L7 – 6.3 Transformations of Exponential and Logarithmic Functions
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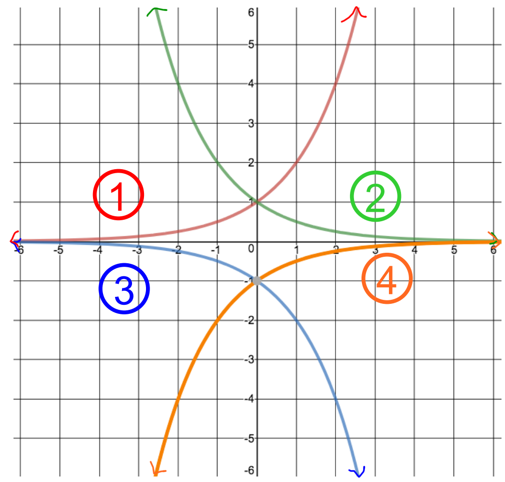
Part 1: Properties of Exponential Functions

General Equation: $y = a(b)^{k(x-d)} + c$ where the base function is $y = b^x$

There are 4 possible shapes for an exponential function

1) $a > 0$ and $b > 1$ (ex. $y = 2^x$)
2) $a > 0$ and $0 < b < 1$ (ex. $y = \left(\frac{1}{2}\right)^x$)
3) $a < 0$ and $b > 1$ (ex. $y = -1(2)^x$)
4) $a < 0$ and $0 < b < 1$ (ex. $y = -1\left(\frac{1}{2}\right)^x$)



To graph the base function $y = b^x$, Find the following key features:

- Horizontal asymptote
    - Starts at $y = 0$ and can be shifted by c
- $y - intercept$
    - set $x = 0$ and solve
- At least one other point to be sure of shape
    - Common to choose $x = 1$ and solve for y

You can then use transformational properties of a, k, d, and c to graph a transformed function

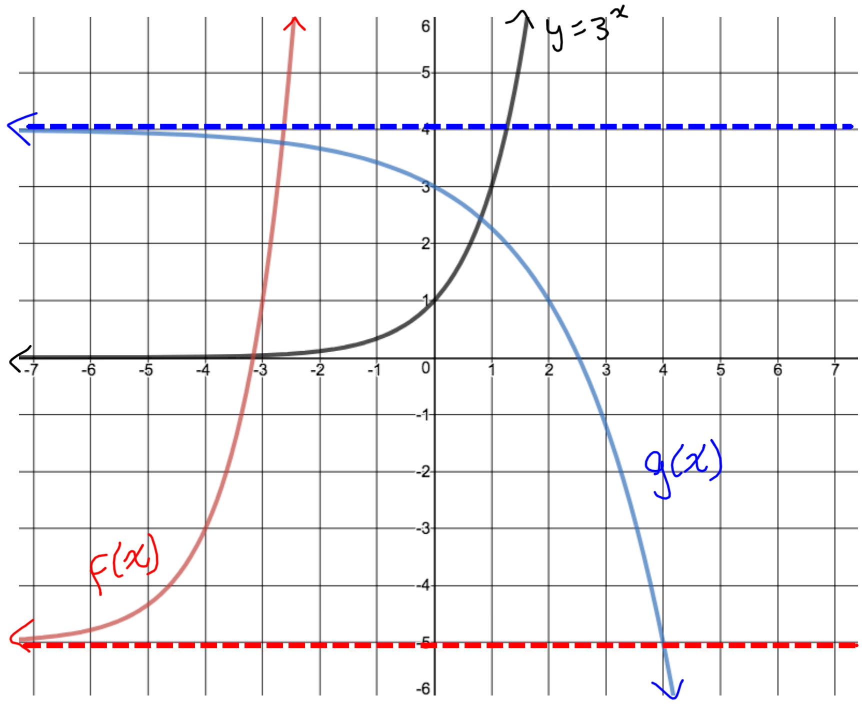
Part 2: Transformations of Exponential Functions

Example 1: Sketch the graph of $f(x) = 2(3)^{x+4} - 5$ and $g(x) = -3^{\frac{1}{2}x} + 4$ using transformations

$y = 3^x$	
x	y
-1	0.33
0	1
1	3
HA	$y = 0$

$f(x) = 2(3)^{x+4} - 5$	
$x - 4$	$2y - 5$
-5	-4.33
-4	-3
-3	1
HA	$y = -5$

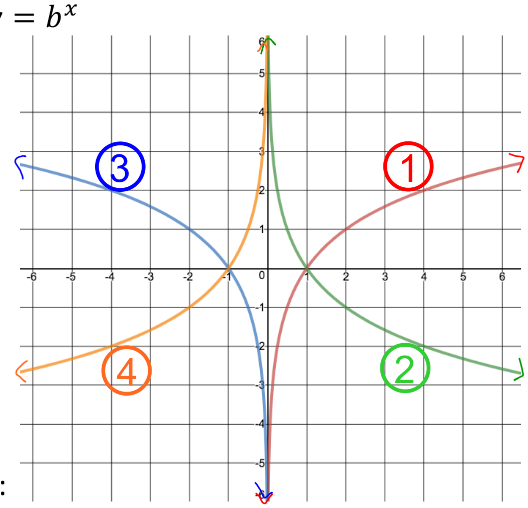
$g(x) = -3^{\frac{1}{2}x} + 4$	
$2x$	$-1y + 4$
-2	3.67
0	3
2	1
HA	$y = 4$



<u>**Part 3: Properties of Logarithmic Functions**</u>

General Equation: $y = a\log_b[k(x-d)] + c$ where the base function is $y = \log_b x$

Remember that $y = \log_b x$ is the inverse of the exponential function $y = b^x$



There are 4 possible shapes for a logarithmic function

1) $k > 0$ and $b > 1$ (ex. $y = \log_2(x)$)
2) $k > 0$ and $0 < b < 1$ (ex. $y = \log_{0.5}(x)$)
3) $k < 0$ and $b > 1$ (ex. $y = \log_2(-x)$)
4) $k < 0$ and $0 < b < 1$ (ex. $y = \log_{0.5}(-x)$)

To graph the base function $y = \log_b x$, Find the following key features:

- Vertical asymptote
  - Starts at $x = 0$ and can be shifted by d
- $x - intercept$
  - set $y = 0$ and solve
- At least one other point to be sure of shape
  - Common to choose $y = 1$ and solve for x

Part 4: Transformations of Logarithmic Functions

Example 2: Sketch the graph of $f(x) = -4\log_3(x) + 2$ and $g(x) = \log_3[-(x+2)] - 4$ using transformations

$y = \log_3(x)$	
x	y
0.33	-1
1	0
3	1
VA	$x = 0$

$f(x) = -4\log_3(x) + 2$	
x	$-4y + 2$
0.33	6
1	2
3	-2
VA	$x = 0$

$g(x) = \log_3[-(x+2)] - 4$	
$-x - 2$	$y - 4$
-2.33	-5
-3	-4
-5	-3
VA	$x = -2$