## Resolving a Vector into its Components

In many situations involving forces, we are interested in a process that is the opposite of composition. This process is called resolution, which means taking a single force and decomposing it into two components. When we resolve a force into two components, it is possible to do this in an infinite number of ways because there are infinitely many parallelograms having a particular single force as the diagonal. However, the most useful and important way to resolve a force vector occurs when this vector is resolved into two components that are at right angles to each other. These components are usually referred to ask the horizontal and vertical components or rectangular vector components.

If $\vec{f}$ is resolved into its respective horizontal and vertical components, $\vec{f}_{x}$ and $\vec{f}_{y}$, then:

$$
\begin{aligned}
\left|\vec{f}_{x}\right| & =|\vec{f}| \cos \theta \\
\left|\vec{f}_{y}\right| & =|\vec{f}| \sin \theta
\end{aligned}
$$

Where $\theta$ is the angle that $\vec{f}$ makes with the $x$-axis.


Example 1: A tow truck is pulling a car from a ditch.
The tension in the cable is 15000 N at an angle of $40^{\circ}$ to the horizontal.
a) Draw a diagram showing the resolution of the force into its rectangular vector components.

b) Determine the magnitudes of the horizontal and vertical vector components of the force.
$\left|\vec{f}_{x}\right|=15000 \cos 40$
$\left|\vec{f}_{x}\right| \cong 11490.7 \mathrm{~N}$
$\left|\vec{f}_{y}\right| \cong 9641.8 \mathrm{~N}$

Example 2: Kayla pulls on a rope attached to her sleigh with a force of 200 N . If the rope makes an angle of $20^{\circ}$ with the horizontal, determine:
a) the force that pulls the sleigh forward $=\left|\vec{f}_{x}\right|$
$\left|\vec{f}_{x}\right|=200 \cos 20$
$\left|\vec{f}_{x}\right| \cong 187.9 \mathrm{~N}$
b) the force that tends to lift the sleigh $=\left|\vec{f}_{y}\right|$

$\left|\vec{f}_{y}\right|=200 \sin 20$
$\left|\vec{f}_{y}\right| \cong 68.4 \mathrm{~N}$

Example 3: A box weighting 140 N is resting on a ramp that is inclined at an angle of $20^{\circ}$. Resolve the weight into rectangular vector components that keep the box at rest.

## Position Diagram:


$|\vec{n}|=140 \cos 20$
$|\vec{n}| \cong 131.6 \mathrm{~N}$
$|\vec{f}|=140 \sin 20$
$|f| \cong 47.9 \mathrm{~N}$

## Vector Diagram:



The force acting perpendicular to the ramp, $\vec{n}$, and the force of friction on the ramp, $\vec{f}$ are holding the box in a state of equilibrium. Therefore, the resultant must be equal to the opposite of the force of the box.

The box is kept at rest by a force of 131.6 N acting perpendicular to the ramp and by friction of 47.9 N parallel to the surface of the ramp.

