L4 – 5.3 Transformations of Trig Functions
MHF4U
Jensen

Part 1: Transformation Properties

$y = a\sin[k(x-d)] + c$ Desmos Demonstration

a	k	d	c
Vertical stretch or compression by a factor of $\lvert a\rvert$. Vertical reflection if $a < 0$ $\lvert a\rvert = amplitude$	Horizontal stretch or compression by a factor of $\frac{1}{\lvert k\rvert}$. Horizontal reflection if $k < 0$. $\frac{2\pi}{\lvert k\rvert} = period$	Phase shift $d > 0; shift\ right$ $d < 0; shift\ left$	Vertical shift $c > 0; shift\ up$ $c < 0; shift\ down$

Example 1: For the function $y = 3\sin\left[\frac{1}{2}\left(\theta + \frac{\pi}{3}\right)\right] - 1$, state the…

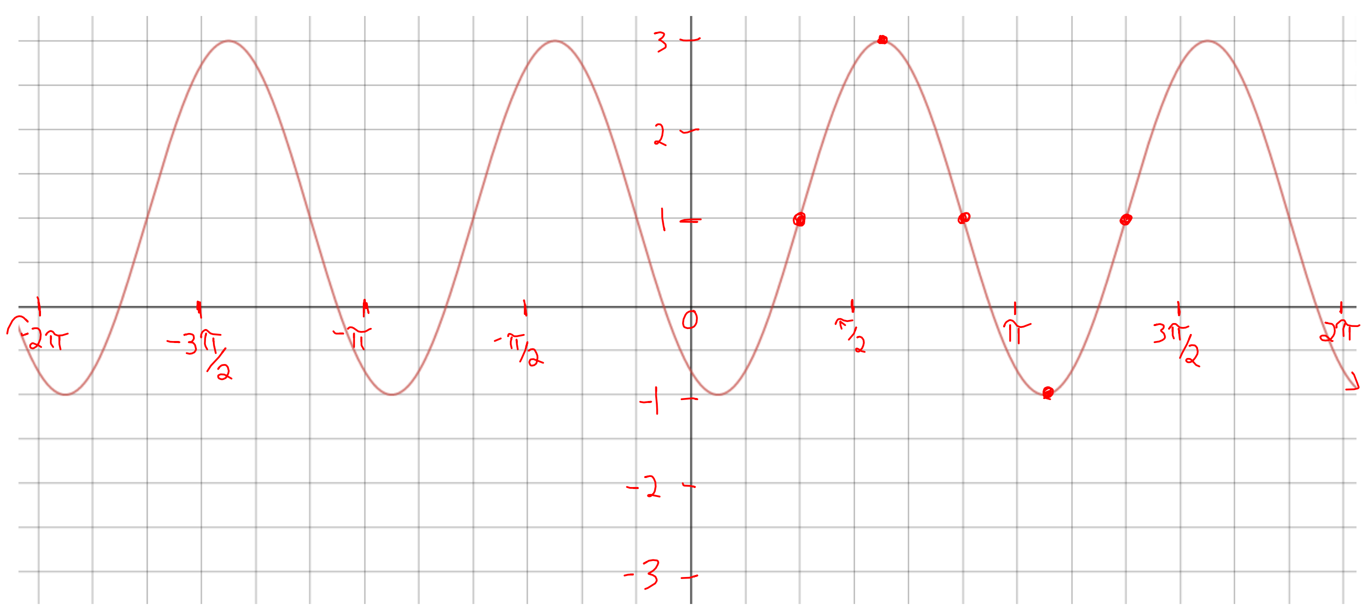
Amplitude: $amp = \lvert a\rvert = 3$	Period: $period = \frac{2\pi}{\lvert k\rvert} = \frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$
Phase shift: $d = -\frac{\pi}{3}$; shift left $\frac{\pi}{3}$	Vertical shift: $c = -1$; shift down 1
Max: $max = c + amp = -1 + 3 = 2$	Min: $min = c - amp = -1 - 3 = -4$

Part 2: Given Equation → Graph Function

Example 2: Graph $y = 2\sin\left[2\left(x - \frac{\pi}{3}\right)\right] + 1$ using transformations. Then state the amplitude and period of the function.

$y = \sin x$	
x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$y = 2\sin\left[2\left(x - \frac{\pi}{3}\right)\right] + 1$	
$\frac{x}{2} + \frac{\pi}{3}$	$2y + 1$
$\frac{\pi}{3} = \frac{2\pi}{6}$	1
$\frac{7\pi}{12} = \frac{3.5\pi}{6}$	3
$\frac{5\pi}{6}$	1
$\frac{13\pi}{12} = \frac{6.5\pi}{6}$	-1
$\frac{4\pi}{3} = \frac{8\pi}{6}$	1



Amplitude: $\frac{max - min}{2} = \frac{3-(-1)}{2} = 2$

Period: π radians

Part 3: Given the Graph → Write the Equation

$y = a\sin[k(x-d)] + c$

a	k	d	c
Find the amplitude of the function: $a = \frac{max - min}{2}$	Find the period (in radians) of the function using a starting point and ending point of a full cycle. $k = \frac{2\pi}{period}$	**for sin x:** x-coordinate of a rising mid-line. **for cos x:** x-coordinate of a maximum point. $d_{sin} = d_{cos} - \frac{\pi}{2k}$ $d_{cos} = d_{sin} + \frac{\pi}{2k}$	Find the vertical shift $c = max - amplitude$ OR $c = \frac{max + min}{2}$ (this finds the 'middle' of the function)

Example 3: Determine the equation of a sine and cosine function that describes the following graph

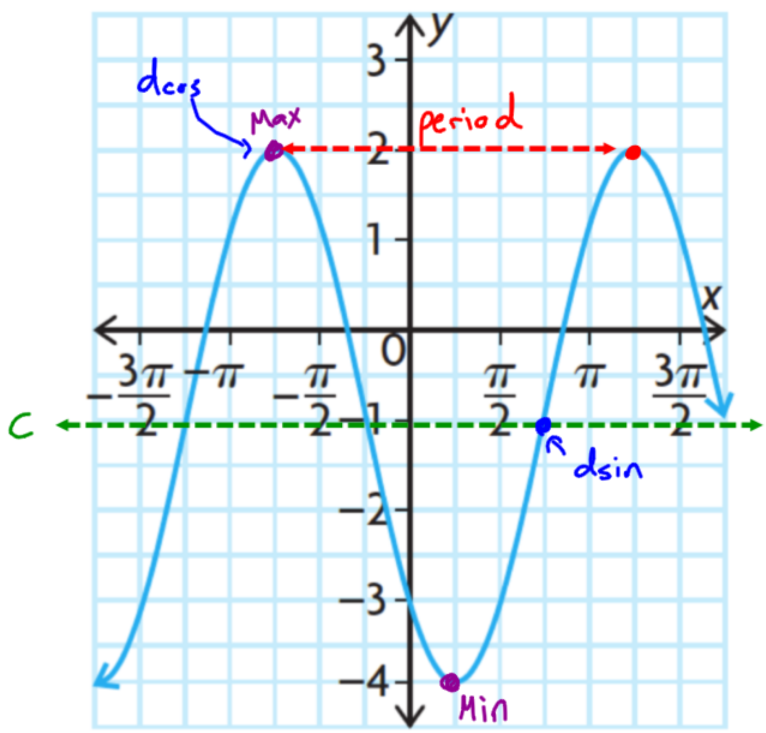
$a = \frac{max - min}{2} = \frac{2-(-4)}{2} = 3$

$k = \frac{2\pi}{period} = \frac{2\pi}{\left(\frac{5\pi}{4} - \left(-\frac{3\pi}{4}\right)\right)} = \frac{2\pi}{2\pi} = 1$

$c = max - |a| = 2 - 3 = -1$

$d_{cos} = \frac{-3\pi}{4}$

$d_{sin} = \frac{3\pi}{4}$



$y = 3\sin\left(x - \frac{3\pi}{4}\right) - 1$

$y = 3\cos\left(x + \frac{3\pi}{4}\right) - 1$

Example 4: Determine the equation of a sine and cosine function that describes the following graph

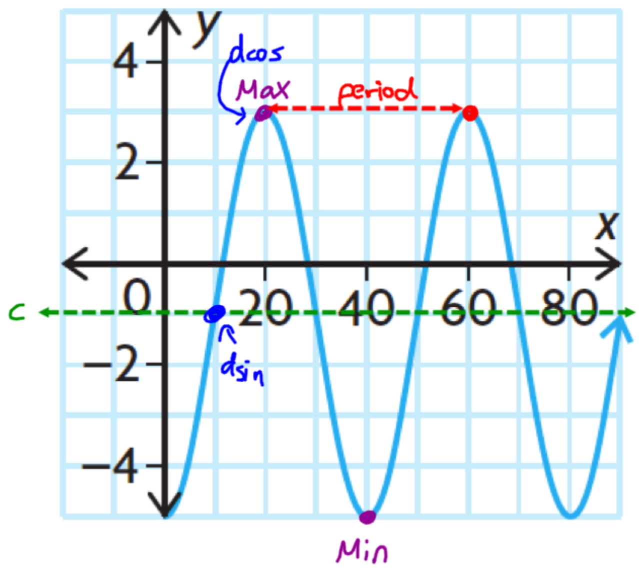
$a = \frac{max - min}{2} = \frac{3-(-5)}{2} = 4$

$k = \frac{2\pi}{period} = \frac{2\pi}{40} = \frac{\pi}{20}$

$c = max - |a| = 3 - 4 = -1$

$d_{cos} = 20$

$d_{sin} = 10$



$y = 4\cos\left[\frac{\pi}{20}(x-20)\right] - 1$

$y = 4\sin\left[\frac{\pi}{20}(x-10)\right] - 1$

Example 5:

a) Create a sine function with an amplitude of 7, a period of π, a phase shift of $\frac{\pi}{4}$ right, and a vertical displacement of -3.

$a = 7$

$k = \frac{2\pi}{period} = \frac{2\pi}{\pi} = 2$

$c = -3$

$d = \frac{\pi}{4}$

$$y = 7\sin\left[2\left(x - \frac{\pi}{4}\right)\right] - 3$$

b) What would be the equation of a cosine function that represents the same graph as the sine function above?

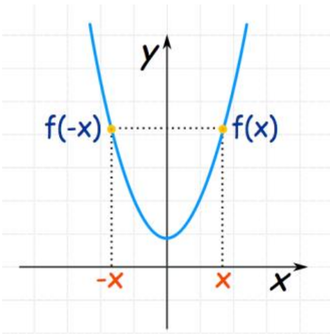
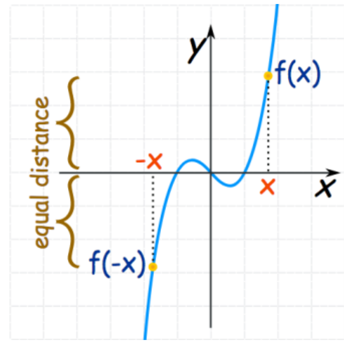
$d_{cos} = d_{sin} + \frac{\pi}{2k} = \frac{\pi}{4} + \frac{\pi}{2(2)} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$y = 7\cos\left[2\left(x - \frac{\pi}{2}\right)\right] - 3$$

Part 4: Even and Odd Functions

<table>
<tr><th>Even Functions</th><th>Odd Functions</th></tr>
<tr><td>EVEN FUNCTION if:

Line symmetry over the <u>y-axis</u></td><td>ODD FUNCTION if:

Point symmetry about the <u>origin (0, 0)</u></td></tr>
<tr><td>Rule:
$f(-x) = f(x)$

</td><td>Rule:
$-f(x) = f(-x)$

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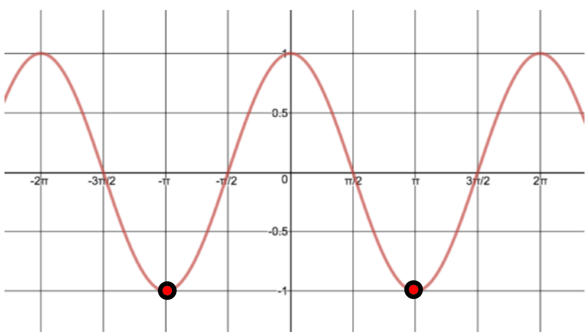
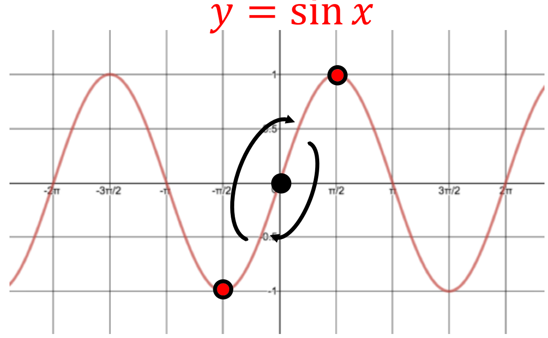
<tr><td>Example:

$y = \cos x$

$f(\pi) = -1$

$f(-\pi) = -1$

Therefore,

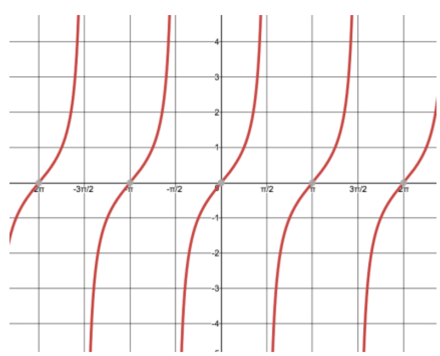
$f(\pi) = f(-\pi)$</td><td>Example:

$y = \sin x$

$f\left(\frac{\pi}{2}\right) = 1$

$f\left(-\frac{\pi}{2}\right) = -1$

Therefore,

$-f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$

$y = \tan x$ is also an odd function
</td></tr>
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