L4 – Applications of Vector Addition | Unit 4
MCV4U
Jensen

Part 1: Resultant Force

When two vectors act on an object, you can use vector addition, the Pythagorean theorem, and trigonometry to find the resultant.

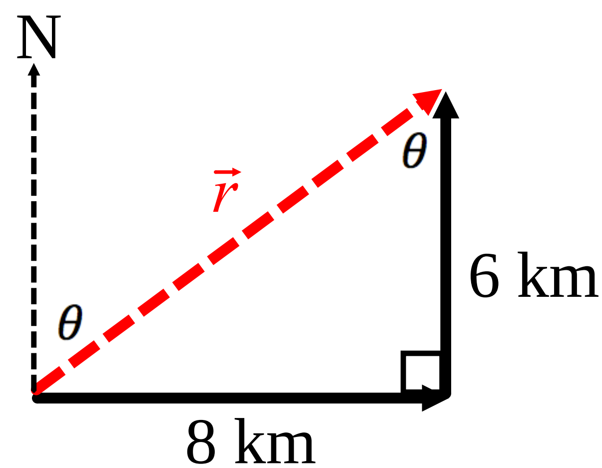
We have shown that if we take any two forces that act at the same point, acting at an angle of θ to each other, the forces may be composed to obtain the resultant of these two forces. Furthermore, the resultant of any two forces is unique because there is only one parallelogram that can be formed with these two forces.

Example 1: A sailboat's destination is 8 km east and 6 km north. Find the magnitude and direction, in bearing notation, of the resultant.

Magnitude:

$|r|^2 = (8)^2 + (6)^2$

$|r| = 10 \text{ km}$



Direction:

$\tan\theta = \frac{8}{6}$

$\theta = \tan^{-1}\left(\frac{8}{6}\right)$

$\theta \cong 53.1°$

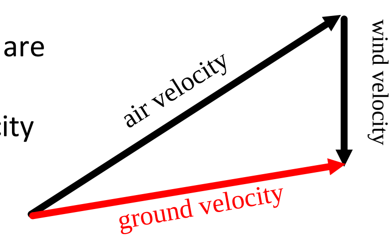
The resultant displacement is 10 km at a true bearing of $053.1°$.

Part 2: Velocity

The resultant velocity of any two velocities is their sum. In all calculations involving resultant velocities, it is necessary to draw a triangle showing the velocities so there is a clear recognition of the resultant and its relationship to the other two velocities.

For velocity questions involving airplanes:

- When the velocity of the airplane is mentioned, it is understood that we are referring to its air speed (speed relative to the air it is flying in).
- When the velocity of the wind is mentioned, we are referring to its velocity relative to the ground.
- The resultant velocity of the airplane is the velocity of the airplane relative to the ground.
- $\vec{v}_{air} + \vec{w} = \vec{v}_{ground}$

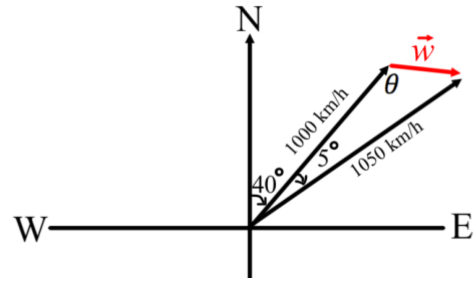


Example 2: A plane travels $N40°E$ at an airspeed of 1000 km/h. Measurement on the ground indicates that the plane is traveling $N45°E$ at a speed of 1050 km/h. What is the velocity of the wind?

Magnitude

$$|\vec{w}|^2 = (1000)^2 + (1050)^2 - 2(1000)(1050)\cos(5)$$

$$|\vec{w}| \cong 102.4 \text{ km/h}$$



Direction:

$$\cos\theta = \frac{1050^2 - 1000^2 - 102.4^2}{-2(1000)(102.4)}$$

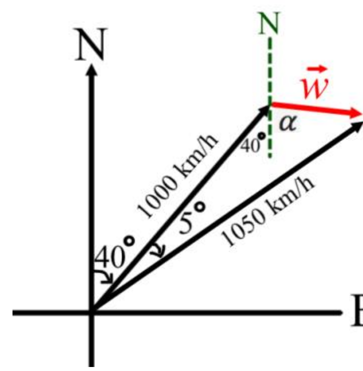
$$\theta \cong 116.7$$

$$\alpha = \theta - 40$$

$$\alpha = 116.7 - 40$$

$$\alpha = 76.7°$$

Note: *Choose cosine law to solve for unknown angle. You could have chosen sine law but would have had to consider the ambiguous case.*

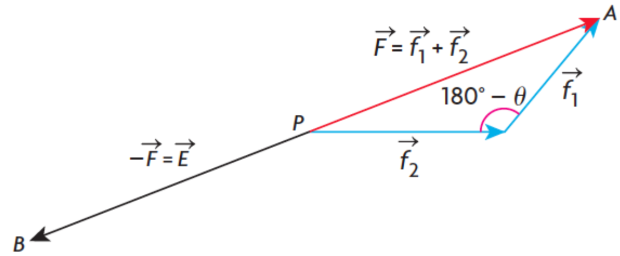


Therefore, the wind speed is 102.4 km/h at a quadrant bearing of $S76.7°E$.

Part 3: Equilibrant Vector

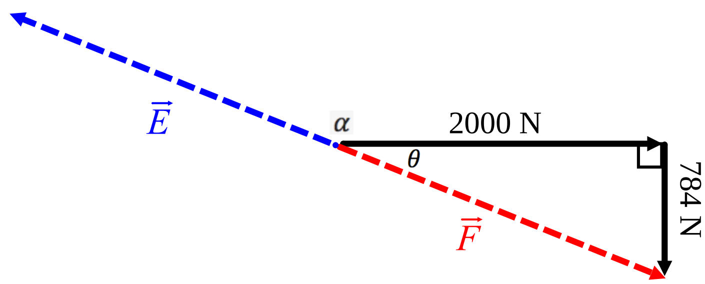
An equilibrant vector is one that balances another vector or a combination of vectors. It is equal in magnitude but opposite in direction to the resultant vector. If the equilibrant is added to a given system of vectors, the sum of all vectors, including the equilibrant, is $\vec{0}$.

The equilibrant of a number of forces is the single force that opposes the resultant of the forces acting on an object. When the equilibrant is applied to the object, this force maintains the object in a state of equilibrium.



Example 3: A clown with mass 80 kg is shot out of a cannon with a horizontal force of 2000 N. The vertical force is the acceleration due to gravity, which is 9.8 m/s^2, times the mass of the clown.

a) Find the magnitude and direction of the resultant force on the clown.



$|\vec{f_g}| = 80(9.8) = 784$ N

$|\vec{F}|^2 = (2000)^2 + (784)^2$

$|\vec{F}| \cong 2148.2$ N

$\tan\theta = \dfrac{784}{2000}$

$\theta \cong 21.4°$

The resultant force has a magnitude of 2148.2 N and a direction of 21.4° below the horizontal.

b) Find the magnitude and direction of the equilibrant force on the clown.

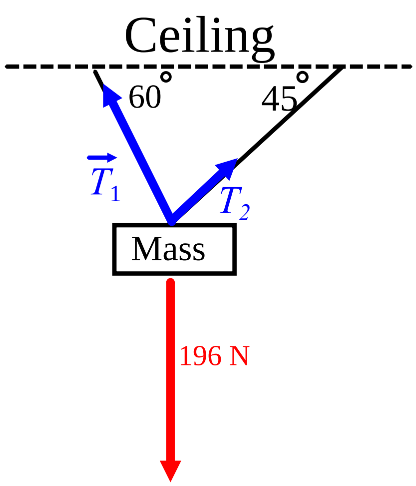
$\alpha = 180 - \theta = 180 - 21.4 = 158.6°$.

The equilibrant force has a magnitude of 2148.2 N and a direction of 158.6° counterclockwise from the horizontal.

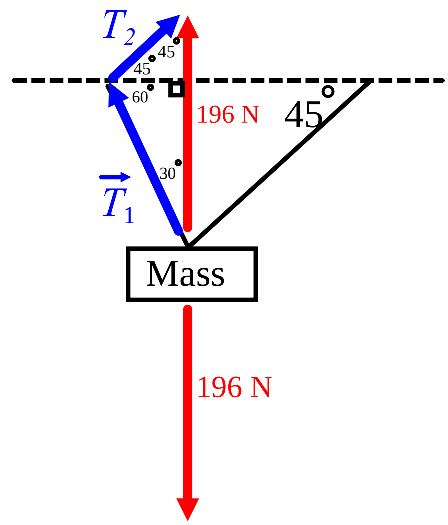
Example 4: A mass of 20 kg is suspended from a ceiling by two lengths of rope that make angles of 60° and 45° with the ceiling. Determine the tension in each of the ropes.

$|\vec{f_g}| = 20(9.8) = 196$ N

Position Diagram:



Vector Diagram:



The resultant of the tensions must be equal in magnitude to the force of gravity but in the opposite direction since the system is in a state of equilibrium.

$|\vec{T}_1|$:

$$\frac{|\vec{T}_1|}{\sin 45} = \frac{196}{\sin 105}$$

$$|\vec{T}_1| = \frac{196 \sin 45}{\sin 105}$$

$$|\vec{T}_1| \cong 143.5 \text{ N}$$

$|\vec{T}_2|$:

$$\frac{|\vec{T}_2|}{\sin 30} = \frac{196}{\sin 105}$$

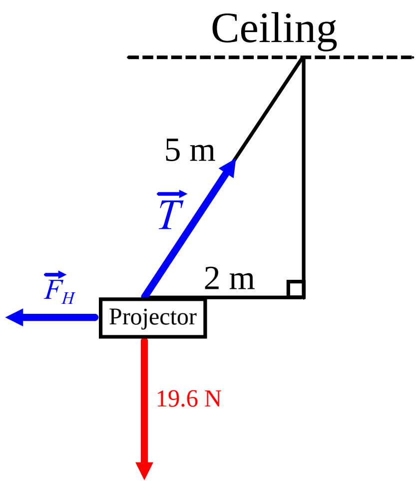
$$|\vec{T}_2| = \frac{196 \sin 30}{\sin 105}$$

$$|\vec{T}_2| \cong 101.5 \text{ N}$$

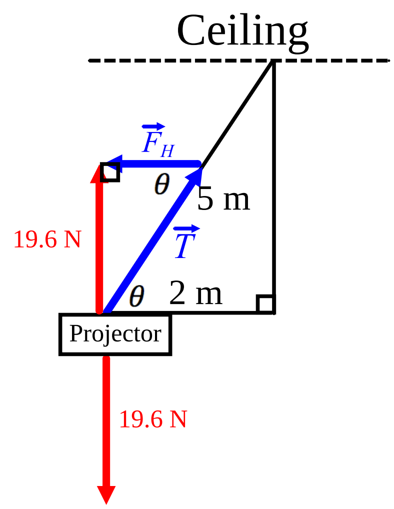
Example 5: A video projector of mass 2 kg is hung by a 5 m rope from the ceiling. The projector is pulled back 2 m (measured horizontally) by a horizontal force. Find the magnitude of the horizontal force and the tension in the rope.

$|\vec{f_g}| = 2(9.8) = 19.6$ N

Position Diagram:



Vector Diagram:



$$\cos\theta = \frac{2}{5}$$

$$\theta \cong 66.4°$$

$$\tan 66.4 = \frac{19.6}{|\vec{F}_H|}$$

$$|\vec{F}_H| = \frac{19.6}{\tan 66.4}$$

$$|\vec{\boldsymbol{F}}_{\boldsymbol{H}}| \cong \mathbf{8.6\ N}$$

$$\sin 66.4 = \frac{19.6}{|\vec{T}|}$$

$$|\vec{T}| = \frac{19.6}{\sin 66.4}$$

$$|\vec{\boldsymbol{T}}| \cong \mathbf{21.4\ N}$$