Part 1: Derivatives of Exponential Functions

Example 1: Find the derivative of each function.

a)
$$y = xe^{x}$$

 $y' = 1e^{x} + e^{x}(x)$
 $y' = e^{x}(1 + x)$

b) $v = e^{2x+1}$

$$y' = e^{2x+1}(2)$$

 $y' = 2e^{2x+1}$

Chain Rule:
If
$$h(x) = f(g(x))$$

 $h'(x) = f'[g(x)] \times g'(x)$
Apply to exponential functions:
If $h(x) = b^{g(x)}$
 $h'(x) = b^{g(x)} \times \ln b \times g'(x)$

c)
$$y = e^x - e^{-x}$$

$$y' = e^{x} - e^{-x}(-1)$$
$$y' = e^{x} + e^{-x}$$

d) $y = 2e^x \cos x$

 $y' = 2[e^x \cos x + (-\sin x)e^x]$ $y' = 2e^x(\cos x - \sin x)$

e)
$$y = x^2 10^x$$

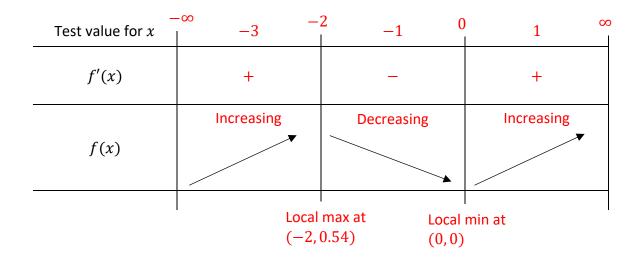
 $y' = 2x(10^x) + 10^x \ln 10(x^2)$
 $y' = x10^x(2 + x \ln 10)$

Example 2: Identify the local extrema of the function $f(x) = x^2 e^x$.

Find the critical numbers:

 $f'(x) = 2xe^{x} + e^{x}x^{2}$ $f'(x) = xe^{x}(2+x)$ $0 = xe^{x}(2+x)$

 $x_1 = 0$ $x_2 = -2$ Note: $e^x \neq 0$



Example 3: The effectiveness of studying for an exam depends on how many hours a student studies. Some experiments show that if the effectiveness, *E*, is put on a scale of 0 to 10, then $E(t) = 0.5 \left[10 + te^{-\frac{t}{20}} \right]$, where *t* is the number of hours spent studying for an examination. If a student has up to 30 hours for studying, how many hours are needed for maximum effectiveness.

Start by finding any critical numbers:

$$E'(t) = 0.5 \left[1e^{-\frac{t}{20}} + e^{-\frac{t}{20}} \left(-\frac{1}{20} \right) t \right]$$
$$E'(t) = 0.5e^{-\frac{t}{20}} \left(1 - \frac{t}{20} \right)$$
$$0 = 0.5e^{-\frac{t}{20}} \left(1 - \frac{t}{20} \right)$$
Note: $0.5e^{-\frac{t}{20}} \neq 0$
$$0 = 1 - \frac{t}{20}$$
$$\frac{t}{20} = 1$$

t = 20 hours

Test endpoints and critical number:

E(0) = 5 $E(20) \approx 8.7$ $E(30) \approx 8.3$

Therefore, studying for 20 hours will yield the maximum effectiveness of studying of about 8.7 out of 10.