Advanced Functions (MHF4U)

Final Exam

Mr. Jensen

Instructions: **There is a 1.5 hour minimum, 3 hour maximum time limit for writing this exam. All questions are to be answered on the exam paper.**

Communicate your solutions clearly.

Calculators are permitted. **Be in Radian Mode!**

Notes: Number of Pages (including cover): 16

The back of this page is left blank intentionally.
The last page is a formula page and extra work page as well.

Number of Questions: 34
Total Marks: 104

Round to 3 decimal places when necessary
(unless otherwise indicated)

Please check over your solutions!

Knowledge	Application	Thinking	Communication
25%	25%	25%	25%

Topic of Study	Points
Multiple Choice	16
Polynomials Functions	26
Exponentials and Logarithms	20
Trigonometry	22
Rational Functions	8
Problem Solving (choose 3 of 4)	12
Total	**104**

Section 1: Multiple Choice

Instructions: *Circle the letter of the most correct answer. Also record you letter of choice in the key at the bottom of the page.*

1) What is the end behavior of the function $P(x) = -5x^4 + x^2 + 7x$

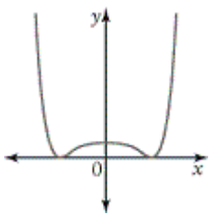
A) Q2 → Q1 **B)** Q3 → Q1 **C)** Q3 → Q4 **D)** Q2 → Q4

2) Which of the following is an odd function

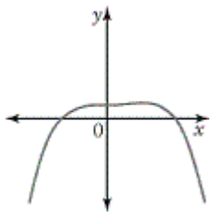
A) $y = 2x^3 - 3x^2$ **B)** $y = -2x^5 - 2x$ **C)** $y = 2x^3 - x + 1$ **D)** All of these

3) Which of the following graphs represents the function $y = -3x^4 + 2x^3 + 1$

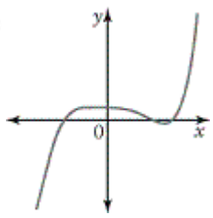
A)



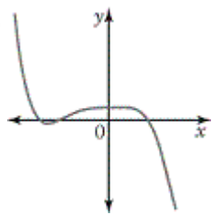
B)



C)

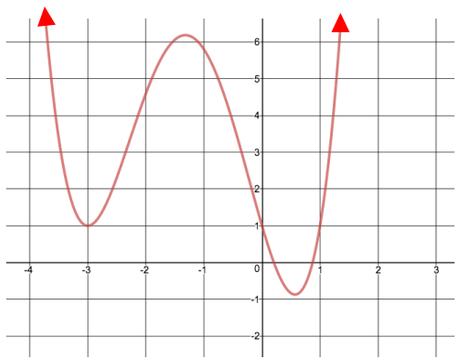


D)



4) What is the least possible degree of the function represented in the graph to right:

A) 2 **B)** 4 **C)** 5 **D)** 6



5) What is the remainder when the polynomial $y = x^2 - 5x + 2$ is divided by $x + 1$

A) 8 **B)** -2 **C)** 6 **D)** 0

6) The polynomial $f(x) = 8x^3 - 27$ will factor into…

A) $(2x - 3)(2x + 3)^2$ **B)** $(2x - 3)(4x^2 + 6x + 9)$

C) $(2x + 3)(4x^2 - 6x + 9)$ **D)** $(2x + 3)(2x - 3)^2$

7) Which of the following divisors is a factor the polynomial $3x^4 + x^3 - 14x^2 - 4x + 8$

A) $x - 1$ **B)** $4x + 3$ **C)** $x + 1$ **D)** $3x + 4$

8) Which logarithm statement below is equivalent to the exponential statement $3^4 = 81$?

A) $\log_3 4 = 81$ **B)** $\log_4 81 = 3$ **C)** $\log_4 3 = 81$ **D)** $\log_3 81 = 4$

1) _____	**2)** _____	**3)** _____	**4)** _____	**5)** _____	**6)** _____	**7)** _____	**8)** _____

9) Evaluate $\log_3 9$

A) 3 **B)** 9 **C)** 0.5 **D)** 2

10) The pH of a solution with a hydronium ion concentration of 0.04 mol/L is

A) 1.40 **B)** 1.09 **C)** 1.04 **D)** 2.62

11) Convert 300° to radian measure.

A) $\frac{5\pi}{3}$ radians **B)** $\frac{4\pi}{3}$ radians **C)** $\frac{5\pi}{6}$ radians **D)** $\frac{7\pi}{6}$ radians

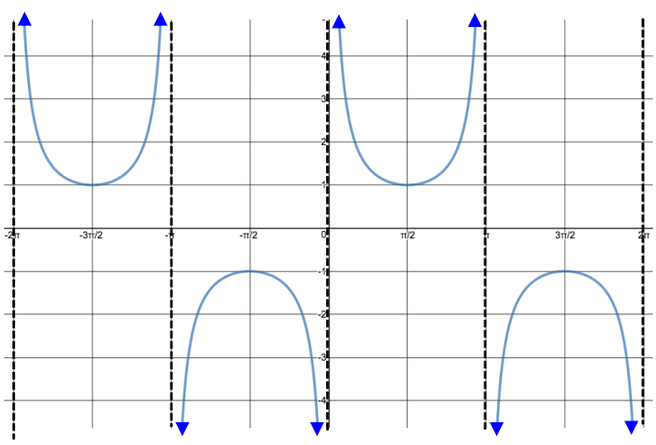
12) Convert $\frac{5\pi}{4}$ radians to degree measure.

A) $135°$ **B)** $225°$ **C)** $45°$ **D)** $315°$

13) Use special triangles and the CAST rule to determine the exact value of $\csc\frac{5\pi}{3}$

A) $-\frac{\sqrt{3}}{2}$ **B)** $-\frac{2}{\sqrt{3}}$ **C)** -2 **D)** $\sqrt{3}$

14) The graph shown is the of the function…



A) $y = \csc x$

B) $y = \sec x$

C) $y = \cot x$

D) $y = \tan x$

15) Which statement below is TRUE about the rational function $y = \frac{3x-2}{x+1}$?

A) VA $x = 3$ and y-int at (0, -2) **B)** VA $x = -1$ and y-int at (0, -2)

C) HA $y = 3$ and x-int at (2, 0) **D)** HA $y = -1$ and x-int at $\left(\frac{3}{2}, 0\right)$

16) Using a surrounding interval, a good estimate for the instantaneous rate of change of the temperature at 10 minutes is…

Time in Minutes	0	5	8	10	13	15	19	21	25
Temp. (in degree Celsius)	25	120	205	250	290	280	290	285	285

A) 250 degrees/min **B)** 15 degrees/min **C)** 10.4 degrees/min **D)** 17 degrees/min

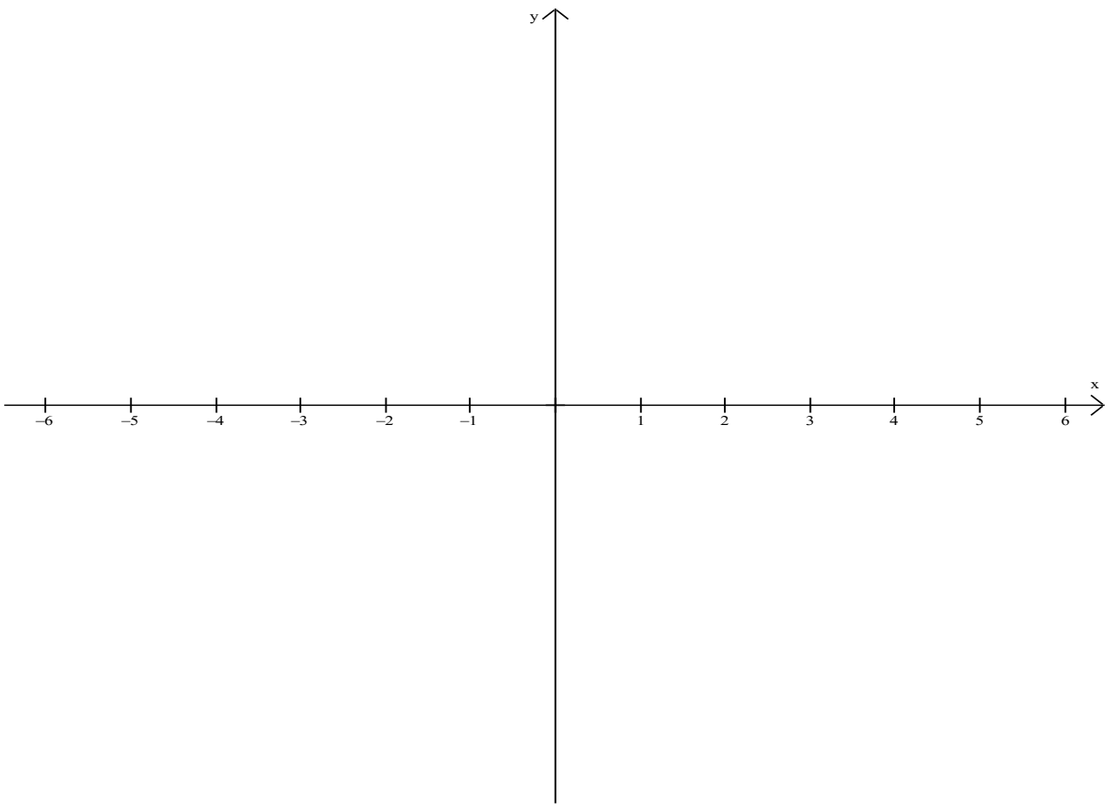
9) _____ **10)** _____ **11)** _____ **12)** _____ **13)** _____ **14)** _____ **15)** _____ **16)** _____

Section 2: Polynomial Functions

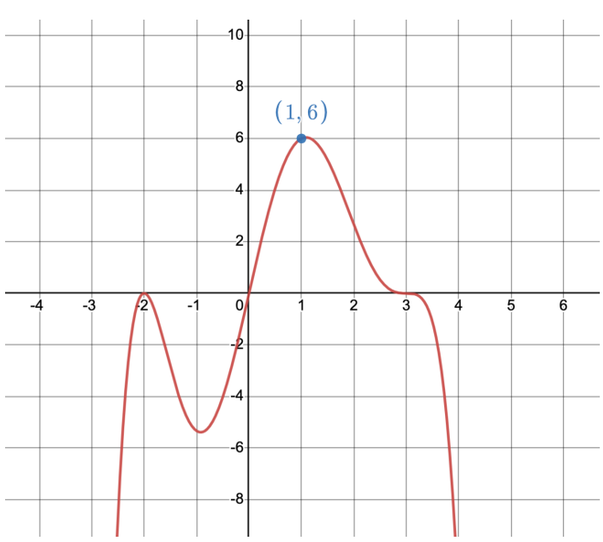
17) Complete the chart and sketch a possible graph of the function labelling the x and y intercepts. [6]

$$f(x) = -\frac{1}{2}(x+4)(2x-3)^2(x+1)^3$$

Degree	Leading Coefficient	End Behaviour	x-intercepts with orders	y-intercept



18) Find an equation in factored form for the quintic polynomial whose graph is shown below. State the final equation in factored form. [3]



Equation:

19) Use long division to divide $6x^3 + x^2 - 13x + 5$ by $2x + 1$. Express your answer in quotient form: [3]

Final Answer in Quotient Form:

20) Use synthetic division to divide $2x^3 + 5x^2 - 5$ by $x + 2$. Express your answer using the multiplication statement that can be used to check the division. [3]

Multiplication Statement to Check Division:

21) Fully factor the polynomial $P(x) = x^3 + x^2 - 10x + 8$. I am looking to see a FULL list of possible zeros, your test of the zero, and your polynomial division. [5]

Possible Zeros:

Test(s):

Factored Form:

$P(x) =$

22) Solve the following equation using any algebraic method. [3]

$2x^3 + 5x^2 - 14x - 8 = 0$

Solution(s):

23) Solve the following inequality using any method. Show your work. [3]

$2x^3 - 6x^2 \geq 18x - 54$

Section 3: Exponential and Logarithmic Functions

24) Rewrite each of the following as a single logarithm and then evaluate. Round to the nearest hundredth if necessary. [6]

a) $\log_5 7 + \log_5 4$ **b)** $2\log 8 + \log 2 - \log 4$ **c)** $\frac{\log_8 80}{\log_8 4}$

25) Solve for x in each of the following equation. Round to 3 decimal places where necessary. Show all of your work. Check for extraneous roots where necessary. [12]

a) $\log(x+2)+\log(x-1)=1$

b) $\ln(2x-10)=6$

c) $4^{2x-5}=8^{x}$

d) $3^{x+5}=5^{2x-1}$

26) The intensities of sound pollution were measured at a small airport runway and a local highway. The airport was 6420.4 times as intense as the highway. If the sound level on the local highway is 91 dB, determine the sound level, in dB, on the runway. [2]

Final Answer:

Section 4: Trigonometry

27) Suppose $\sin\theta = \frac{21}{29}$ and $\frac{\pi}{2} < \theta < \pi$. State an exact value for $\sin(2\theta)$. (Hint: use an identity, DO NOT SOLVE FOR θ) [3]

$\sin(2\theta) =$

28) Use a compound angle formula to find an exact value for $\cos\left(\frac{7\pi}{12}\right)$. Show all of your work. [4]

$\cos\left(\frac{7\pi}{12}\right) =$

29) Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible. [6]

a) $2\sin x + \sqrt{3} = 0$

Solution(s):

b) $2\cos^2 x - \cos x = 0$

Solution(s):

30) Prove the following trig identity. Show ALL of your work. [3]

$\frac{1-\cos(2x)}{\sin(2x)} = \tan x$

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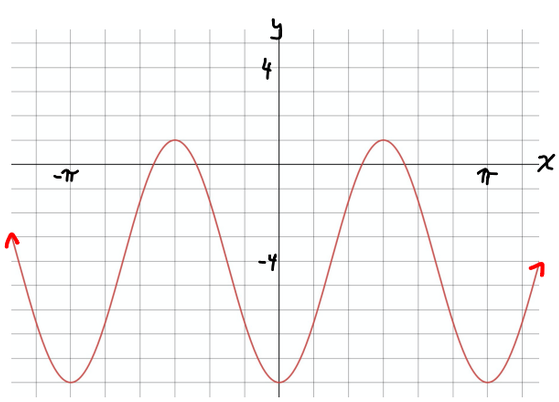
31) Find two equations (one sine and one cosine) to represent the function on the graph below. Show your calculations for full marks. [5]



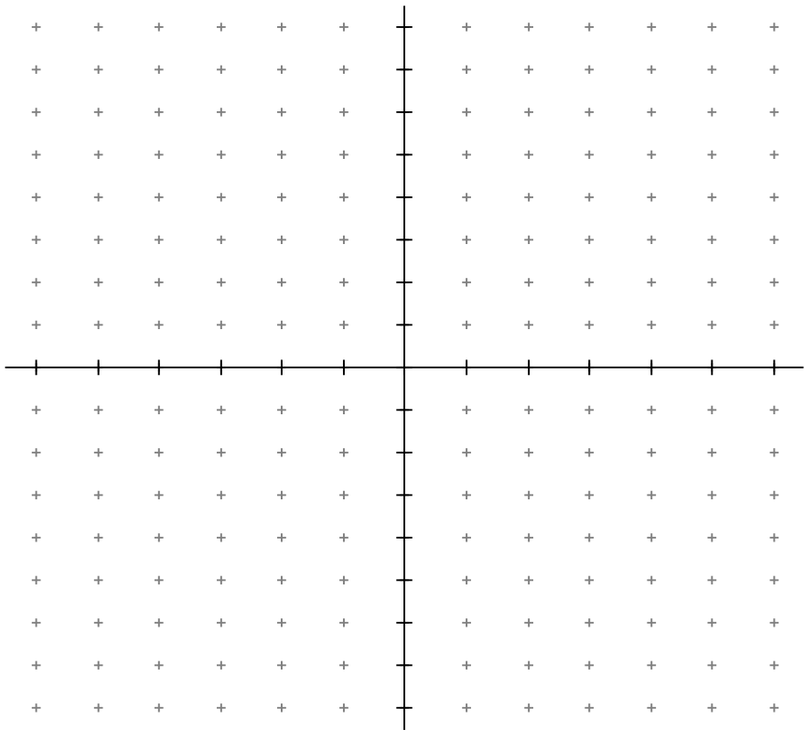
Equation 1: _____ Equation 2: _____

Section 6: Rational Functions

32) Graph $f(x)$ OR $g(x)$, NOT BOTH. Circle your function of choice. Show your work and any key information that you used to graph your function. Label any asymptotes and label your x and y scales appropriately. [4]

$f(x) = \frac{2x-4}{x+1}$

$g(x) = \frac{1}{x^2+2x-8}$



33) Solve the inequality $\frac{2x-5}{x-1} \geq 1$. Show your work including a factor table. [4]

Solution:

34) Choose 3 of the following 4 questions to complete. It must be clear which 2 questions you chose. [12]

Choice 1: Determine ALL solutions for the equation $\cos(2x) = \frac{\sqrt{3}}{2}$ in the interval $0 \leq x \leq 2\pi$. Give exact answers where possible.

Choice 2: Solve the equation $4^{2x} - 2(4^x) - 15 = 0$. Round to 3 decimal places where necessary. Make sure to check for extraneous routes where necessary.

Choice 3: Given the equation $f(x) = x^4 - 2x^3 + kx - 5$, solve for k given that $f(x)$ divided by $x + 1$ has a remainder of -6.

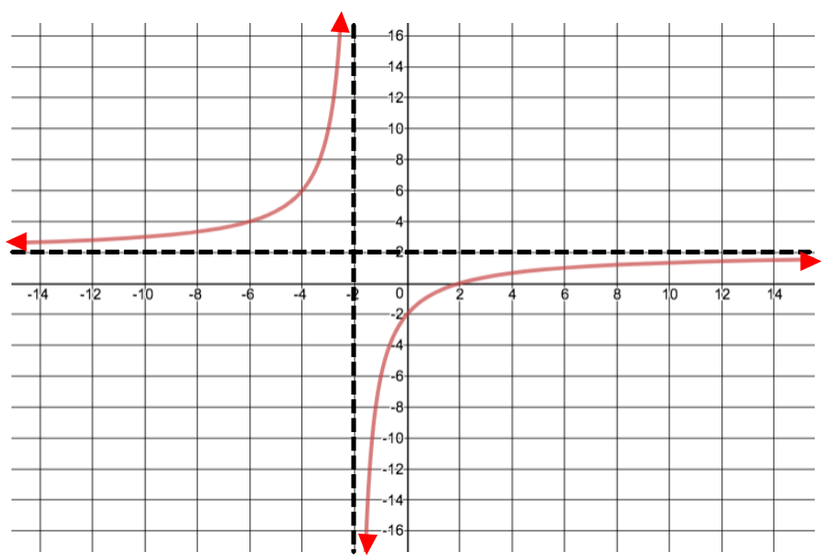
Choice 4: Use the given graph to find the following limits:

a) $\lim_{x \to \infty} f(x)$

b) $\lim_{x \to -2^+} f(x)$

c) $\lim_{x \to -2^-} f(x)$

d) $\lim_{x \to -2} f(x)$



Formula Page

TRIG IDENTITIES

Sine Law: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ **Cosine Law:** $c^2 = a^2 + b^2 - 2ab\cos C$

Reciprocal Functions: $\csc x = \frac{1}{\sin x}; \sec x = \frac{1}{\cos x}; \cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}; \quad \cot x = \frac{\cos x}{\sin x}$

Even/Odd Functions: $\cos(-x) = \cos x; \quad \sin(-x) = -\sin x$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1; \quad \cos^2 x = 1 - \sin^2 x; \quad \sin^2 x = 1 - \cos^2 x$

$\tan^2 x + 1 = \sec^2 x; \quad 1 + \cot^2 x = \csc^2 x$

Transformation Identities: $\cos\left(x - \frac{\pi}{2}\right) = \sin x; \ \sin\left(x + \frac{\pi}{2}\right) = \cos x$

Co-function Identities: $\cos\left(\frac{\pi}{2} - x\right) = \sin x; \ \sin\left(\frac{\pi}{2} - x\right) = \cos x$

Compound Angle Formulas:

$\sin(x + y) = \sin x \cos y + \cos x \sin y; \quad \sin(x - y) = \sin x \cos y - \cos x \sin y$

$\cos(x + y) = \cos x \cos y - \sin x \sin y; \quad \cos(x - y) = \cos x \cos y + \sin x \sin y$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}; \quad \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double Angle Formulas:

$\sin(2x) = 2\sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x; \quad \cos(2x) = 2\cos^2 x - 1; \quad \cos(2x) = 1 - 2\sin^2 x$

$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$

OTHER FORMULAS

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **Newton Quotient:** $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Difference of Squares: $a^2 - b^2 = (a - b)(a + b)$ **Perfect Square Trinomial**: $a^2 + 2ab + b^2 = (a + b)^2$

Sum of Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ **Difference of Cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Exponential Formulas		
$\boldsymbol{A(t) = A_0(1 + i)^t}$ where i is percent growth (+) or decay (-)	$\boldsymbol{A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}}$ where H is the half-life period	$\boldsymbol{A(t) = A_0(2)^{\frac{t}{D}}}$ where D is the doubling period
Logarithmic Formulas		
$\boldsymbol{pH = -\log[H^+]}$ where pH is acidity and $[H^+]$ is concentration of hydronium ions in mol/L	$\boldsymbol{\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)}$ where β is the loudness in dB and I is the intensity of sound in W/m^2	$\boldsymbol{M = \log\left(\frac{I}{I_0}\right)}$ Where M is the magnitude measured by richters and I is intensity.

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