L6 – Applications of Rates of Change Unit 1
MCV4U
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Part 1: Rates of Change Applications

Example 1: Suppose the function $V(t) = \frac{50\,000 + 6t}{1+0.4t}$ represents the value, V, in dollars, of a new car t years after it is purchased.

a) What is the rate of change of the value of the car at 2 years? 5 years? And 7 years?

b) What was the initial value of the car?

Example 2: Kinetic energy, K, is the energy due to motion. When an object is moving, its kinetic energy is determined by the formula $K(v) = 0.5mv^2$, where K is in joules, m is the mass of the object, in kilograms; and v is the velocity of the object, in meters per second.

Suppose a ball with a mass of 0.35 kg is thrown vertically upward with an initial velocity of 40 m/s. Its velocity function is $v(t) = 40 - 9.8t$, where t is time, in seconds.

a) Express the kinetic energy of the ball as a function of time.

b) Determine the rate of change of the kinetic energy of the ball at 3 seconds.

Linear Density:

The linear density of an object refers to the mass of an object per unit length. Suppose the function $f(x)$ gives the mass, in kg, of the first x meters of an object. For the part of the object that lies between x_1 and x_2, the average linear density $= \frac{f(x_2)-f(x_1)}{x_2-x_1}$. The corresponding derivative function $f'(x)$ is the linear density, the rate of change of mass at a particular length x.

Example 3: The mass, in kg, of the first x meters of wire can be modelled by the function $f(x) = \sqrt{3x+1}$.

a) Determine the average linear density of the part of the wire from $x = 5$ to $x = 8$.

b) Determine the linear density at $x = 5$ and $x = 8$. What do these results tell you about the wire.

Part 2: Business Applications

Terminology:

- The demand functions, or price function, is $p(x)$, where x is the number of units of a product or service that can be sold at a particular price, p.
- The revenue function is $R(x) = x \cdot p(x)$, where x is the number of units of a product or service sold at a price per unit of $p(x)$.
- The cost function, $C(x)$, is the total cost of producing x units of a product or service.
- The profit function, $P(x)$, is the profit from the sale of x units of a product or service. The profit function is the difference between the revenue function and the cost function: $P(x) = R(x) - C(x)$

Economists use the word marginal to indicate the derivative of a business function.

- $C'(x)$ is the marginal cost function and refers to the instantaneous rate of change of total cost with respect to the number of items produced.
- $R'(x)$ is the marginal revenue function and refers to the instantaneous rate of change of total revenue with respect to the number of items sold.
- $P'(x)$ is the marginal profit function and refers to the instantaneous rate of change of total profit with respect to the number of items sold.

Example 4: A company sells 1500 movie DVDs per month at $10 each. Market research has shown that sales will decrease by 125 DVDs per month for each $0.25 increase in price.

a) Determine a demand (or price) function.

b) Determine the marginal revenue when sales are 1000 DVDs per month.

c) The cost of producing x DVDs is $C(x) = -0.004x^2 + 9.2x + 5000$. Determine the marginal cost when production is 1000 DVDs per month.

d) Determine the actual cost of producing the 1001^{st} DVD.

e) Determine the Profit and Marginal Profit for the monthly sales of 1000 DVDs.