L4 – Derivatives of Exponential Functions Unit 3

MCV4U

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Part 1: Review of e and $\ln x$

Properties of e:

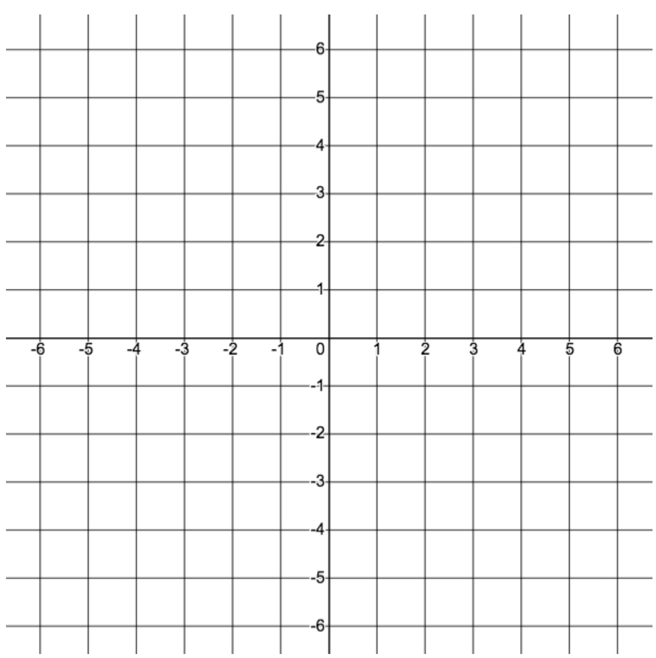
- $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx$
- e is an ____________________ number, similar to π. They are non-terminating and non-repeating.
- $\log_e x$ is known as the ____________________ and can be written as __________.
- Many naturally occurring phenomena can be modelled using base-e exponential and logarithmic functions.
- $\log_e e = \ln e =$

Example 1: Graph the functions $y = e^x$ and $y = \ln x$

$y = e^x$	
x	y

$y = \ln x$	
x	y

Note: $y = \ln x$ is the inverse of $y = e^x$



Example 2: The population of a bacterial culture as a function of time is given by the equation $P(t) = 200e^{0.094t}$, where P is the population after t days.

a) What is the initial population of the bacterial culture?

b) Estimate the population after 3 days.

c) How long will the bacterial culture take to double its population?

d) Re-write this function as an exponential function having base 2.

Use the exponential growth formula:

$$A(t) = A_0(2)^{\frac{t}{D}}$$

where D is the doubling period

Part 2: Derivatives of Exponential Functions

Rule: If $f(x) = b^x$, $f'(x) = b^x \ln b$

Example 3: Determine the derivative of each function

a) $y = 2^x$

b) $y = e^x$

c) $y = 3(2)^x$

d) $y = 3^x + 1$

Notice that $\frac{d}{dx} e^x = e^x$

Example 4: Find the equation of the line that is tangent to the curve $y = 2e^x$ at $x = \ln 3$.

Example 5: A biologist is studying the increase in the population of a particular insect in a provincial park. The population triples every week. Assume the population continues to increase at this rate. Initially, there are 100 insects.

a) Determine the number of insects present after 4 weeks.

b) How fast is the number of insects increasing

i) when they are initially discovered?

ii) at the end of 4 weeks?