L5 – Cross Product of Vectors Unit 5

MCV4U

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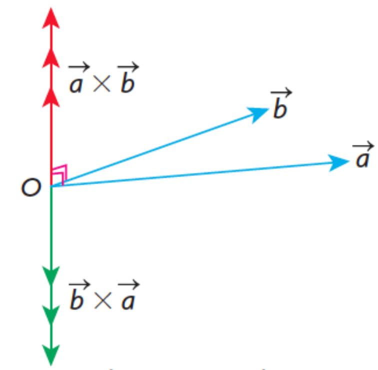
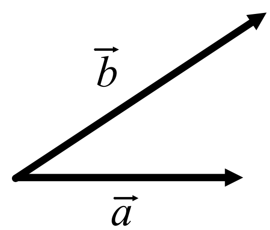
The cross product of two vectors $\vec{a}$ and $\vec{b}$ in R^3 is the vector that is ___________________________ to these vectors such that the vectors $\vec{a}$, $\vec{b}$, and $\vec{a} \times \vec{b}$ form a right-handed system.

Right handed system tells you to point your hand along vector $\vec{a}$ and curl your fingers towards vector $\vec{b}$. Your thumb will be pointing in the direction of $\vec{a} \times \vec{b}$. Notice that $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$ point in opposite directions.



The Cross Product
(Vector Product)

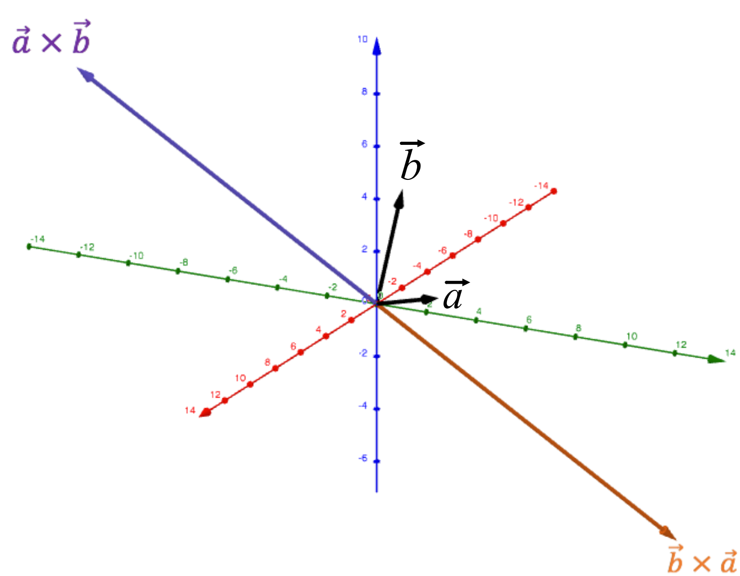
Sometimes the direction of the cross product is defined by either 'in to the page' or 'out of the page':



$\vec{a} \times \vec{b}$	$\vec{b} \times \vec{a}$
"Out of the page"	"In to the page"

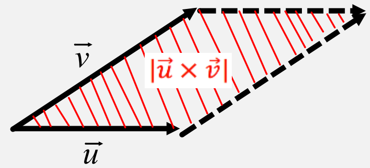
3D visualization:

https://www.geogebra.org/3d/jyqcr3bf



Properties of Cross Product:

- $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
- $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$
- $(\vec{u} + \vec{v}) \times \vec{w} = \vec{u} \times \vec{w} + \vec{v} \times \vec{w}$
- If $\vec{u} \times \vec{v} = 0$, and $\vec{u}$ and $\vec{v}$ are non-zero, then $\vec{u}$ and $\vec{v}$ are collinear.
- $k(\vec{u} \times \vec{v}) = (k\vec{u}) \times \vec{v} = \vec{u} \times (k\vec{v})$
- $|\vec{u} \times \vec{v}| =$ the area of the parallelogram defined by $\vec{u}$ and $\vec{v}$



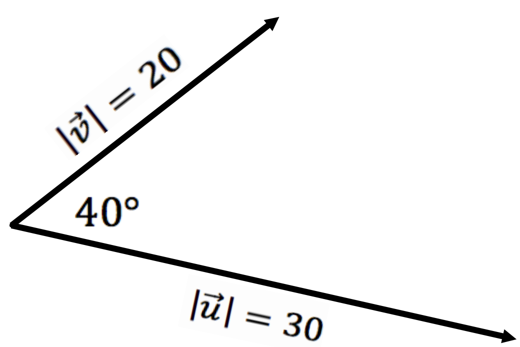
Part 2: Cross Product of Geometric Vectors

Formula: $\vec{a} \times \vec{b} = \left(|\vec{a}||\vec{b}|\sin\theta\right)\hat{n}$

θ is the angle between the vectors
$\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

Example 1: If $|\vec{u}| = 30$, $|\vec{v}| = 20$, the angle between $\vec{u}$ and $\vec{v}$ is $40°$, and $\vec{u}$ and $\vec{v}$ are in the plane of the page, find…

a) $\vec{u} \times \vec{v}$



b) $\vec{v} \times \vec{u}$

<u>**Part 3: Cross Product of Algebraic Vectors**</u>

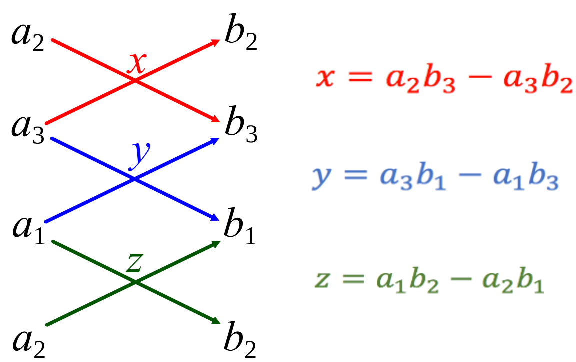
Formula:

If $\vec{a} = [a_1, a_2, a_3]$ and $\vec{b} = [b_1, b_2, b_3]$

$\vec{a} \times \vec{b} = [\ a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1]$

$= (a_2b_3 - a_3b_2)\vec{i} + (a_3b_1 - a_1b_3)\vec{j} + (a_1b_2 - a_2b_1)\vec{k}$

How to set it up:



Example 2: If $\vec{p} = [-1,3,2]$ and $\vec{q} = [2,-5,6]$, calculate each of the following:

a) $\vec{p} \times \vec{q}$

b) $\vec{q} \times \vec{p}$

Example 3a: Determine the area of the parallelogram defined by the vectors $\vec{u} = [4,5,2]$ and $\vec{v} = [3,2,7]$.

Example 3b: Determine the angle between the vectors $\vec{u}$ and $\vec{v}$.