L8 – The Natural Logarithm
MHF4U
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Part 1: What is $'e'$?

Example 1: Suppose you invest \$1 at 100% interest for 1 year at various compounding levels. What is the highest amount of money you can have after 1 year?

Note: the formula used for compound interest of \$1 at 100% interest annually compounded n times during the year is:

$$A = 1\left(1 + \frac{1}{n}\right)^n$$

Compounding Level, n	Amount, A in dollars
Annualy (once a year)	
Semi-annually (2-times)	
Quarterly (4-times)	
Monthly (12-times)	
Daily (365-times)	
Secondly (31 536 000-times)	
Continuously (1 000 000 000-times)	

Properties of e:

- $e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$ ____________________________
- e is an _______________ number, similar to π. They are non-terminating and non-repeating.
- $\log_e x$ is known as the _________________________ and can be written as ______
- Many naturally occurring phenomena can be modelled using base-e exponential and logarithmic functions.
- $\log_e e = \ln e =$ ___

Part 2: Reminder of Log Rules

Power Law of Logarithms	$\log_b x^n = n \log_b x$ for $b > 0, b \neq 1, x > 0$
Product Law of Logarithms	$\log_b(mn) = \log_b m + \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$
Quotient Law of Logarithms	$\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$
Change of Base Formula	$\log_b m = \frac{\log m}{\log b}, m > 0, b > 0, b \neq 1$
Exponential to Logarithmic	$y = b^x \rightarrow \mathrm{x} = \log_b y$
Logarithmic to Exponential	$y = \log_b x \rightarrow x = b^y$
Other useful tips	$\log_a(a^b) = b$ $\quad \log a = \log_{10} a$ $\quad \log_b b = 1$

Part 2: Solving Problems Involving *e*

Example 2: Evaluate each of the following

a) e^3

b) $\ln 10$

c) $\ln e$

Example 3: Solve each of the following equations

a) $20 = 3e^x$

b) $e^{1-2x} = 55$

c) $2\ln(x-3)-7=3$

d) $\ln(4e^{x})=2$

Part 3: Graphing Functions Involving *e*

Example 4: Graph the functions $y=e^{x}$ and $y=\ln x$

Note: $y=\ln x$ is the inverse of $y=e^{x}$

$y=e^{x}$	
x	y

$y=\ln x$	
x	y