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L8 – The Natural Logarithm		•
MHF4U		
Jensen		•
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Part 1: What is e'?

Example 1: Suppose you invest \$1 at 100% interest for 1 year at various compounding levels. What is the highest amount of money you can have after 1 year?

Note: the formula used for compound interest of \$1 at 100% interest annually compounded n times during the year is:

$$A = 1\left(1 + \frac{1}{n}\right)^n$$

Compounding Level, n	Amount, A in dollars
Annualy (once a year)	$A = 1\left(1 + \frac{1}{1}\right)^1 = 2$
Semi-annually (2-times)	$A = 1\left(1 + \frac{1}{2}\right)^2 = 2.25$
Quarterly (4-times)	$A = 1\left(1 + \frac{1}{4}\right)^4 = 2.4414$
Monthly (12-times)	$A = 1\left(1 + \frac{1}{12}\right)^{12} = 2.61304$
Daily (365-times)	$A = 1\left(1 + \frac{1}{365}\right)^{365} = 2.71457$
Secondly (31 536 000-times)	$A = 1\left(1 + \frac{1}{31536000}\right)^{31536000} = 2.718281785$
Continuously (1 000 000 000-times)	$A = 1\left(1 + \frac{1}{100000000}\right)^{1000000000} = 2.718281827$

Properties of e:

- $e = \lim_{n \to \infty} \left(1 + \frac{1}{n} \right)^n \approx 2.718\ 281\ 828\ 459$
- *e* is an <u>irrational</u> number, similar to π . They are non-terminating and non-repeating.
- $\log_e x$ is known as the <u>natural logarithm</u> and can be written as $\ln x$
- Many naturally occurring phenomena can be modelled using base-*e* exponential and logarithmic functions.
- $\log_e e = \ln e = 1$

Part 2: Reminder of Log Rules

Power Law of Logarithms	$\log_b x^n = n \log_b x \text{for } b > 0, b \neq 1, x > 0$		
Product Law of Logarithms	$\log_b(mn) = \log_b m + \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$		
Quotient Law of Logarithms	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n \text{for } b > 0, b \neq 1, m > 0, n > 0$		
Change of Base Formula	$\log_b m = \frac{\log m}{\log b}$, $m > 0, b > 0, b \neq 1$		
Exponential to Logarithmic	$y = b^x \rightarrow x = \log_b y$		
Logarithmic to Exponential	$y = \log_b x \rightarrow x = b^y$		
Other useful tips	$\log_a(a^b) = b \qquad \qquad \log_a a = \log_{10} a \qquad \qquad \log_b b = 1$		

Part 2: Solving Problems Involving e

Example 2: Evaluate each of the following

a) $e^3 \cong 20.086$

b) $\ln 10 \cong 2.303$

c) $\ln e = 1$

Example 3: Solve each of the following equations

a)
$$20 = 3e^{\chi}$$

 $20 = 3e^{\chi}$
 $\frac{20}{3} = e^{\chi}$
 $\ln \left(\frac{20}{3}\right) = \ln \left(e^{\chi}\right)$
 $\ln \left(\frac{20}{3}\right) = \chi \cdot \ln(e)$
 $\ln \left(\frac{20}{3}\right)$
 $\ln \left(\frac{20}{3}\right) = \chi \cdot \ln(e)$
 $\ln \left(\frac{20}{3}\right)$
 $\ln \left(\frac{20}{3}\right) = \chi \cdot \ln(e)$
 $\ln \left(\frac{20}{3}\right)$
 $\ln \left(\frac{20}{$

$$2\ln(x-3) - 7 = 3$$

 $2\ln(x-3) = 10$
 $\ln(x-3) = 5$
 $e^{5} = x - 3$
 $e^{5} + 3 = 2$
 $\chi = 151.413$

$$ln(4e^{x}) = 2$$

$$e^{2} = 4e^{x}$$

$$\frac{e^{2}}{4} = e^{x}$$

$$ln(\frac{e^{2}}{4}) = ln(e^{x})$$

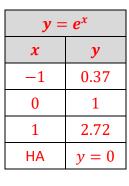
$$ln(\frac{e^{2}}{4}) = x \cdot ln(e)^{x}$$

$$\chi = 0.614$$

: Graphing Functions Involving e

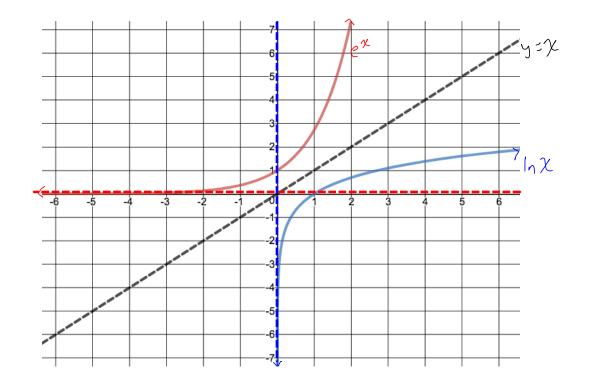
<u>Part 3</u>

Example 4: Graph the functions $y = e^x$ and $y = \ln x$



$y = \ln x$		
x	у	
0.37	-1	
1	0	
2.72	1	
VA	x = 0	

Note: $y = \ln x$ is the inverse of $y = e^x$



d) $\ln(4e^x) = 2$