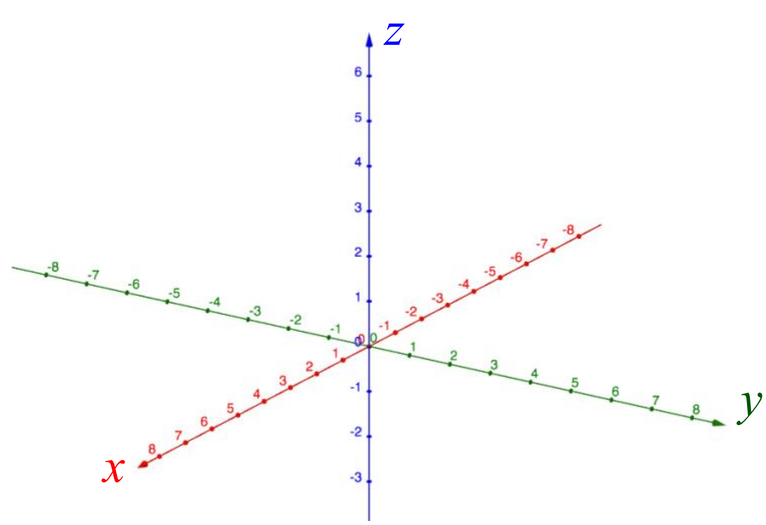
$$\vec{v} = [a, b, c] = [a, 0,0] + [0, b, 0] + [0,0, c] = a\vec{\imath} + b\vec{\jmath} + c\vec{k}$$

Part 3: Magnitude of Vectors in R^3



Example 2: For $\vec{u} = [3, -1, 2]$…

a) sketch the position vector



b) write the vector in terms of $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$

c) find the magnitude

Example 3: For the points $A(1,3,1)$ and B(5, 4, -2)

a) Find the magnitude of $\overrightarrow{AB}$

Vector between 2 points:

$$\overrightarrow{P_1P_2} = [x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1]$$

b) Find a unit vector, $\vec{u}$, in the same direction as $\overrightarrow{AB}$

Tools for 2-D vectors modified for 3-D vectors:

Vector Addition: $\vec{u} + \vec{v} = [u_x + v_x,\ u_y + v_y,\ u_z + v_z]$

Vector Subtraction: $\vec{u} - \vec{v} = [u_x - v_x,\ u_y - v_y,\ u_z - v_z]$

Vector between 2 points: $\overrightarrow{P_1P_2} = [x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1]$

Magnitude of a vector between 2 points: $\left|\overrightarrow{P_1P_2}\right| = \sqrt{(x_2 - x_1)^2 + (\ y_2 - y_1)^2 + (\ z_2 - z_1)^2}$

Dot Product: for $\vec{u} = [u_1, u_2, u_3]$ and $\vec{v} = [v_1, v_2, v_3]$, $\quad \vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$

Example 4: Given the vectors $\vec{u} = [2,3,-5]$, $\vec{v} = [8,-4,3]$, and $\vec{w} = [-6,-2,0]$, simplify each vector expression.

a) $-3\vec{v}$

b) $\vec{u} + \vec{v} + \vec{w}$

c) $|\vec{u} - \vec{v}|$

d) $\vec{u} \cdot \vec{v}$

Example 5: Determine if the vectors $\vec{a} = [6,2,4]$ and $\vec{b} = [9,3,6]$ are collinear.

Example 6: Find a such that $[1,2,3]$ and $[2, a, 6]$ are collinear.

Example 7: Calculate the angle between $\vec{u} = [0, -1, -4]$ and $\vec{v} = [6,1, -2]$

Angle between 2 vectors can be found using:

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

Example 8: Find a vector that is orthogonal to $[3,4,5]$

Example 9: Find the magnitude of the projection of $\vec{u} = [3,-3,2]$ onto $\vec{v} = [5,2,0]$

Formula reminder:

$$\left|proj_{\vec{b}}\,\vec{a}\right| = \left|\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right|$$