

## L5 – 5.4 Solve Linear Trigonometric Equations

MHF4U

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In the previous lesson we have been working with identities. Identities are equations that are true for ANY value of  $x$ . In this lesson, we will be working with equations that are not identities. We will have to solve for the value(s) that make the equation true.

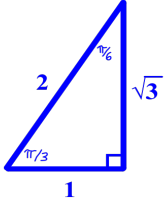
Remember that 2 solutions are possible for an angle between 0 and  $2\pi$  with a given ratio. Use the reference angle and CAST rule to determine the angles.

When solving a trigonometric equation, consider all 3 tools that can be useful:

1. Special Triangles
2. Graphs of Trig Functions
3. Calculator

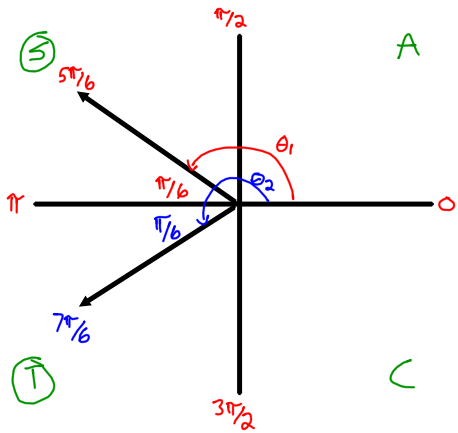
**Example 1:** Find all solutions for  $\cos \theta = -\frac{\sqrt{3}}{2}$  in the interval  $0 \leq x \leq 2\pi$

$-\frac{\sqrt{3}}{2} \rightarrow$  Graph  $\rightarrow$  NO  $\rightarrow$  Yes  $\rightarrow$   $\triangle$



$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Put reference angle in to quadrants 2 and 3 to make the ratio negative



$\theta_1 = \pi - \frac{\pi}{6} = 5\frac{\pi}{6}$

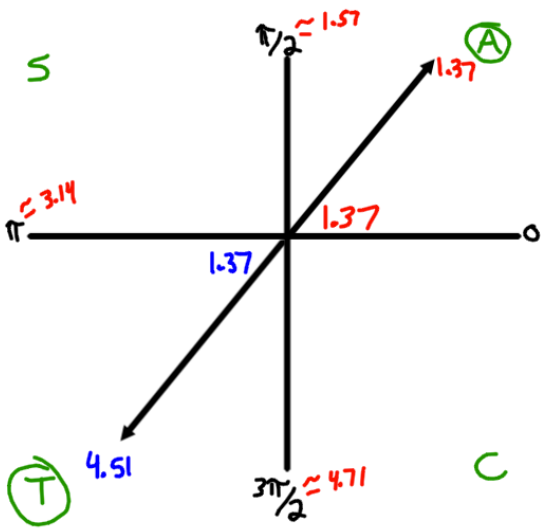
$\theta_2 = \pi + \frac{\pi}{6} = 7\frac{\pi}{6}$

$\cos 5\frac{\pi}{6} = \cos 7\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

**Example 2:** Find all solutions for  $\tan \theta = 5$  in the interval  $0 \leq x \leq 2\pi$

$\tan \theta = 5 \rightarrow$  Graph NO  $\rightarrow$   $\Delta$  NO  $\rightarrow$  CALCULATOR

NOTE: Tan is  $\oplus$  in Q1 and Q3



$\tan \theta = 5$   
 $\theta_1 = \tan^{-1}(5)$   
 $\theta_1 \approx 1.37$  radians  
 $\theta_2 = \pi + 1.37$   
 $\theta_2 \approx 4.51$  radians

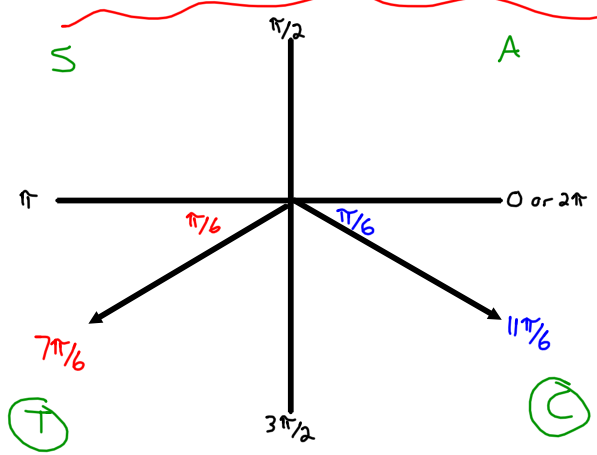
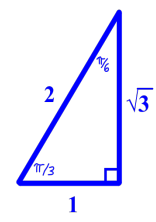
$\tan 1.37 = \tan 4.51 = 5$

**Example 3:** Find all solutions for  $2 \sin x + 1 = 0$  in the interval  $0 \leq x \leq 2\pi$

$2 \sin x + 1 = 0$   
 $2 \sin x = -1$   
 $\sin x = -1/2$

Graph NO  $\rightarrow$   $\Delta$  Yes

$\sin \pi/6 = 1/2$   
 use as reference angle in Q3 + Q4 to make ratio  $\ominus$



$\theta_1 = \pi + \pi/6$   
 $= 7\pi/6$   
 $\theta_2 = 2\pi - \pi/6$   
 $= 11\pi/6$

$\sin 7\pi/6 = \sin 11\pi/6 = -1/2$

**Example 4:** Solve  $3(\tan x + 1) = 2$ , where  $0 \leq x \leq 2\pi$

$$3(\tan x + 1) = 2$$

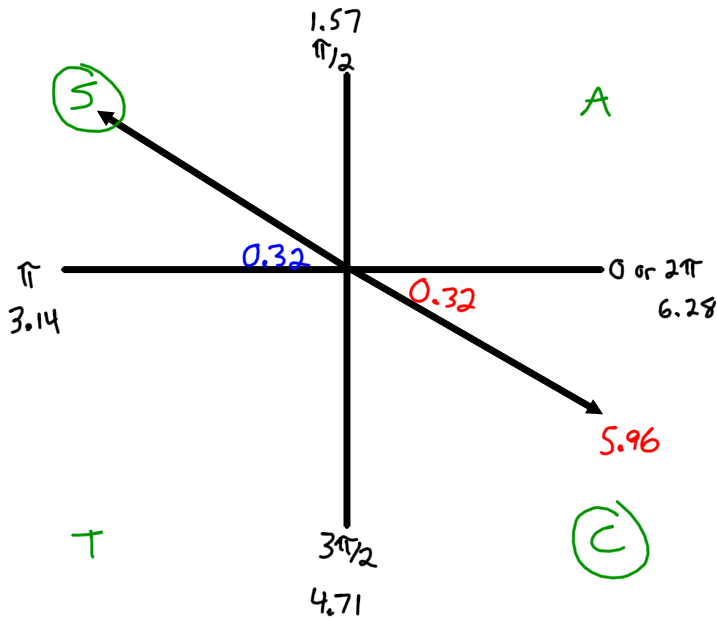
$$\tan x + 1 = \frac{2}{3}$$

$$\tan x = \frac{2}{3} - 1$$

$$\tan x = -\frac{1}{3}$$

NOTE:  $\tan$  is  $\ominus$   
in Q2 + Q4

Graph  $\rightarrow$  NO  $\rightarrow$  CALCULATOR  
NO



$$\tan x = -\frac{1}{3}$$

$$x_1 = \tan^{-1}\left(-\frac{1}{3}\right)$$

$$x_1 = -0.32175$$

$$x_1 = 2\pi - 0.32175$$

$$x_1 \approx 5.96 \text{ radians}$$

$$x_2 = \pi - 0.32$$

$$x_2 \approx 2.82 \text{ radians}$$

$$\tan 5.96 = \tan 2.82 = -\frac{1}{3}$$