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<mark>L8 – The Natural Logarithm</mark>						
MHF4U						
Jensen						

#### Part 1: What is e'?

**Example 1:** Suppose you invest \$1 at 100% interest for 1 year at various compounding levels. What is the highest amount of money you can have after 1 year?

Note: the formula used for compound interest of \$1 at 100% interest annually compounded n times during the year is:

$$A = 1\left(1 + \frac{1}{n}\right)^n$$

Compounding Level, <i>n</i>	Amount, A in dollars
Annualy (once a year)	
Semi-annually (2-times)	
Quarterly (4-times)	
Monthly (12-times)	
Daily (365-times)	
Secondly (31 536 000-times)	
Continuously (1 000 000 000-times)	

#### **Properties of** *e*:

- $e = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n$
- e is an \_\_\_\_\_ number, similar to  $\pi$ . They are non-terminating and non-repeating.
- $\log_e x$  is known as the \_\_\_\_\_ and can be written as \_\_\_\_\_
- Many naturally occurring phenomena can be modelled using base-*e* exponential and logarithmic functions.
- $\log_e e = \ln e = \_$

## Part 2: Reminder of Log Rules

Power Law of Logarithms	$\log_b x^n = n \log_b x  \text{for } b > 0, b \neq 1, x > 0$					
Product Law of Logarithms	$\log_b(mn) = \log_b m + \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$					
Quotient Law of Logarithms	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n  \text{for } b > 0, b \neq 1, m > 0, n > 0$					
Change of Base Formula	$\log_b m = \frac{\log m}{\log b}$ , $m > 0, b > 0, b \neq 1$					
Exponential to Logarithmic	$y = b^x \rightarrow x = \log_b y$					
Logarithmic to Exponential	$y = \log_b x \rightarrow x = b^y$					
Other useful tips	$\log_a(a^b) = b \qquad \qquad \log_a a = \log_{10} a \qquad \qquad \log_b b = 1$					

## Part 2: Solving Problems Involving e

Example 2: Evaluate each of the following

**a)** e<sup>3</sup>

**b)** ln 10

**c)** ln *e* 

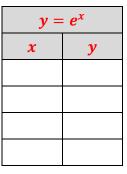
**Example 3:** Solve each of the following equations

**a)**  $20 = 3e^x$ 

**b)**  $e^{1-2x} = 55$ 

# Part 3: Graphing Functions Involving e

**Example 4:** Graph the functions  $y = e^x$  and  $y = \ln x$ 



ln x
у

**Note:**  $y = \ln x$  is the inverse of  $y = e^x$ 

