L5 – Differentiation Rules for Exponential Functions

Unit 3

MCV4U

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Part 1: Derivatives of Exponential Functions

Chain Rule:

If $h(x) = f(g(x))$

$h'(x) = f'[g(x)] \times g'(x)$

Apply to exponential functions:

If $h(x) = b^{g(x)}$

$h'(x) = b^{g(x)} \times \ln b \times g'(x)$

Example 1: Find the derivative of each function.

a) $y = xe^x$

$y' = 1e^x + e^x(x)$

$y' = e^x(1 + x)$

b) $y = e^{2x+1}$

$y' = e^{2x+1}(2)$

$y' = 2e^{2x+1}$

c) $y = e^x - e^{-x}$

$y' = e^x - e^{-x}(-1)$

$y' = e^x + e^{-x}$

d) $y = 2e^x \cos x$

$y' = 2[e^x \cos x + (-\sin x)e^x]$

$y' = 2e^x(\cos x - \sin x)$

e) $y = x^2 10^x$

$y' = 2x(10^x) + 10^x \ln 10(x^2)$

$y' = x10^x(2 + x\ln 10)$

Example 2: Identify the local extrema of the function $f(x) = x^2e^x$.

Find the critical numbers:

$f'(x) = 2xe^x + e^x x^2$

$f'(x) = xe^x(2 + x)$

$0 = xe^x(2 + x)$

$x_1 = 0$ $\quad$ $x_2 = -2$ $\quad$ Note: $e^x \neq 0$

$-\infty$ $\quad -2 \quad 0 \quad \infty$

Test value for x	-3	-1	1
$f'(x)$	$+$	$-$	$+$
$f(x)$	Increasing	Decreasing	Increasing

Local max at $(-2, 0.54)$

Local min at $(0, 0)$

Example 3: The effectiveness of studying for an exam depends on how many hours a student studies. Some experiments show that if the effectiveness, E, is put on a scale of 0 to 10, then $E(t) = 0.5\left[10 + te^{-\frac{t}{20}}\right]$, where t is the number of hours spent studying for an examination. If a student has up to 30 hours for studying, how many hours are needed for maximum effectiveness.

Start by finding any critical numbers:

$$E'(t) = 0.5\left[1e^{-\frac{t}{20}} + e^{-\frac{t}{20}}\left(-\frac{1}{20}\right)t\right]$$

$$E'(t) = 0.5e^{-\frac{t}{20}}\left(1 - \frac{t}{20}\right)$$

$0 = 0.5e^{-\frac{t}{20}}\left(1 - \frac{t}{20}\right)$ Note: $0.5e^{-\frac{t}{20}} \neq 0$

$$0 = 1 - \frac{t}{20}$$

$$\frac{t}{20} = 1$$

$t = 20$ hours

Test endpoints and critical number:

$E(0) = 5$
$E(20) \cong 8.7$
$E(30) \cong 8.3$

Therefore, studying for 20 hours will yield the maximum effectiveness of studying of about 8.7 out of 10.