L4 – Derivatives of Exponential Functions | Unit 3
MCV4U
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Part 1: Review of e and $\ln x$

Properties of e:

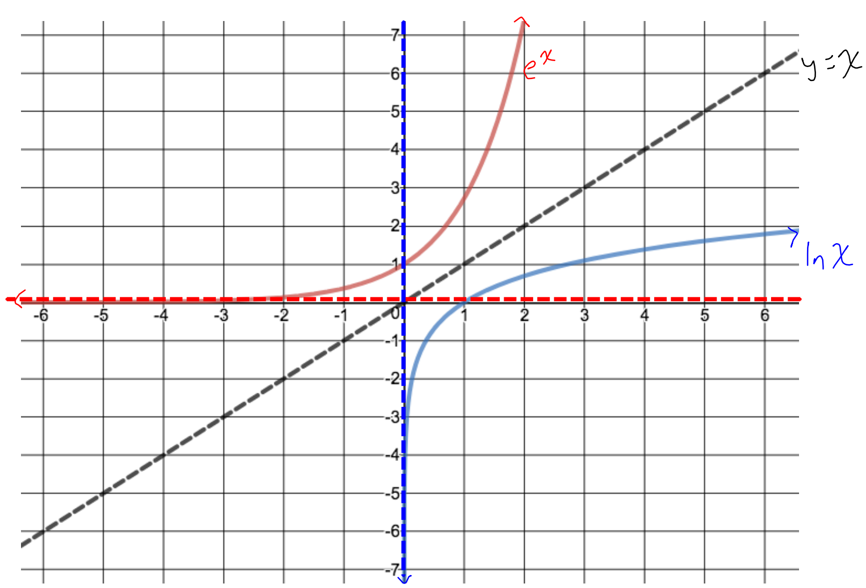
- $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.718\ 281\ 828\ 459$
- e is an **irrational** number, similar to π. They are non-terminating and non-repeating.
- $\log_e x$ is known as the **natural logarithm** and can be written as $\ln x$
- Many naturally occurring phenomena can be modelled using base-e exponential and logarithmic functions.
- $\log_e e = \ln e = 1$

Example 1: Graph the functions $y = e^x$ and $y = \ln x$

$y = e^x$	
x	y
-1	0.37
0	1
1	2.72
HA	$y = 0$

$y = \ln x$	
x	y
0.37	-1
1	0
2.72	1
VA	$x = 0$

Note: $y = \ln x$ is the inverse of $y = e^x$



Example 2: The population of a bacterial culture as a function of time is given by the equation $P(t) = 200e^{0.094t}$, where P is the population after t days.

a) What is the initial population of the bacterial culture?

$P(0) = 200e^{0.094(0)}$

$P(0) = 200$

b) Estimate the population after 3 days.

$P(3) = 200e^{0.094(3)}$

$P(3) \cong 265.2$

c) How long will the bacterial culture take to double its population?

$400 = 200e^{0.094t}$

$2 = e^{0.094t}$

$\ln 2 = \ln e^{0.094t}$

$\ln 2 = 0.094t \ln e$

$\ln 2 = 0.094t$

$$t = \frac{\ln 2}{0.094}$$

$t \cong 7.37$ days

d) Re-write this function as an exponential function having base 2.

$P(t) = 200(2)^{\frac{t}{7.37}}$

Use the exponential growth formula:

$$\boldsymbol{A(t) = A_0(2)^{\frac{t}{D}}}$$

where D is the doubling period

Part 2: Derivatives of Exponential Functions

Rule: If $f(x) = b^x$, $f'(x) = b^x \ln b$

Example 3: Determine the derivative of each function

a) $y = 2^x$

$y' = 2^x \ln 2$

b) $y = e^x$

$y' = e^x \ln e$

$y' = e^x(1)$

$y' = e^x$

c) $y = 3(2)^x$

$y' = 3(2)^x \ln 2$

d) $y = 3^x + 1$

$y' = 3^x \ln 3$

Notice that $\frac{d}{dx}e^x = e^x$

Example 4: Find the equation of the line that is tangent to the curve $y = 2e^x$ at $x = \ln 3$.

Slope of tangent line:

$y' = 2e^x$

$y'(\ln 3) = 2e^{\ln 3}$

$y'(\ln 3) = 2(3)$

$y'(\ln 3) = 6$

$m = 6$

Point on tangent line:

$y = 2e^x$

When $x = \ln 3$

$y = 2e^{\ln 3}$

$y = 2(3)$

$y = 6$

Equation of tangent line:

$y = mx + b$

$6 = 6(\ln 3) + b$

$b = 6 - 6\ln 3$

$\mathbf{y = 6x + 6 - 6\ln 3}$

Example 5: A biologist is studying the increase in the population of a particular insect in a provincial park. The population triples every week. Assume the population continues to increase at this rate. Initially, there are 100 insects.

a) Determine the number of insects present after 4 weeks.

$P(t) = 100(3)^t$

$P(4) = 100(3)^4$

$P(4) = 8100$

b) How fast is the number of insects increasing

i) when they are initially discovered?

$P'(t) = 100(3)^t \ln 3$

$P'(0) = 100(3)^0 \ln 3$

$P'(0) \cong 109.9$

At the beginning, it is increasing by 109.9 insects per week

ii) at the end of 4 weeks?

$P'(4) = 100(3)^4 \ln 3$

$P'(4) \cong 8898.8$ insects per week