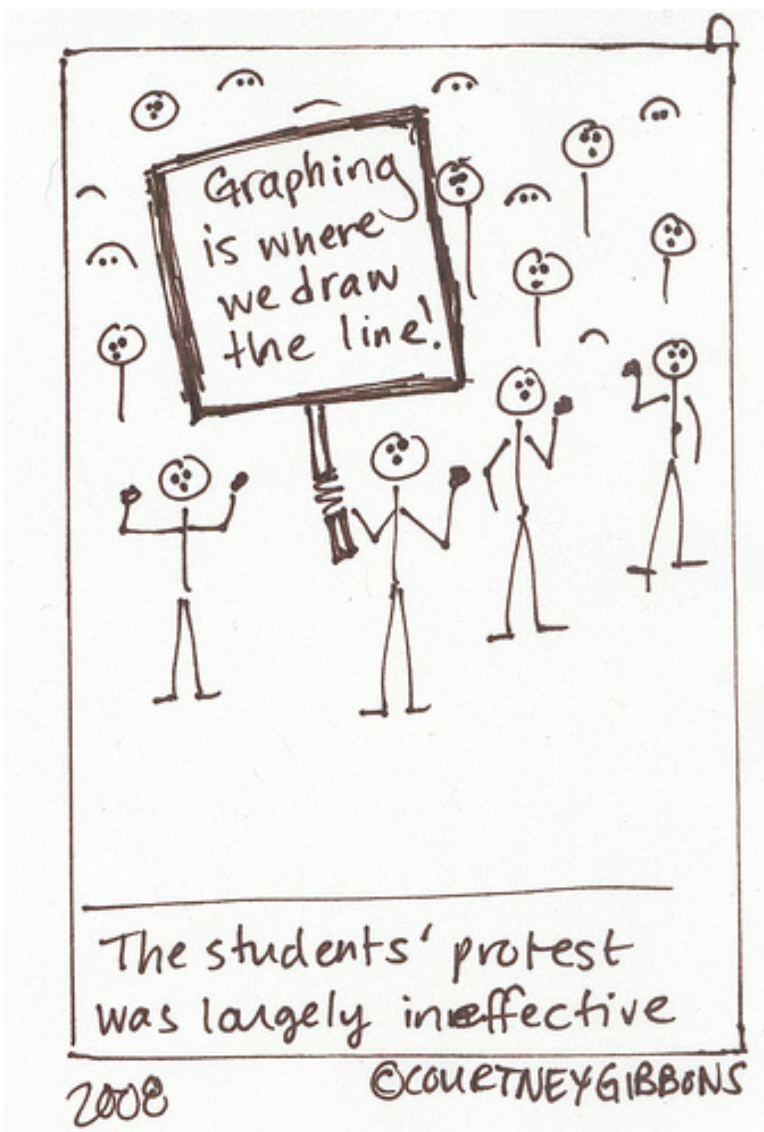
Unit 2 – Linear Relations

Chapter 2 – Relations
Chapter 5 – Modeling With Graphs
Chapter 6 – Analyze Linear Relations

MPM1D



Chapter 2

Linear and Non-Linear Relations

2.1 Hypotheses and Sources of Data

Hypothesis: The Pittsburgh Penguins are the most popular sports franchise.

Discuss the validity of this hypothesis:

Not valid because there is no evidence (proof).

How could this hypothesis be verified?

Survey a variety of people from different towns.

What is a hypothesis?....

A hypothesis is a theory or statement that is either true or false

Practice making hypotheses about the relationship between each pair of variables :

1. The number of texts sent per day and the age of a person?

The older the person, the fewer texts sent.

2. How much a person likes the Penguins and their IQ?

The higher a person's IQ, the more they like the Penguins.

3. The size of an animal and its lifespan?

The bigger the size, the longer the lifespan.

Hypotheses and their opposites

Write a hypothesis about a relationship between the variables in each pair. Then, give the opposite hypothesis

4) A driver's age and the risk of having an accident

Hypothesis: As drivers age, their risk of having an accident increases.

Opposite: As drivers age, their risk of having an accident does not increase.

or

As drivers age, their risk of having an accident decreases or stays the same.

5) Homework completion and marks.

Hypothesis: Less homework completion results in lower test marks.

Opposite: Less homework completion does not result in lower test marks.

or

Less homework completion results in higher or the same test marks.

Sources of Data

Primary Data: Original data that a researcher gathers specifically for a particular experiment or survey

Secondary Data: Data that someone else has already gathered for some other purpose

6)

Which of the following is a primary source of data:

a) an article in a magazine
b) a database
c) conducting an experiment to test the effectiveness of a new medication
d) an entry from an online encyclopedia

---

7)

Which of the following is a secondary source of data:

a) conducting a survey amongst your classmates
b) conducting an experiment to study the effects of pollution
c) data collected 100 years ago by the Canadian government
d) counting the makes of cars in a mall parking lot

Explain whether each set of data is primary or secondary. What are the advantages and disadvantages of each person's choice of data source

8) Daniel phoned 100 families in his town to ask them how many pets they owned:

Primary data because Daniel performed the survey himself. The telephone survey is easy to do but time consuming.

9) Cathy used data from Statistics Canada to determine the proportion of households in Canada that have at least one car.

Cathy is using a secondary source since Statistics Canada gathered the survey data. Stats Canada is an excellent source because it collects data from a huge number of families all across Canada. Cathy could never gather that much data all by herself. Collecting secondary data is usually more time efficient than collecting primary data.

2.2 Sampling Principles

DO IT NOW!

King's Christian Collegiate wants to get a mascot for sporting events. They want the mascot to appeal to all of the students in the school. The school administration wants to ask student's what they want but they are unsure how to decide fairly. Should they:

a) Ask all of the grade 9 students
b) Let the first 100 students who arrive at school fill out a survey
c) Ask 6 students from every period 1 class
d) Ask all students whose last name starts with A,D, or Y
e) Let Mr. Jensen decide, he will think of something awesome

Which one did you pick and why?

This sample best represents the entire school body. It will have the least amount of bias

Definitions

Population - The whole group of people or items being studied.

Sample - Any group of people or items selected from a population.

Cencus - A survey of all members of a population.

Why is the whole population not always surveyed when a hypothesis about the population is to be verified?

Populations being studied are often very large. It would take too long to do a census in most cases.

For which of the following is a sample suitable?

a. Find the most common make of car in the school parking lot.

b. Find your family's favourite food.

c. Find the most popular video game among grade 9 students in your class.

d. Find the favourite video game among grade 9 students in Canada.

Types of Sampling

Random Sample - a sample in which all members of a population have an equal chance of being chosen

Example: Drawing names from a hat

Non-random sampling - using a method that is not random to choose a sample from a population

Example: Asking only your best friends

Types of Random Sampling

Simple Random Sampling - choosing a specific number of members randomly from the entire population

Example: Having a computer randomly choose 100 students from our school population

Systematic Random Sampling - choosing members of a population at fixed intervals from a randomly selected member.

Example: Using a list of all students at the school, pick a random starting point and then choose every 8th student until you have 100 students for the survey.

Stratified Random Sampling - dividing a population into distinct groups and then choosing the same fraction of members from each group

Example: The principal chooses to survey 10% of the students from each grade.

Classify the sampling technique used in each survey as simple random, systematic random, stratified random, or non-random sampling.

a) The principal selects people that work in the cafeteria to interview about the quality of cafeteria food.

non-random.

b) A computer is programmed to randomly select 100 names from a club's membership list.

simple random

c) Students are selected at random, with the number of students in each age group selected proportional to the size of the age group.

stratified random

d) To select 100 people who can buy concert tickets, the ticket agent randomly selects one wristband number and then every 10th number after that.

systematic random

Hobson's Company surveyed its 2000 customers by generating 200 random numbers between 1 and 2000, and then selecting names from the customer list corresponding to these numbers. This is an example of:

a. systematic random sampling
c. non-random sampling
b. stratified random sampling
d. simple random sampling

Which of the following is not an example of random sampling?

a. Use a random number generator to pick 10% of the players in each division of a hockey league.
b. Use a randomly generated number between 1 and 10 to pick a name on a list, and then select every 8th person on the list.
c. Ask every 10th person entering a mall for an opinion on government spending on health care.
d. Write names on slips of paper, and then pick the names out of a hat, making sure the pieces of paper are well mixed.

Which of the following is a systematic random sample?

a. A name is randomly selected from a list of a store's customers and every 10th person is selected before and after it.

b. A Member of Parliament randomly selects phone numbers from a city directory to survey citizen's opinions on government taxation.

c. The principal selects the same fraction of students from each class for a survey.

d. The Human Resources department of Acme Manufacturing Company sends out surveys to 50 employees randomly selected from the entire list of employees.

Bias

Bias - error resulting from choosing a sample that does not represent the whole population.

A sample could be biased if it is:

a. too small
b. only based on one gender and age group
c. not randomly drawn
d. all of the above

2.3 & 2.4 Scatter Plots and Trends in Data

Independent Variable: a variable that affects the value of another variable

Dependent Variable: a variable that is affected by some other variable

Example:

Independent - time spent practicing free throws

Dependent - free throw percentage in games

Your free throw percentage ***depends on*** the amount of time you spend practicing free throws

x # of Hours John Studies	John's Test Score y
0	75
.5	80
1	85
1.5	90
2	95
2.5	100

Independent Variable: # of Hours John Studies

Dependent Variable: John's Test Score

How are they related?

The more you study, the higher your test score will be.

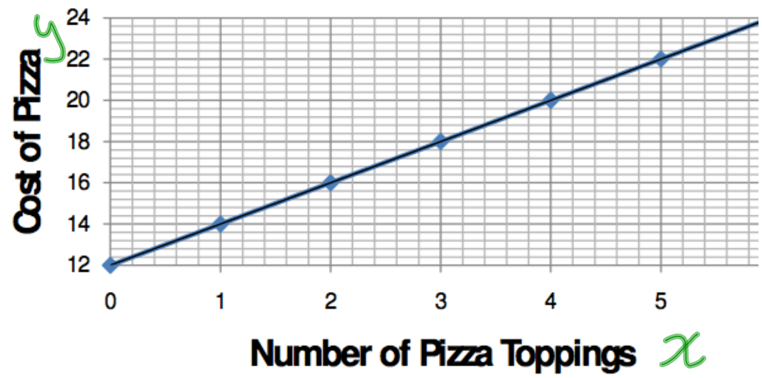
x Number of Guests	Meal Preparation Time (min) y
3	25
4	33
5	41
6	49
7	57
8	65

Independent Variable: Number of Guests

Dependent Variable: Meal Prep Time

How are they related?

The more guests you have attending dinner, the longer it will take to prepare the meal.



Independent Variable:

Number of Pizza Toppings

Dependent Variable:

Cost of Pizza

How are they related?

The more toppings you put on your pizza, the more it will cost

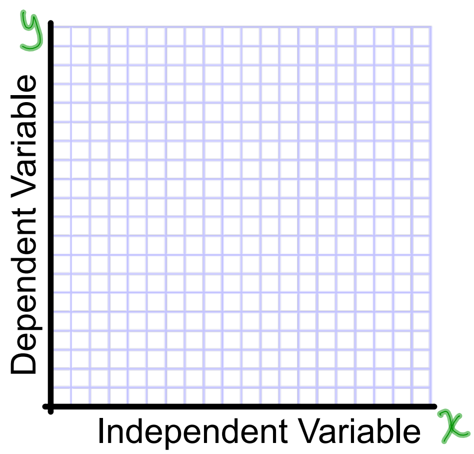
Now fill in the following the chart using your understanding of each type of variable:

x **Independent Variable**	y **Dependent Variable**
Number of gallons in your gas tank	How far you can drive
How much you read	Your IQ
Number of calories you eat each day	How much you weigh
How physically active you are	Your level of happiness
Number of hours you study for a test	Your test mark

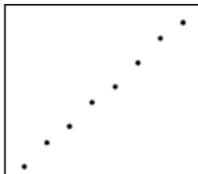
Scatter Plots

A Scatter plot is a graph that shows the correlation between two variables.

The Independent variable goes on the horizontal (x) axis, and the dependent variable goes on the vertical (y) axis.

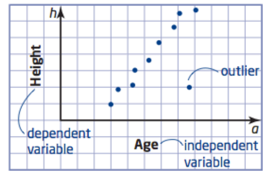


Types of correlations:

	A scatter plot shows a positive correlation when the pattern rises up to the right. *This means that the two quantities increase together.*
	A scatter plot shows a negative correlation when the pattern falls down to the right. *This means that as one quantity increases the other decreases.*
	A scatter plot shows no correlation when no pattern appears. *Hint:* *If the points are roughly enclosed by a circle, then there is no correlation.*

Correlations can also be strong or weak depending on how close or spread out the points on the scatter plot are.

Define an outlier:



measurement that differs significantly from the rest of the data.

When should you include an outlier in your data set?

If you can't show that it is inaccurate or unrepresentative

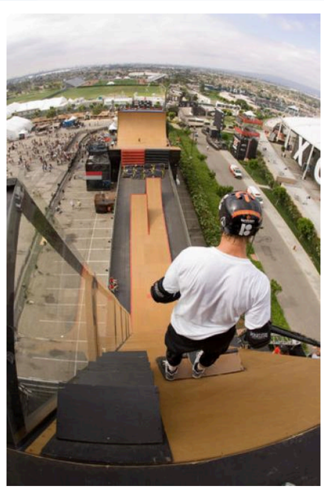
When shouldn't you?

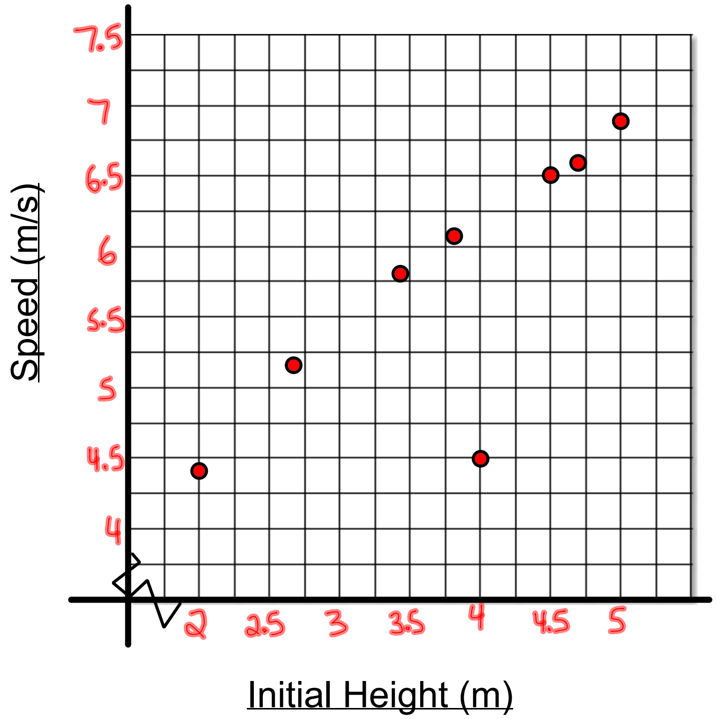
If you can show that it is inaccurate or unrepresentative

Make a Scatter Plot

A skateboarder starts from various points along a steep ramp and coasts to the bottom. This table lists the initial height and his speed at the bottom of the ramp.

Initial Height (m)	2.0	2.7	3.4	3.8	4.0	4.5	4.7	5.0
Speed (m/s)	4.4	5.2	5.8	6.1	4.5	6.5	6.6	6.9





Independent Variable: Initial Height

Dependent Variable: Speed

Describe the relationship:

The higher the initial height, the faster the speed.

There is a strong positive linear correlation

Are there any outliers? If so what are possible reasons for the outlier?

Yes, (4.0, 4.5) is an outlier. Maybe the skateboarder fell...

Line of Best Fit

A line of best fit can help you see the relationship between variables and also to make interpolations and extrapolations

Properties of a line of best fit:

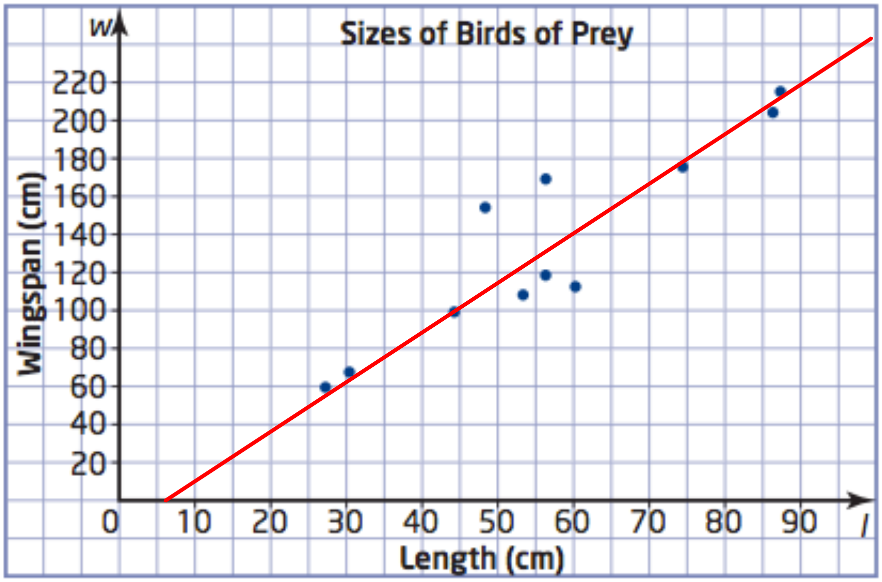
1. Straight line that passes through or close to as many points as possible

2. Any points that are not on the line should be evenly distributed above and below it

Interpolation: estimate a value between two measurements in a set of data

Extrapolation: estimate a value beyond the range of a set of data

Practice drawing a line of best fit:

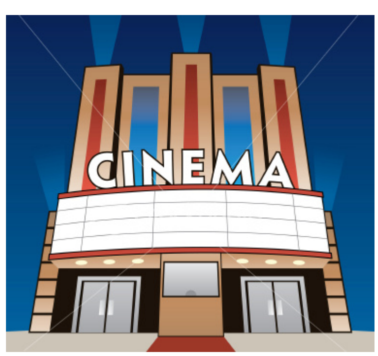


This table shows the number of paid movie admissions in Canada for 12 month periods

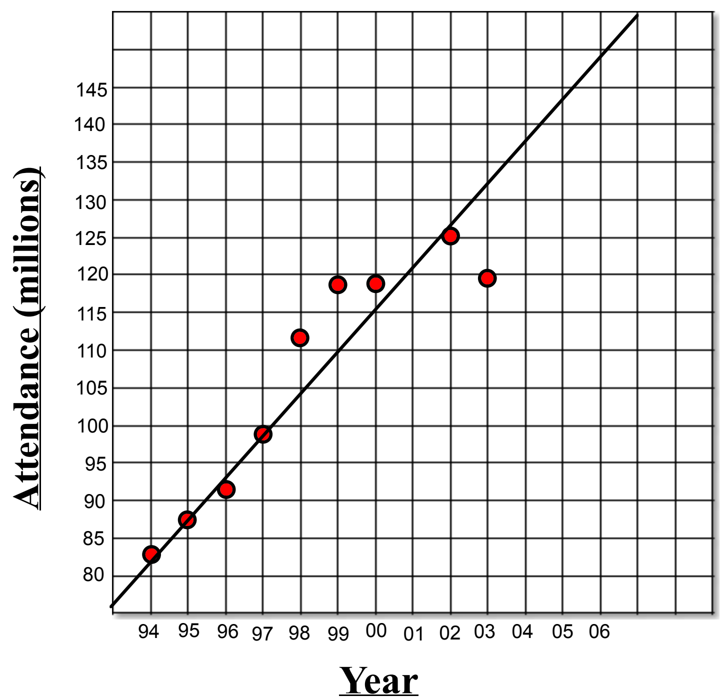
Fiscal Year	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Attendance (millions)	83.8	87.3	91.3	99.1	111.6	119.3	119.3	no data	125.4	119.6

Independent Variable: Fiscal Year

Dependent Variable: Attendance



Graph the data and draw a line of best fit:



Describe the correlation:

There is a strong positive linear correlation between the year and movie attendance.

Movie attendance increases as the year increases.

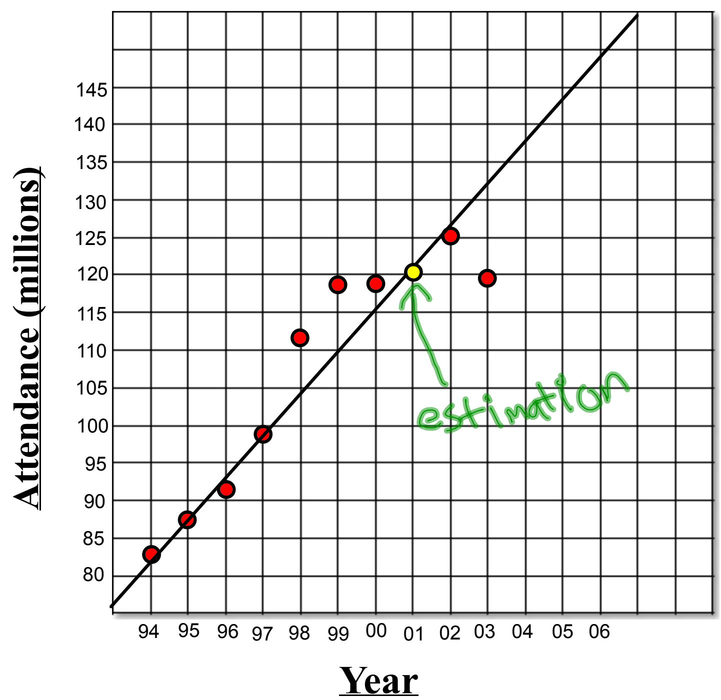
There is no data for 2001, estimate the movie attendance for this year using your line of best fit?

120 million

Did you use interpolation or extrapolation to estimate this data?

Interpolation

Graph the data and draw a line of best fit:



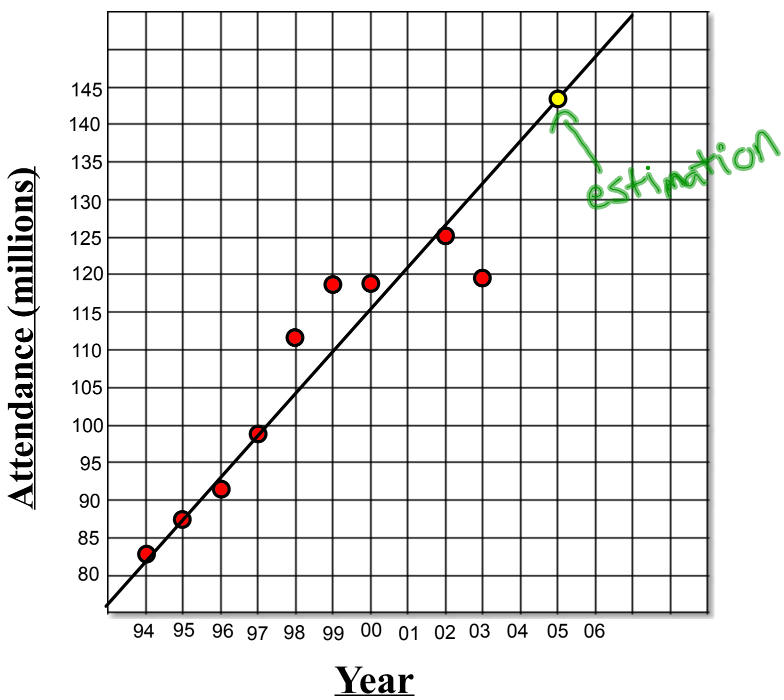
Estimate the movie attendance for 2005 by extending your line of best fit:

144 million

Did you use interpolation or extrapolation to estimate this data?

Extrapolation

Graph the data and draw a line of best fit:



Homework

Complete Worksheet

2.5 Linear and Non Linear Relationships

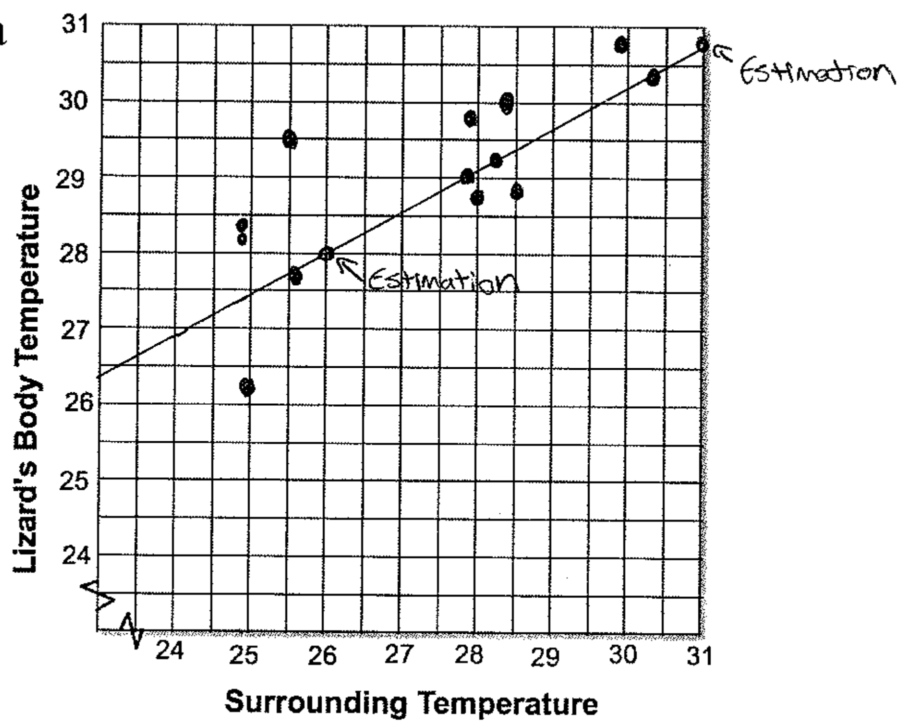
MPM1D
Jensen

ANSWERS

Do It Now The gymnophthalmid lizard lives in the Amazon rainforest. Recent research found that this lizard keeps its body temperature close to the temperature of its surroundings. The table lists data from this research.

Surrounding Temperature (°C)	25.0	24.8	27.9	30.3	28.2	24.8	25.6	29.9	25.5	28.4	28.5	28.0	27.9
Lizard's Body Temperature (°C)	26.2	28.2	29.7	30.3	29.8	28.3	27.6	30.8	29.5	30.0	28.8	28.7	29.0

a) Graph the data



b) Draw a line of best fit

c) Estimate the lizard's body temperature if the surrounding temperature is 26°C. Is this interpolation or extrapolation?

28°C. This is interpolation.

d) Estimate the lizard's temperature if the surrounding temperature is 31 degrees Celcius. Is this interpolation or extrapolation?

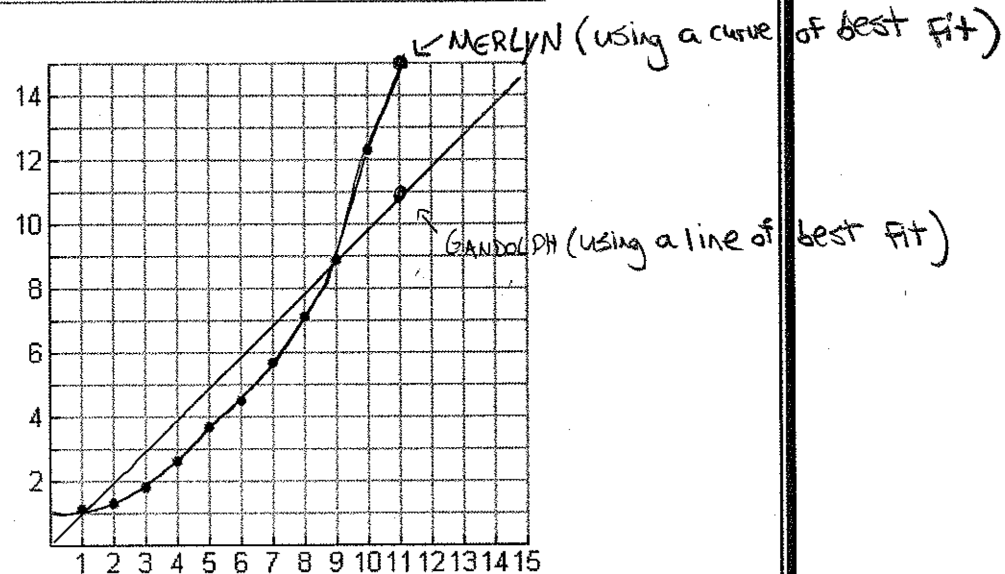
30.75°C. This is extrapolation.

Are all relationships linear? Is a line of best fit always appropriate?

Example 1:

A scatter plot of the following ten points is given below.

x	1	2	3	4	5	6	7	8	9	10
y	1.1	1.3	1.8	2.6	3.7	4.5	5.7	7.1	8.9	12.3



Gandolf predicts that when x is 11, y will be 11.
Merlyn predicts that when x is 11, y will be 15.

Who is correct? Why?

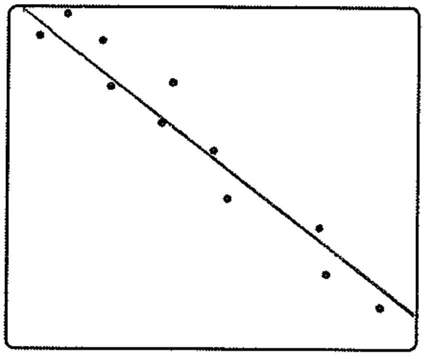
Merlyn is correct because the data does not follow a linear trend.

Many non-linear relations can be modeled with a curve of best fit.
You can draw curves of best fit using the same method as for a line of best fit. A curve of best fit should:

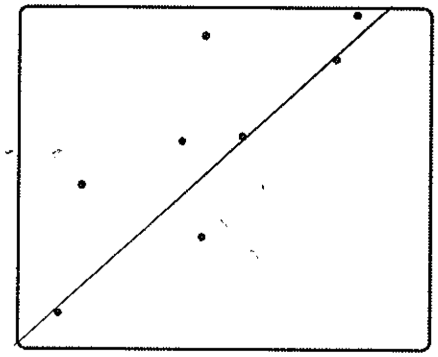
1. Pass through or close to as many points as possible,
2. Any points that are not on the curve should be distributed evenly above and below it.

Example 2: Describing Scatter Plots and Lines/Curves of Best Fit

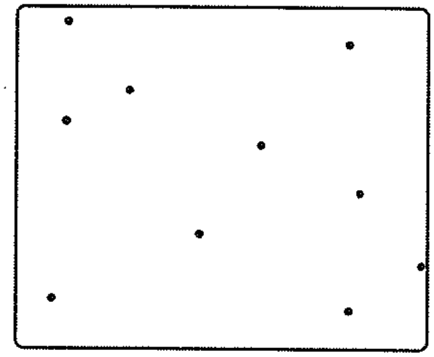
Draw a line or curve of best fit for each of the scatter plots below, if possible. Write two or three key words to describe each relation on the line below the scatter plot. (*positive relationship, negative relationship, no relationship, strong, weak, linear, non-linear*)



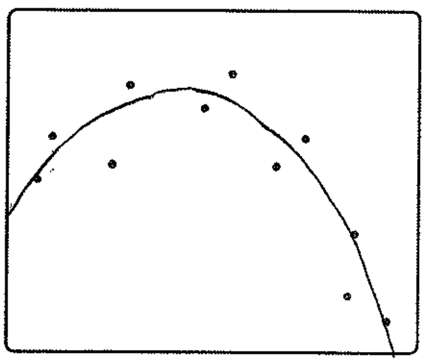
a) negative, strong, linear



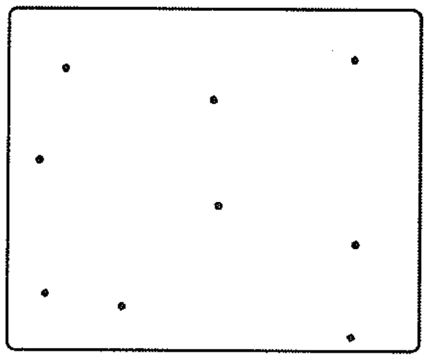
b) Positive, weak, linear



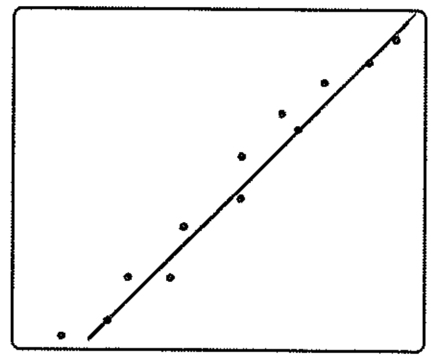
c) no relationship



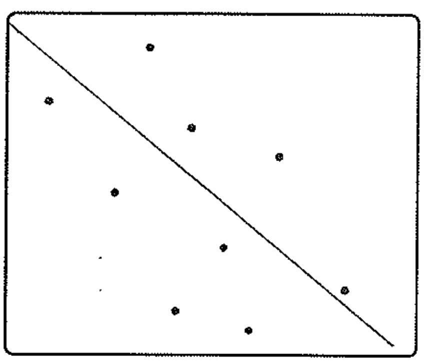
d) non-linear



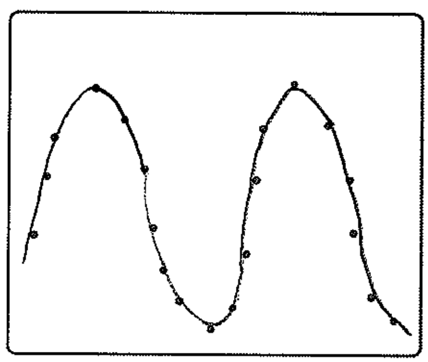
e) no relationship



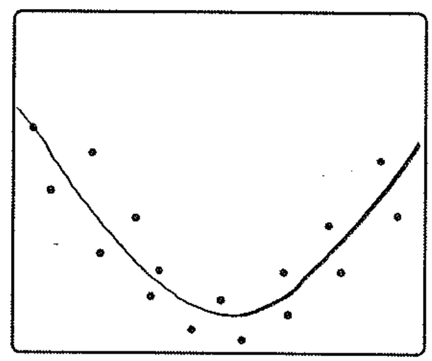
f) strong, positive, linear



g) weak, negative, linear



h) non-linear

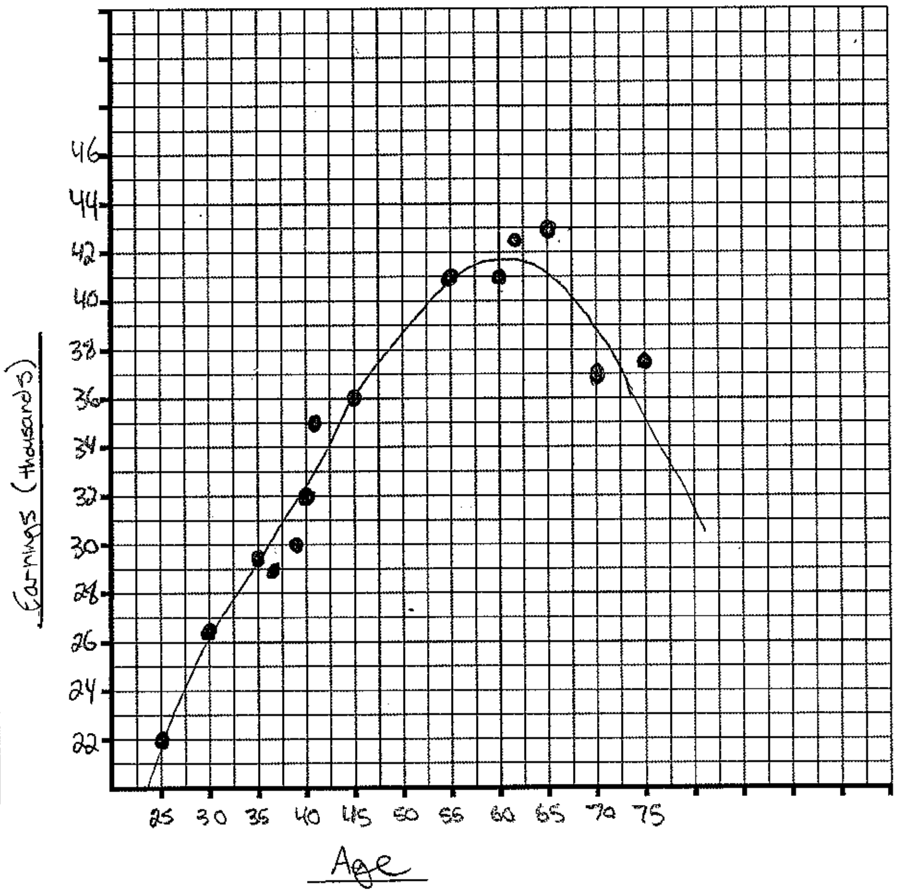


i) non-linear

Example 3: Test the hypothesis: The older you are, the more money you earn.

Plot the data on the scatter plot below, choosing appropriate scales and labels.

Age	Earnings ($)
25	22000
30	26500
35	29500
37	29000
38	30000
40	32000
41	35000
45	36000
55	41000
60	41000
62	42500
65	43000
70	37000
75	37500



a) Draw a curve of best fit. Describe the trend in the data.

Non-linear. Earnings increase up to age 65, then they decrease with age.

b) Does the data support the hypothesis? Give reasons to support your answer.
(Refer to the scatter plot.)

No, after the age of 65 you start to earn less money.

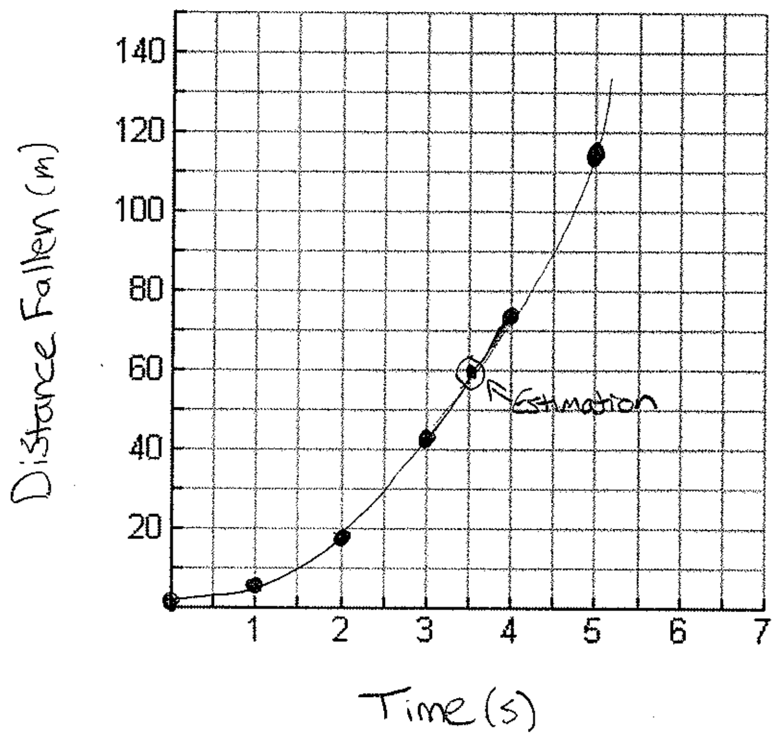
c) Explain why the data for ages over 65 do not correspond with the hypothesis.

That is a common time for retirement.

Example 4: A skydiver jumps from an airplane. The distance fallen and time taken are recorded in the table.

Time (s)	Distance (m)
0	0
1	5
2	19
3	42
4	74
5	115

a) Draw a scatter plot of the relation and draw a line or curve of best fit.



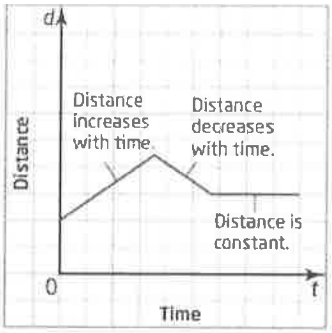
b) Classify the relation as linear or non-linear. Explain your choice.

Non-linear. As time increases, the rate of distance fallen is increasing. This causes the data to form a curve.

c) How far will the skydiver have fallen in 3.5 s?

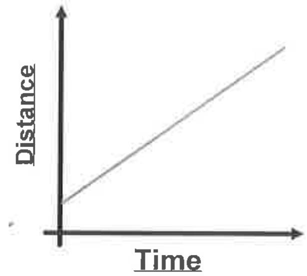
60m. This is interpolation.

2.6 – Distance Time Graphs



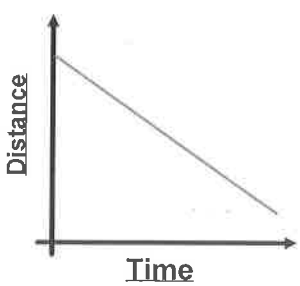
A distance-time graph shows an object's distance from a fixed point over a period of time.

A rising line shows that distance from a point increases as time increases.



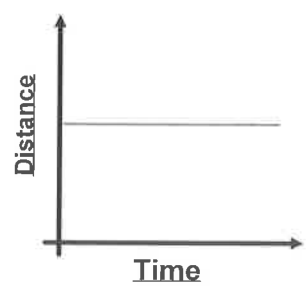
MOVING AWAY FROM SENSOR

A falling line shows that distance from a point decreases as time increases.



* MOVING TOWARDS SENSOR *

A horizontal line shows that distance from a point remains constant



* NO MOVEMENT *

Rate of Movement

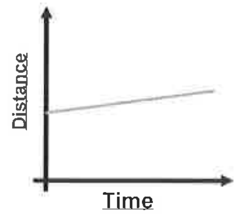
The **speed** of a person affects the steepness of the graph.

The steeper the line is, the faster the rate of movement.

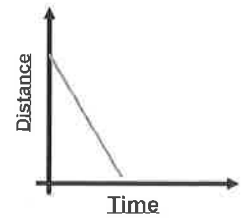
Remember:

$$speed = \frac{\Delta distance}{\Delta time}$$

Which graph represents slow movement? Which one represent fast movement?



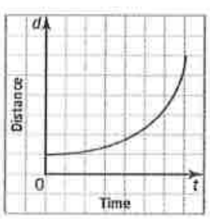
SLOW



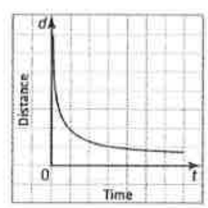
FAST

Changes of Rate of Movement

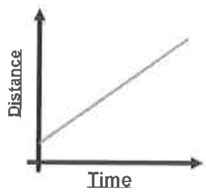
A curve may represent an increase in rate of movement (acceleration)



A curve may represent a decrease in rate of movement (deceleration)



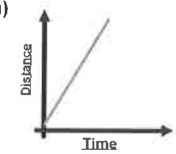
A straight line represents an object moving at a constant rate or steady pace.



Practice Describing Distance Time Graphs

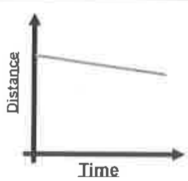
A person walks in front of a motion sensor. Describe the motion that would produce each of the following graphs:

a)



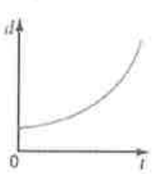
This line is steep. The person moves at a fast, steady pace away from the sensor.

b)



The person moves at a slow; steady pace towards the sensor.

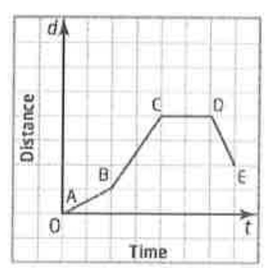
c)



The curve indicates a change in speed. the person moves away from the sensor; slowly at first, and then gradually picks up speed. (acceleration away from the sensor)

Analyzing a Distance-Time Graph

Describe the following graph that represents a person's distance from home over a period of time:



AB: Away from home at a slow steady pace.

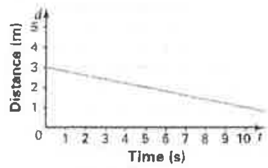
BC: Away from home at a fast steady pace.

CD: No movement

DE: Towards home at a fast steady pace.

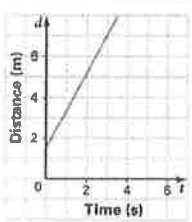
Describe the motion represented by each of the following graphs:

a)



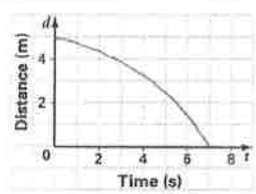
The person starts at a distance of 3 meters from the sensor and walks towards it at a slow; steady pace.

b)



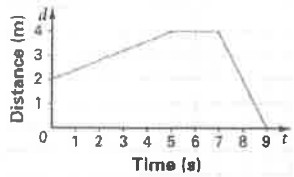
The person starts at a distance of 1.5 meters from the sensor and walks away from it at a fast, steady pace.

c)



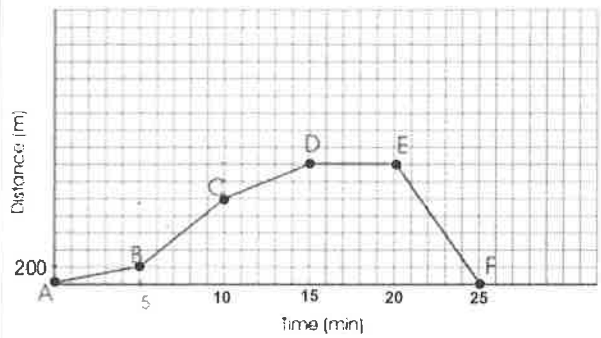
The person starts at a distance 5 meters from the sensor and walks toward it, slowly at first, and then gradually picks up speed.

d)



The person starts at a distance of 2 meters from the sensor, takes 5 seconds to walk at a slow steady pace away from the sensor, pauses for 2 seconds, and then walks at a fast steady pace towards the sensor.

Chris walks each day as part of his daily exercise. The graph shows his distance from home as he walks his route.



Using the graph, give an explanation of what is occurring over Chris' walk. Include information about time, distance, direction and speed during each segment

AB: speed $= \frac{200}{5} = 40$ m/min.

Chris walks away from home at a constant speed of 40 m/min.

BC: speed $= \frac{800}{5} = 160$ m/min

Chris walks quickly away from his house at a speed of 160 m/min.

CD: speed $= \frac{400}{5} = 80$ m/min

Chris walks away from his house at a constant speed of 80 m/min

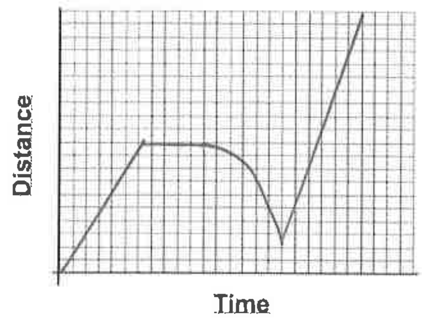
DE: Chris stops for a 5 minute break.

EF: Chris walks home quickly at a speed of 280 m/min

Creating a Distance Time Graph

Create a graph that shows Mr. Jensen's **distance from his own team's net** while he is playing hockey based on the following scenario:

Mr. Jensen starts with the puck in his own end and skates away from his net towards the other teams end at a steady pace. At centre ice he gets the puck stolen from him. Mr. Jensen is furious and stops for a couple seconds to slam his stick on the ice in frustration. He then decides to chase down the guy who stole the puck from him. He accelerates back towards his own net and steals the puck. He then slams on the brakes and skates at a fast steady pace away from his net towards the other team's net. He is so fast that he gets a breakaway and scores (HON candidate for sure).



5.1 Direct Variation

DO IT NOW!!

You and a friend decide to have a friendly bicycle race on Saturday. How far ahead will you be after one hour, given that you travel at the following speeds?

a) Heading North   your speed is 12km/h, your friend's speed is 10 km/h

b) Heading South   you travel 5 km in 30 minutes, your friend travels 5 km in 20 minutes

Speed Calculations:

$$speed = \frac{distance}{time} = \frac{km}{hour}$$

a) My speed = 12 km/h

Friend = 10 km/h

∴ after one hour, I will be 2 km ahead of my friend.

b) Me $= \frac{5\text{ km}}{0.5\text{ hrs}} = 10$ km/h

Friend $= \frac{5\text{ km}}{\frac{1}{3}\text{ hour}} = 15$ km/h

∴ my friend will be 5 km ahead of me after 1 hour.

Speed is an example of a rate of change

Part A: Investigate Direct Variation using Tables

<u>Example 1</u>: Use a table to organize the information in the following problem.

a) How long does it take to fill a 3 000 litre hot tub if a water truck supplies water at a rate of 250 litres per hour and the tub is initially empty?

Time (h)	Water (l)
0	0
1	250
2	500
3	750
4	1000
5	1250
6	1500
7	1750
8	2000
9	2250
10	2500
11	2750
12	3000

b) What if the water truck was able to double the flow rate?

Time (h)	Water (l)
0	0
1	500
2	1000
3	1500
4	2000
5	2500
6	3000

Which variable is dependent? (If a variable is dependent its value cannot be known without knowing the other variable's value). **To determine the dependent variable ask yourself the following key questions:**

Does the amount of water DEPEND on time?

OR

Does time DEPEND on the amount of water ?

∴ Dependent (y) = water
Independent (x) = time

$$Rate\ of\ Change = \frac{\Delta dependent\ variable}{\Delta independent\ variable} = \frac{\Delta y}{\Delta x} = \frac{\Delta water}{\Delta time}$$

a) Calculate the rate of change of the amount of water in the hot tub between the following times, using the first table:

i) between 0 and 1 hour:

$$rate = \frac{250\ L}{1\ h} = 250\ L/h$$

ii) between 0 and 2 hours:

$$\text{rate} = \frac{\Delta \text{water}}{\Delta \text{time}} = \frac{500L - 0L}{2h - 0h} = \frac{500L}{2h} = 250\ L/h$$

iii) between 2 and 5 hours:

$$\text{rate} = \frac{\Delta \text{water}}{\Delta \text{time}} = \frac{1250L - 500L}{5h - 2h} = \frac{750L}{3h} = 250\ L/h$$

What do you notice about these rates?

They are all the same. The rate is constant.

When the rate is ***constant,*** it is often referred to as the ***constant of variation***.

A **direct variation** is a relationship between two variables in which one variable is a constant multiple of the other. The rate of change is constant (constant of variation). A direct variation is a situation in which two quantities, such as hours and pay, increase or decrease at the same rate. That is, as one quantity doubles, the other quantity also doubles. The variables are said to be directly proportional.

For example: *if you get paid hourly, when you work twice as many hours, you will make twice as much money. How much you make is a constant multiple of how many hours you work.*

<u>Example 2</u>: Identify the dependent and independent variables in the following rates of change.

	Dependent variable (y)	Independent variable (x)
The car travelled at 75 km/h.	km	hours
In November the temperature drops 1.2 degrees Celsius per day.	degrees Celsius	days

<u>Example 3</u>:

Determine the constant of variation (rates of change) given the data in the following tables.

Recall: Rate of change (constant of variation) m $= \dfrac{\Delta\text{dependent } y}{\Delta\text{independent } x}$

a)

Hours worked (h) x	Money made (\$) y
0	**0**
1	35
2	70
3	105

Δx Δy

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{35-0}{1-0}$$

$$= \frac{\$35}{1h}$$

$$= \$35\ /\text{hour}$$

b)

Mass of grain (kg) x	Cost ($) y
0	**0**
20	125
40	250
60	375

Δx Δy

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{125 - 0}{20 - 0}$$

$$= \frac{\$125}{20\text{ kg}}$$

$$= \$6.25/\text{kg}$$

i) What is the same in the relationships given in parts a) and b)?

Initial value of (0,0)

Both tables represent **direct variations**, the independent variable directly affects the dependent variable.

Direct variation will always have (0 , 0) as the initial value!

ii) Write an equation to model the data in the tables given in Example 3. Remember that 'y' varies directly with 'x'; $\boldsymbol{y = mx}$

a)

hours

$y = 35x$

$ made

b)

kg

$y = 6.25x$

cost

iii) Which is the independent variable (x)?

a) hours b) kg

Which is the dependent variable (y)?

a) $ made b) cost

In general, the equation of a **direct variation** is always in the form $y = mx$, where m is the constant of variation

Part B: Investigate Direct Variations using Graphs

<u>Example 4</u>: The cost y to do electrical work varies directly with time. x Electric company charges $25 per hour to do electrical work while AC-DC electrical charges $50 per hour.

a) Write equations to model each relationship.

Electric company:

$m = \$25/h$

hours

$y = 25x$

cost

AC-DC electrical:

$m = \$50/h$

$y = 50x$ hours

cost

b) Use the equations to complete the tables for 0 to 4 hours.

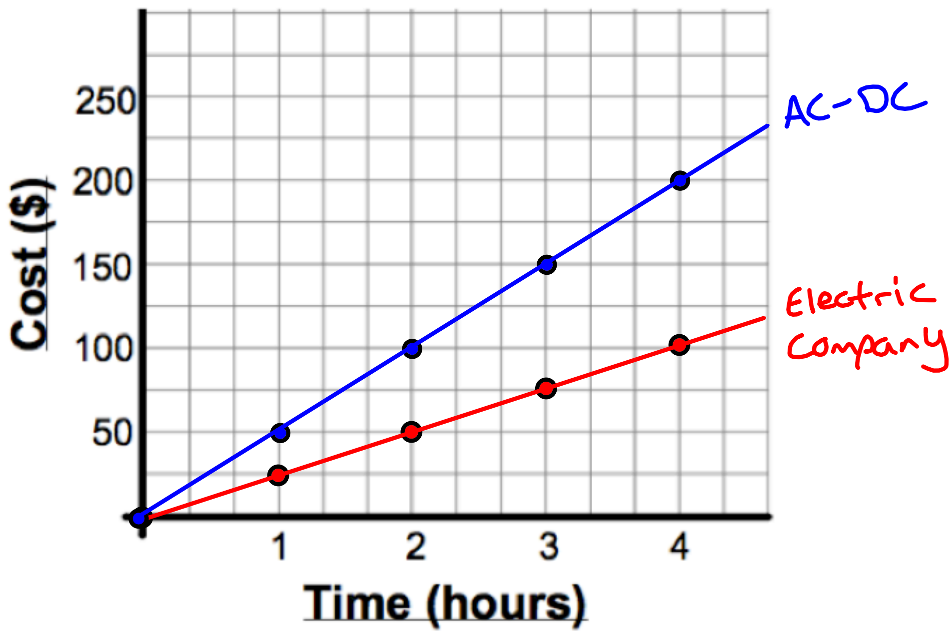
Electric company:

Hours (h)	Cost ($)
0	0
1	25
2	50
3	75
4	100

AC-DC electrical:

Hours (h)	Cost ($)
0	0
1	50
2	100
3	150
4	200

c) Graph the data for both companies on the same Cartesian coordinate grid.



d) Looking at the **graph or the table,** which company should I choose if I have a job that requires 3 hours of electrical work?

Electric Company

e) How can we show the rate of change on the graph? How does the steepness of the line relate to the rate of change?

A steeper line shows a higher rate of change (m).

f) What is the same for both graphs?

- Lines are straight (constant rate of change)
- Start at the origin (0,0)

In general, the graph of a **direct variation** is a straight line which always passes through the origin

Part C: Interpreting Direct Variation Word Problems

Example 5:

The distance travelled by Mr. Jensen when he drives varies directly with time. His car travels 630 km in 3 hours.

y x

i) What is the constant of variation? $constant\ of\ variation = \frac{\Delta dependent}{\Delta independent} = \frac{\Delta y}{\Delta x}$

$$m = \frac{\Delta y}{\Delta x} = \frac{630\ km}{3\ hrs} = 210\ km/h$$

ii) Write an equation to represent the relationship between the variables

$$y = 210x$$

time

distance

iii) Which variable is the dependent/independent variable?

dependent (y) = distance

independent (x) = time

Example 6:

The total cost varies directly with the number of MP3's downloaded. 13 MP3 downloads costs $12.87

x y

i) What is the constant of variation?

$$m = \frac{\Delta y}{\Delta x} = \frac{\$12.87}{13 \text{ MP3's}} = \$0.99 / \text{MP3}$$

ii) Write an equation to represent the relationship between the variables

MP3's

$$y = 0.99x$$

cost

iii) Which variable is the dependent/independent variable?

dependent (y) = cost

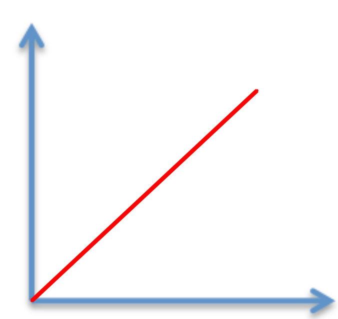
independent (x) = # of MP3's

Consolidate:

Direct variation occurs when the dependent variable is a constant multiple of the independent variable.

Direct variation can be defined algebraically as $y = mx$ where *m* is the constant of variation.

The graph of a direct variation is a straight line that passes through the origin.



5.2 Partial Variation

Part 1: DO IT NOW

The Keg Restaurant charges $100 to reserve a private dining room plus $40 per person.

a) Write an equation to show the relationship between the cost of the reservation and the number of people attending.

$y = 40x + 100$

y is cost
x is # of people.

b) What is different about this equation and the equation of a direct variation ($y = mx$)?

There is an additional cost added.

c) How much will it cost to reserve the room if

i) An extended family of 25 want to have dinner to celebrate a recent birth of twins?

$y = 40(25) + 100$
$y = 1000 + 100$
$y = 1100$

It would cost \$1100.

ii) The Pittsburgh Penguins want to celebrate their 2009 Cup Victory. There are 24 players and 6 coaches attending the celebration.

$y = 40(30) + 100$
$y = 1200 + 100$
$y = 1300$

It would cost \$1300

Part 2: Recall properties of direct variations

A direct variation is a relationship between two variables in which one variable is a constant multiple of the other.

Model a direct variation in an equation: $y = mx$

Constant of variation is defined as: $m = rate\ of\ change = \frac{\Delta y}{\Delta x}$

Direct variations are linear relations that always pass through which point on the Cartesian coordinate grid? The origin (0,0)

Part 3: Compare direct variations to partial variations

The Tesla electrical company charges $25 per hour to do electrical work plus a fee of $50 for the estimate on the proposed work. AC-DC electrical charges $50 per hour. Write equations to model each relationship. Let x represent the number of hours and let y represent the total cost.

Tesla Electric company:

$y = 25x + 50$

AC-DC electrical:

$y = 50x$

Use the equations to create tables to organize the data for 0 to 4 hours.

Tesla electric company:

Hours (h)	Cost ($)
0	50
1	75
2	100
3	125
4	150

AC-DC electrical:

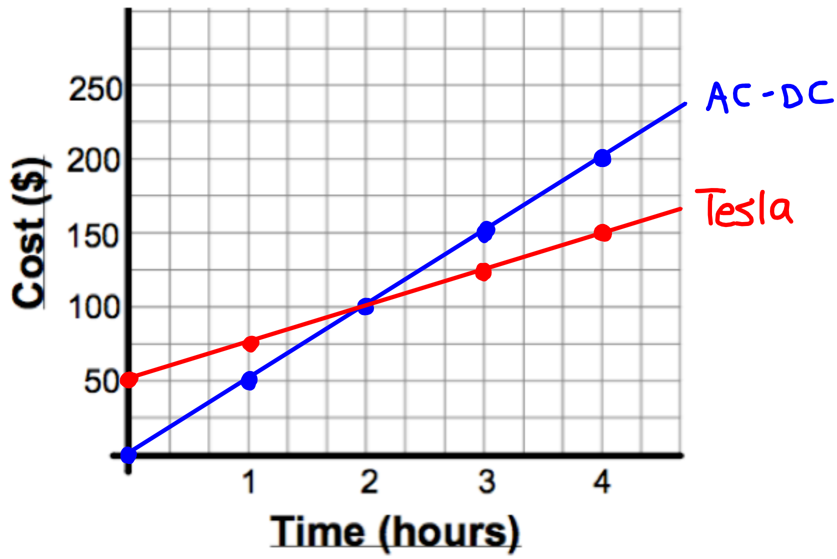
Hours (h)	Cost ($)
0	0
1	50
2	100
3	150
4	200

Which relation is a direct variation and how do you know?

AC-DC is a direct variation because its initial value is 0.

The graph of AC-DC would pass through the origin.

Now graph the data for both companies on the same Cartesian coordinate grid.



Looking at the **graph or the table**, we should use TESLA for 3 hours of electrical work. Does this company always offer the best deal? Explain.

Tesla only offers a better deal if the work takes longer than 2 hours.

What is different about the two relations?

Tesla is a PARTIAL variation (initial cost $\neq 0$)

AC-DC is a DIRECT variation (initial cost $= 0$)

A **PARTIAL VARIATION** is a relationship between two variables in which the dependent variable is the sum of a constant number and a constant multiple of the independent variable.

In general, the graph of a **partial variation** has the following properties:

- it is a straight line which does not pass through the origin (0,0)
- the equation of a partial variation is always in the form $y = mx + b$
- 'b' is the initial value (y-intercept, fixed cost)
- 'm' is the constant of variation (rate of change, variable cost)

Part 4: Working with Partial Variation

a) Complete the following chart given that y varies partially with x (you may need to determine the constant of variation)

x	y
0	6
1	9
2	12
3	15
4	18
7	27

Δx Δy

b) What is the initial value of 'y' (y-intercept)?

when $x=0$; $y=?$

$\boxed{b=6}$

c) What is the constant of variation (rate of change)?

$m = \frac{\Delta y}{\Delta x} = \frac{9-6}{1-0} = \frac{3}{1} = 3$

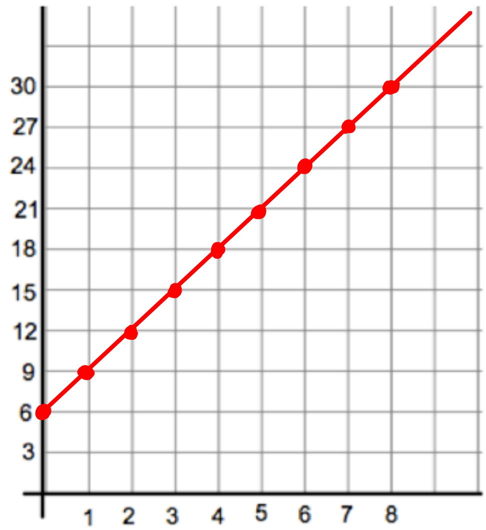
Remember: $m = \frac{\Delta y}{\Delta x}$

$\boxed{m=3}$

d) Write an equation relating y and x in the form $y = mx + b$

$y = 3x + 6$

e) Graph the relation



Part 5: Application of Partial Variation

A school is planning an awards banquet. The cost of renting the banquet facility and hiring serving staff is $675. There is an additional cost of $12 per person for the meal.

a) Identify the fixed cost (initial value; b) and the variable cost (constant of variation; m)

$m = 12$

$b = 675$

b) Write an equation to represent this relationship in the form $y = mx + b$

$y = 12x + 675$

cost

# of people

c) Use your equation to determine the total cost if 500 people attend the banquet.

x

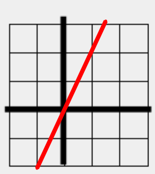
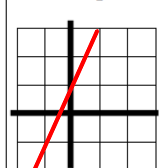
$y = 12x + 675$

$y = 12(500) + 675$

$y = 6675$

It would cos $6675.

Consolidate:

Direct variation			**Partial variation**		
Table	Graph	Equation	Table	Graph	Equation
Has (0,0) as the initial value	Passes through the origin	$y = mx$	Has an initial value other than zero	Crosses the dependent axis (y-axis) at an initial value other than 0	$y = mx + b$
Create an example: x y 0 0 1 2 2 4 3 6	**Create an example:** 	**Create an example:** $y=2x$	**Create an example:** x y 0 1 1 3 2 5 3 7	**Create an example:** 	**Create an example:** $y=2x+1$

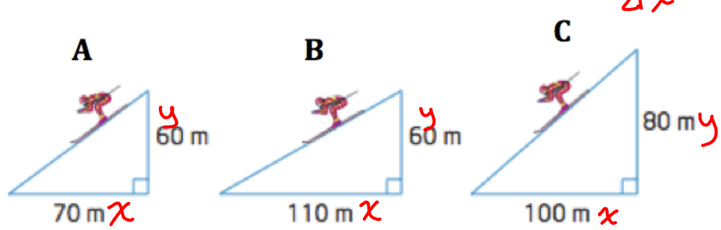
5.3a Slope

Investigation

Slope: A measurement of the steepness of a line.

Remember: $m = \frac{\Delta y}{\Delta x}$

The following diagrams represent ski hills.



$m = \frac{60}{70}$ $m = \frac{60}{110}$ $m = \frac{80}{100}$

$m = 0.857$ $m = 0.545$ $m = 0.8$

1. Rank the hills in order of their steepness, from least to greatest.

i. B **ii.** C **iii.** A

2. A hill rises 2 meters over a horizontal run of 8 meters. A second hill rises 4 meters over a horizontal run of 10 meters. Which is the steeper hill?

Hill 1

$m = \frac{2}{8}$

$m = 0.25$

Hill 2

$m = \frac{4}{10}$

$m = 0.4$

Hill 2 is steeper

3. Describe your method for determining steepness:

Calculated the rate of change.

A larger rate of change = steeper slope.

Part 1: How do we find the slope of a line?

The steepness of a line segment is measured by its SLOPE. The slope is the ratio of the RISE to the RUN and is often represented by the letter m.

You should maybe be starting to make a connection; what else did we use the letter m to represent?

RISE: the vertical distance between two points (Δy)

RUN: the horizontal distance between two points (Δx)

$$\boxed{\textbf{\textit{Slope}} = \boldsymbol{m} = \frac{\boldsymbol{rise}}{\boldsymbol{run}} \quad \boldsymbol{or} \quad \frac{\Delta \boldsymbol{y}}{\Delta \boldsymbol{x}}}$$

When determining the rise and run of a line from its graph you must know that:

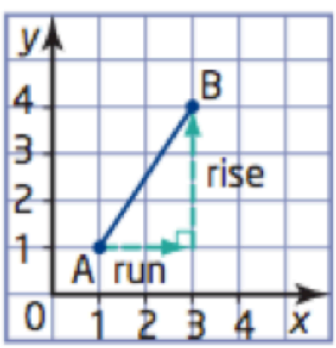
Counting units in the upward direction gives a POSITIVE rise

Counting units in the downward direction gives a NEGATIVE rise

Counting units to the right gives a POSITIVE run

Counting units to the left gives a NEGATIVE run

Example 1: Count the units on the grid to determine the rise and run.



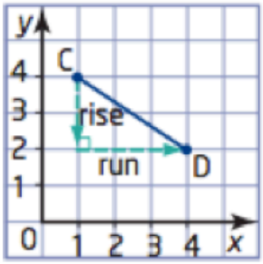
rise = 3

run = 2

What's the slope of this line?

$$m = \frac{\text{rise}}{\text{run}} = \frac{3}{2}$$

Example 2: Count the units on the grid to determine the rise and run



rise = -2

run = 3

What's the slope of this line?

$m = \frac{rise}{run} = \frac{-2}{3}$

Looking at example 1:

Is the slope positive or negative? POSITIVE

What direction does the line go? Up to the right

Looking at example 2:

Is the slope positive or negative? NEGATIVE

What direction does the line go? Down to the right

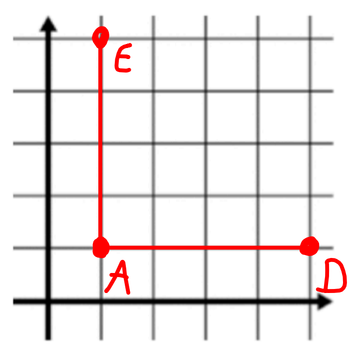
Conclusion about positive and negative slopes:

A line that goes up to the right has a positive slope.

A line that goes down to the right has a negative slope.

Part 2: Finding the slope of vertical and horizontal lines

Step 1: Plot the points A(1,1) and D(5,1) on the graph provided. Connect the points to form the line segment AD.



Step 2: Determine the rise and the run of line AD

rise = 0 run = 4 $m = \frac{0}{4} = 0$

The slope of any horizontal line is 0

Step 3: Plot the point E(1,5) on the same grid. Connect it to point A to form the line segment AE.

Step 4: Determine the rise and the run of line AE

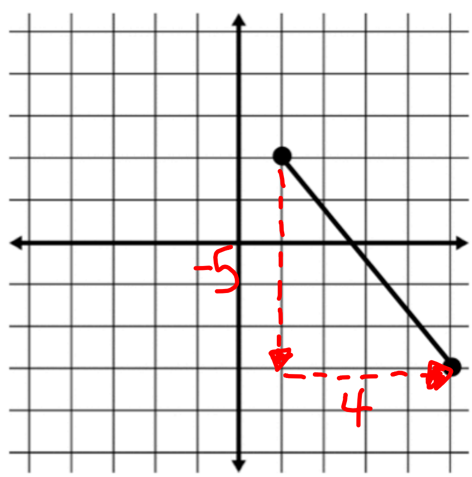
rise = 4 run = 0 $m = \frac{4}{0}$ = undefined

The slope of any vertical line is undefined

Part 3: Practice Finding Slopes

Calculate the slope of each line segments

Example 3:



rise is: -5

run is: 4

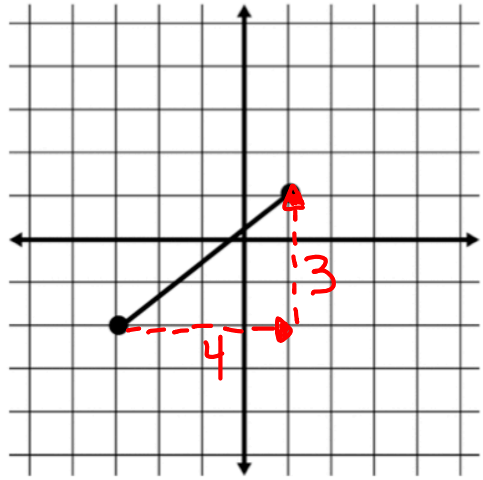
$m = \frac{\text{rise}}{\text{run}} = \frac{-5}{4}$

Example 4:

rise is: 3

run is: 4

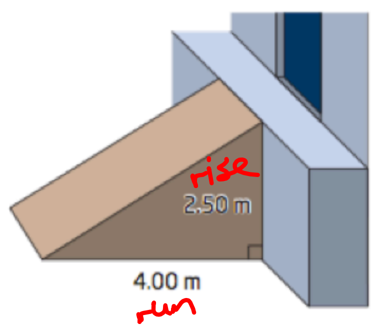
$m = \frac{3}{4}$



Example 5: The ramp at a loading dock rises 2.5 meters over a run of 4 meters.

What is the slope of the ramp?

$m = \frac{rise}{run} = \frac{2.5}{4} = 0.625$



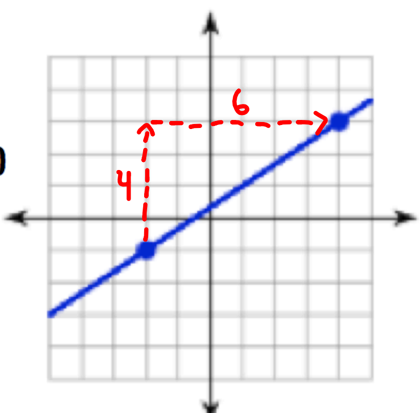
5.3b Slope

Part 1: Do It Now

Find the slope of each of the following lines by looking at the graph and determining the rise and the run.

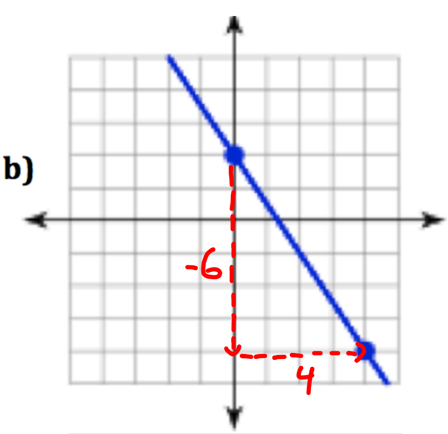
Remember: $slope = \dfrac{rise}{run}$

a)



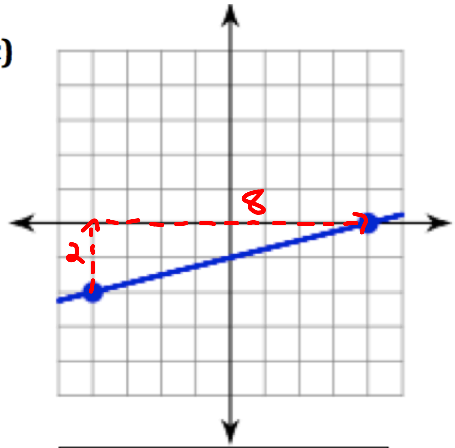
$m = \frac{4}{6} = \frac{2}{3}$

b)



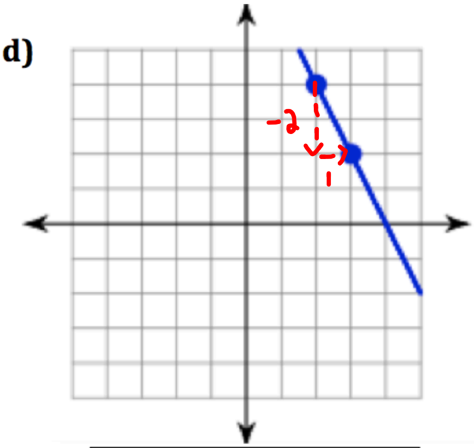
$m = \frac{-6}{4} = \frac{-3}{2}$

c)



$m = \frac{2}{8} = \frac{1}{4}$

d)



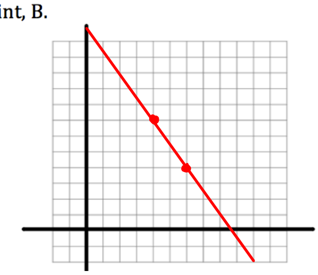
$m = \frac{-2}{1} = -2$

Part 2: Draw a graph to find another point on a line

Example 1: A line segment has one endpoint, A(4,7), and slope of $-\frac{3}{2}$. Find the coordinates of another possible endpoint, B.

Step 1: Plot the point A(4,7).

Step 2: Use the slope $-\frac{3}{2}$ to find another endpoint. (rise / run)



Note: $-\frac{3}{2} = \frac{-3}{2}$, therefore the line has a rise of __-3__ and run of __2__

To plot another point, start at point A and use the slope of the line to plot another point.

The rise of -3 tells us we should go __DOWN__ 3 units.

The run of 2 tells us we should go __RIGHT__ 2 units.

Another possible endpoint is: (6,4)

Note: There are an infinite number of solutions!!! What would have happen if you used a slope of $\frac{3}{-2}$? Why does this happen?

Rise of 3, run of -2. Using this would give you another point on the same line but to the opposite side.

Example 2:

If a line has slope of $-\frac{1}{2}$, and the line passes through the point (4 , 5) determine the coordinates of two points to the left, and two points to the right on the same line.

Note: $-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$

Graphical solution:
('move' to other points according to the slope)

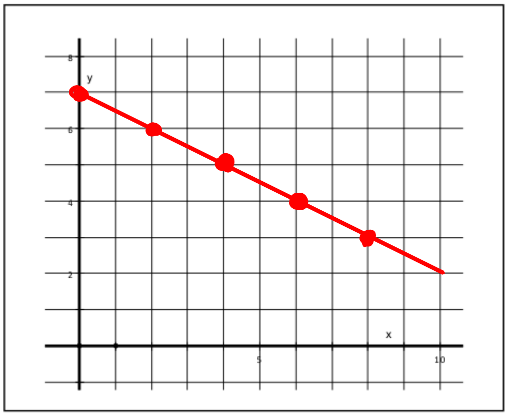


Table solution:
('move' to other points according to the slope)

x	y
0	7
2	6
4	5
6	4
8	3

Example 3:

If a line has slope of $m = \frac{1}{3}$, and the line passes through the point (-2 , -1) determine the coordinates of a point to the left and right on the same line.

Graphical solution:
('move' to other points according to the slope)

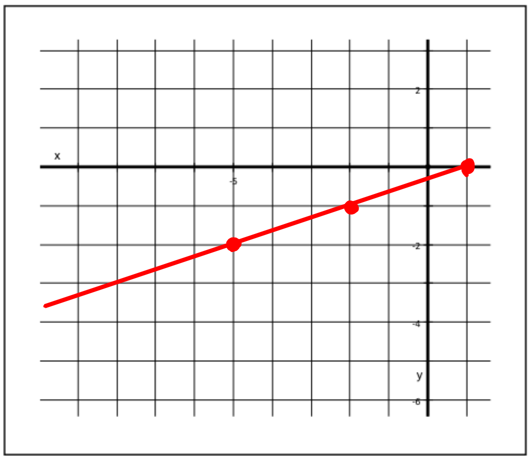


Table solution:
('move' to other points according to the slope)

x	y
-5	-2
-2	-1
1	0

Part 3: Use the coordinates to find another point on the line

Example 4: A line segment has one endpoint A(-2,7) and a slop of $-\frac{4}{3}$. Find the coordinates of another point on the line.

$-\frac{4}{3} = \frac{-4}{3}$ Therefore the line has a rise of __-4__ and a run of __3__.

Add the rise to the y-coordinate and the run to the x-coordinate to find another possible point.

Other endpoint = (-2 + 3, 7 + (-4)) = (1, 3)

> **Note: you could also subtract the rise and run to find a point to the other side on the line.**

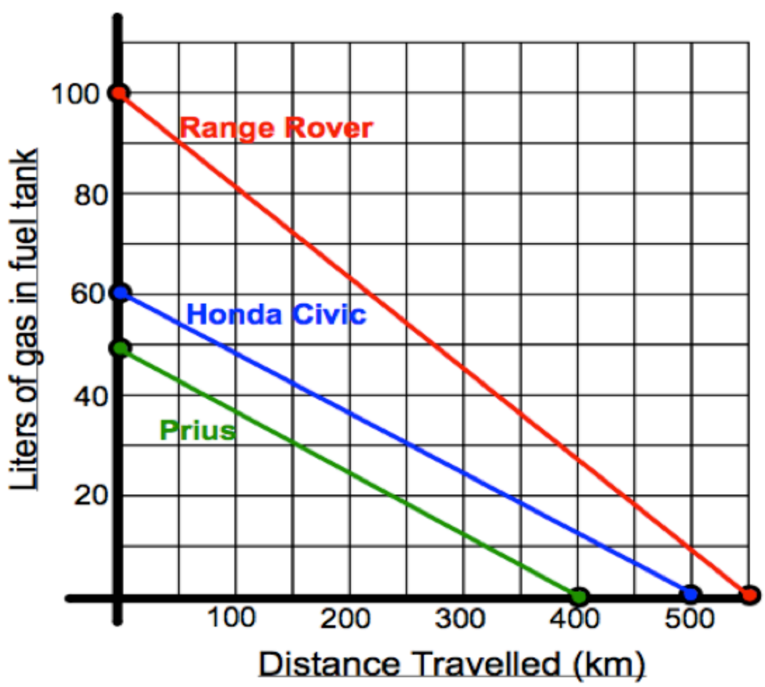
Example 5: A line segment has one endpoint A($\overset{x}{3}$,$\overset{y}{-5}$) and a slope of $-\frac{7}{2}$. Find the coordinates of another point on the line.

$-\frac{7}{2} = \frac{-7}{2}$ ← Rise Δy / ← Run Δx

Other point = (3+2 , -5+(-7)) = (5, -12)

5.4 Slope as a Rate of Change

Part 1: Do It Now



1. The independent variable is DISTANCE TRAVELLED
The dependent variable is LITRES OF GAS IN TANK

2. How can you determine the rate of fuel consumption using this data?

Find the slope of each line.

$m = \frac{rise}{run}$

3. Determine the rate of gas consumption for each vehicle and then rank them in order of efficiency.

Vehicle	2010 Range Rover	2011 Prius	2007 Honda Civic
Rate of gas consumption	$m = \frac{-100}{550}$ $= -0.182$ L/km	$m = \frac{-50}{400}$ $= -0.125$ L/km	$m = \frac{-60}{500}$ $= -0.12$ L/km
Efficiency ranking	3	2	1

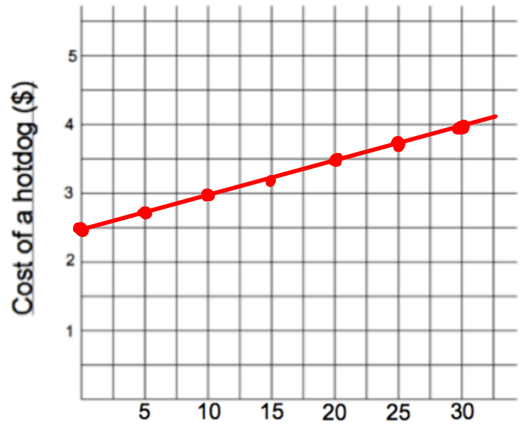
Part 2: Connecting Slope and Rate of Change

From the Do It Now question we have discovered that slope = rate of change. Look at the following table to see further how they are connected. How the linear equation is represented determines the terminology we use describe the slope.

Word problem	Table	Graph	Equation
m is the rate of change	$m = \frac{\Delta y}{\Delta x}$	$m = \frac{rise}{run}$	$m = slope$

Years since July 1980	Cost of a hotdog ($)
0	2.50
5	2.75
10	3.00
15	3.25
20	3.50
25	3.75
30	4.00

Example 1:
The cost of a hot dog at the Rogers Centre has been going up for several years. Graph the data. Let x be the number of years since July 1980.



a) Determine the slope using the graph

Rise = 0.25 Run = 5 Slope = $\frac{0.25}{5} = 0.05$

b) Determine the rate of change of the cost of hot dogs using the table.

rate of change $= m = \frac{\Delta y}{\Delta x} = \frac{2.75-2.50}{5-0} = \frac{0.25}{5} = 0.05$

Remember:

Rate of change = $\frac{\Delta y}{\Delta x}$

c) Write an equation to represent the cost of a hot dog based on the number of years since July 1980. What part of the equation represents the slope?

$m = 0.05$

$b = 2.50$

$\boxed{y = 0.05x + 2.50}$

Example 2: Mr. Jensen is training for a triple marathon and runs every day before school. This morning he ran 5 km in 20 minutes.

a) Calculate the rate of change of Mr. Jensen's distance from his starting point. (in this case rate of change is = average speed)

Rate of change = $\frac{\Delta dependent\ variable}{\Delta independent\ variable}$

Dependent variable: Distance (km)

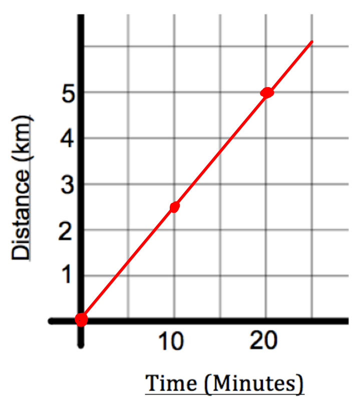
Independent variable: Time (minutes)

Rate of change (speed) = $\frac{\Delta \text{distance}}{\Delta \text{time}} = \frac{5 \text{ km}}{20 \text{ min}} = 0.25 \text{ km/min}$

b) Graph distance as it relates to time

$y = mx$

$y = 0.25x$



c) Calculate the slope of the line from the graph

Slope = $\frac{rise}{run}$

$m = \frac{rise}{run} = \frac{2.5}{10} = 0.25$

d) Explain the meaning of the rate of change and how it relates to the slope of the graph

The rate of change is the speed of the runner in this scenario. The steeper the slope, the higher the rate of change (faster the speed).

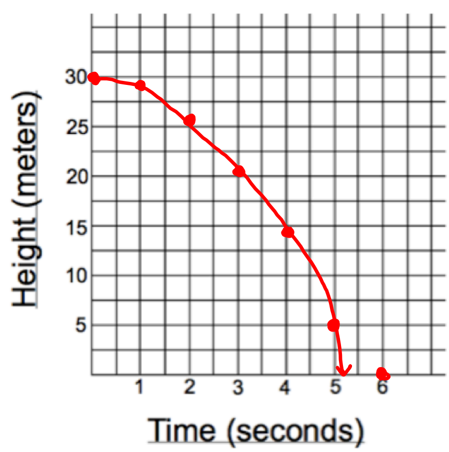
5.5 First Differences

DO IT NOW

If a tennis ball falls out of the third story window of a building will its motion be linear? The height of the ball over time is recorded in the following table.

Graph the relation and determine if it represents linear motion.

Time (seconds)	*Height (meters)*
0	*30*
1	*29*
2	*26*
3	*21*
4	*14*
5	*5*
6	*0*



NOT LINEAR.
Rate of change is not costant.

Part 1:Recall

We know from graphing lines that **if the slope (rise and the run) is constant** then the relation will **form a straight line**.

$$slope = m = \frac{rise}{run} = \frac{\Delta x}{\Delta y}$$

Therefore, we need to determine if the changes in x and y are constant in a table to determine if a relation is linear.

Part 2: What are First differences

First differences are the differences between consecutive y-values in tables of values with evenly spaced x-values.

If the first differences of a relation are constant, the relation is LINEAR

If the first differences of a relation are not constant, the relation is NON-LINEAR

Notice that the differences between consecutive y-values are constant! This means it is a linear relation

Notice that the x-values change by a constant amount. This is a requirement to work with first differences!

x	y
0	0
1	3
2	6
3	9
4	12

First Differences
3-0 = 3
6-3 = 3
9-6 = 3
12-9 = 3

Part 3: Calculating First Differences

Complete a table of values for each equation given. Then determine if the first differences are constant and state whether the relation is linear or non linear.

Example 1:

$y = -2x + 7$

x	y
0	7
1	5
2	3
3	1
4	-1

First Differences
5-7=-2
3-5=-2
1-3=-2
-1-1=-2

Conclusion:
the first differences are Constant

therefore the relationship is Linear

Example 2:

$y = x^2$

x	y
0	0
1	1
2	4
3	9
4	16

First Differences
1-0=1
4-1=3
9-4=5
16-9=7

Conclusion:
the first differences are Not Constant

therefore the relationship is Non-Linear

Example 3:

$y = 2^x$

x	y
0	1
1	2
2	4
3	8
4	16

First Differences

2-1 = 1
4-2 = 2
8-4 = 4
16-8 = 8

Conclusion:
the first differences are Not Constant

therefore the relationship is Non Linear

Part 4: Check Your Understanding

Use first differences to determine which of these relations are linear and which are non linear.

Example 4:

x	y
0	7
1	3
2	-1
3	-5
4	-9

First Differences

3-7 = -4
-1-3 = -4
-5-(-1) = -4
-9-(-5) = -4

Type of relation: Linear

Example 5:

x	y
2	-5
3	10
4	25
5	40
6	55

First Differences
10-(-5)=15
25-10=15
40-25=15
55-40=15

Type of relation: Linear

Example 6:

x	y
-2	-10
-1	-2
0	0
1	2
2	10

First Differences
-2-(-10) = 8
0-(-2) = 2
2-0 = 2
10-2 = 8

Type of relation: Non-linear

5.6 - Connecting Slope, Rate of Change, and First Differences

Part 1: Do It Now

a) Calculate the first differences

x	y	First Differences
0	-3	
3	1	$1-(-3)=4$
6	5	$5-1=4$
9	9	$9-5=4$
12	13	$13-9=4$

Type of relation:

LINEAR

b) Using the table of values, what is the constant of variation (slope)?

Point 1: $(\overset{x_1}{0}, \overset{y_1}{-3})$ Point 2: $(\overset{x_2}{3}, \overset{y_2}{1})$

Remember: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{3-0} = \frac{4}{3}$$

c) What is the initial value (y-intercept)?

when $x=0$, $y=-3$. $\therefore b=-3$

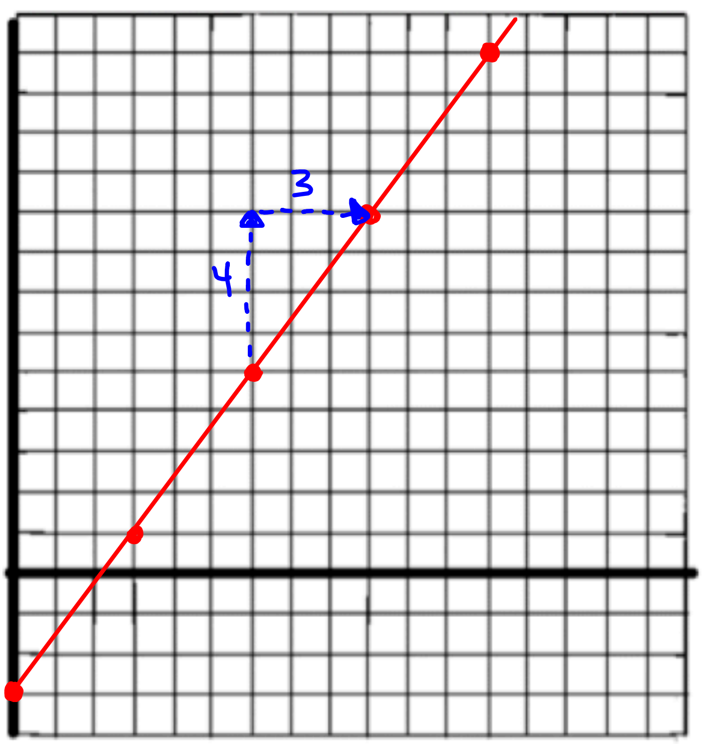
d) Is this a direct variation or partial variation?

Partial because $b \neq 0$

e) Write an equation for the relation in the form $y = mx + b$ using the constant of variation (m) and the initial value (b)

$$y = \frac{4}{3}x - 3$$

f) Graph the relation



g) Find the slope of the line from the graph. How does this relate to the constant of variation?

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{4}{3}$$

This is the same as the constant of variation.

h) What is the y-intercept? How does this relate to the initial value?

$$\text{y-intercept} = b = -3$$

This is the same as the initial value.

i) Write the equation of the line in the form $y = mx + b$ using the slope and y-intercept

$$y = \frac{4}{3}x - 3$$

<u>Part 2: The Rule of Four</u>

A relation can be represented in a variety of ways so that it can be looked at from different points of view. A mathematical relation can be described in four ways:

1. Using words
2. Using a graph
3. Using a table of values
4. Using an equation

Part 3: Write an equation when the relation is represented in words

Remember that the equation of a line is $y = mx + b$

Considering that a line is really just a set of ordered pairs, (x , y), it makes sense that the equation of a line needs to contain the variables x and y. These variables will define the coordinates that make up the line.

This means that the only 2 values that need to be determined in order to write the equation of a linear relation are *m* and *b*.

When a linear relation is represented in words m is the rate of change and b is the initial value.

Linear relation represented in words: ***m*** **= rate of change (slope)**
b **= initial value (y-intercept)**

Example 1: Write an equation for the following relationship by first identifying the value of m and b.

The Copy Centre charges $75 to design a poster plus 25 cents **for each copy**.

$m = 0.25$ $b = 75$

And the equation of this linear relation is:

cost → $y = 0.25x + 75$

\# of copies

If The Copy Centre changed their cost per flyer to 35 cents for each copy the equation would become:

$y = 0.35x + 75$

If The Copy Centre changed their design cost to $125 the equation will become: $y = 0.25x + 125$

Example 2: y varies directly with x. When $x = 2, y = 8$.

a) What is the initial value? (y-intercept)

All direct variations have an initial value of 0.

$\therefore b = 0$

b) What is the slope of the line? Point 1: $(\overset{x_1}{0}, \overset{y_1}{0})$ Point 2: $(\overset{x_2}{2}, \overset{y_2}{8})$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{2-0} = \frac{8}{2} = 4$$

c) Write an Equation for this relationship

$$y = 4x$$

Example 3: y varies partially with x. When $x = 0, y = 3$, and when $x = 2, y = 8$.

a) What is the initial value? (y-int)

when $x=0, y=3$. $\therefore b = 3$

b) What is the slope of the line? Point 1: $(\overset{x_1}{0}, \overset{y_1}{3})$ Point 2: $(\overset{x_2}{2}, \overset{y_2}{8})$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-3}{2-0} = \frac{5}{2}$$

c) Write an Equation for this relationship

$$y = \frac{5}{2}x + 3$$

Part 4: Write an equation when the relation is represented in a table of values

Remember:

$$slope = rate\ of\ change = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b = initial\ value = y - intercept = value\ of\ y\ when\ x\ is\ 0$$

Example 4: Determine the equation of the following linear relations using the tables provided:

a)

x	y
x_1 0	y_1 6
x_2 1	y_2 9
2	12
3	15
4	18

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9-6}{1-0}$

$= \frac{3}{1}$

$= 3$

$b = 6$

Equation: $y = 3x + 6$

b)

x	y
x_1 -2	y_1 3
x_2 0	y_2 5
2	7
4	9
6	11

$m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{5-3}{0-(-2)}$

$= \frac{2}{2}$

$= 1$

$b = 5$

Equation:

$y = x + 5$

What should we do if the initial value isn't in the table?

c)

x	y
0	-17
1	-14
2	-11
x_1 3	y_1 -8
x_2 4	y_2 -5

$m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-5-(-8)}{4-3}$

$= \frac{3}{1}$

$= 3$

$b = -17$

Equation: $y = 3x - 17$

d)

x	y
-8	-5
x_1 -6	y_1 -10
x_2 -4	y_2 -15
-2	-20
0	-25

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-15-(-10)}{-4-(-6)}$

$= \dfrac{-5}{2}$

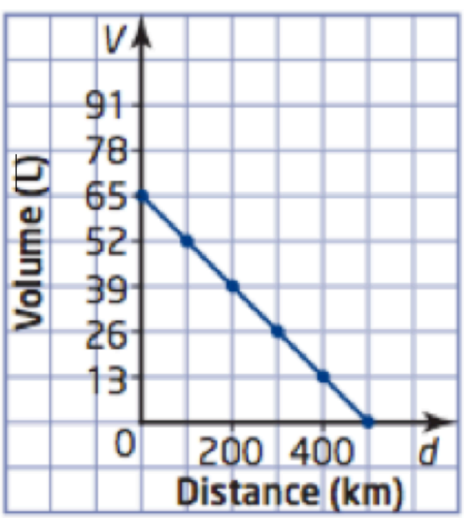
$b = -25$

Equation: $y = -\dfrac{5}{2}x - 25$

Part 5: Write an equation when the relation is represented as a graph

Example 5: The graph shows the relationship between the volume of gasoline remaining in a car's fuel tank and the distance driven.

> **Remember:** $m = slope = \frac{rise}{run}$
>
> $b = initial\ value = y - intercept$



Slope: $m = \dfrac{rise}{run}$

$= \dfrac{-65}{500}$

$= \dfrac{-13}{100}$

***y*-intercept:** $b = 65$

Equation: $y = -\dfrac{13}{100}x + 65$

6.1a Equation of a Line in Slope y-Intercept Form

Example 1: Complete the following chart

$y = mx + b$

slope

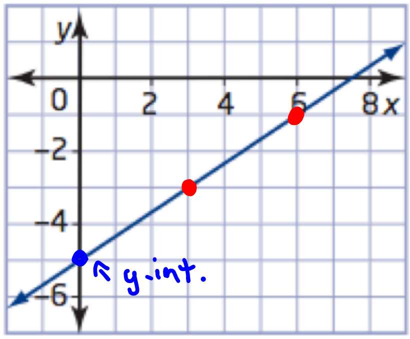
y-int.

Equation	Slope	y-intercept
$y = -2x - 5$	$m = -2$	$b = -5$
$y = x + 2$	$m = 1$	$b = 2$
$y = \frac{2}{5}x + 8$	$m = 2/5$	$b = 8$
$y = -\frac{1}{2}x$	$m = -1/2$	$b = 0$
$y = 4$	$m = 0$	$b = 4$

$y = 0x + 4$

Example 2: Identify the slope and y-intercept of each line

a)



Slope: to find the slope use two points on the line and the formula $m = \frac{rise}{run}$ OR $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Point 1: $(\overset{x_1}{3}, \overset{y_1}{-3})$

Point 2: $(\overset{x_2}{6}, \overset{y_2}{-1})$

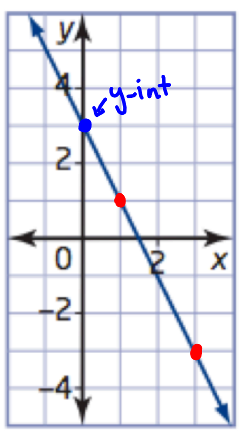
$m = \frac{-1-(-3)}{6-3}$

$= \frac{2}{3}$

y-intercept: you can find the y-intercept by looking at the graph and checking where the line crosses the y-axis. (When $x = 0, y = ?$)

Slope: $m = \frac{2}{3}$	**y-intercept:** $b = -5$
Equation of the line: $y = \frac{2}{3}x - 5$	

b)



Slope: to find the slope use two points on the line and the formula $m = \frac{rise}{run}$ OR $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Point 1: $(\overset{x_1}{1}, \overset{y_1}{1})$

Point 2: $(\overset{x_2}{3}, \overset{y_2}{-3})$

$m = \frac{-3-1}{3-1}$

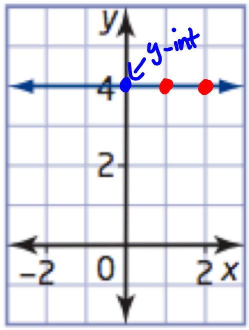
$= \frac{-4}{2}$

$= -2$

y-intercept: you can find the y-intercept by looking at the graph and checking where the line crosses the y-axis. (When $x = 0, y = ?$)

Slope: $m = -2$	**y-intercept:** $b = 3$
Equation of the line: $y = -2x + 3$	

c)



Slope: to find the slope use two points on the line and the formula $m = \frac{rise}{run}$ OR $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Point 1: $(\overset{x_1}{1}, \overset{y_1}{4})$

Point 2: $(\overset{x_2}{2}, \overset{y_2}{4})$

$m = \frac{4-4}{2-1}$

$= \frac{0}{1}$

$= 0$

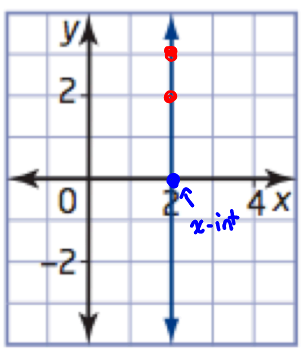
y-intercept: you can find the y-intercept by looking at the graph and checking where the line crosses the y-axis. (When $x = 0, y = ?$)

Slope: 0 **y-intercept:** 4

Equation of the line: $y = 0x + 4 \rightarrow y = 4$

Note: all horizontal lines have a slope of _0_________ and an equation of the form _$y = b$______, where b is the _y-intercept.__________.

d)



Slope: to find the slope use two points on the line and the formula $m = \frac{rise}{run}$ OR $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Point 1: $(\overset{x_1}{2}, \overset{y_1}{2})$

Point 2: $(\overset{x_2}{2}, \overset{y_2}{3})$

$m = \frac{3-2}{2-2}$

$= \frac{1}{0}$

= undefined

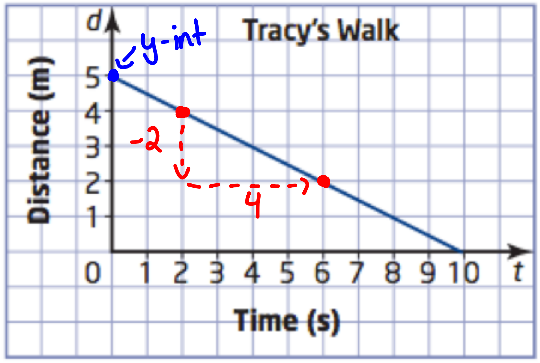
y-intercept: you can find the y-intercept by looking at the graph and checking where the line crosses the y-axis.

Slope: undefined **y-intercept:** none

Equation of the line: $x = 2$

Note: All vertical lines have an _undefined______ slope and an equation of the form _$x = a$__________, where a is the _x-intercept__________.

Example 3: Interpreting a Linear Relation



Identify the slope and the vertical intercept of the linear relation and explain what they mean.

Slope: $m = \frac{\text{rise}}{\text{run}}$ **y-intercept:** $b = 5$

$m = \frac{-2}{4}$

$m = \frac{-1}{2}$ OR -0.5

The slope represents Tracy's speed. The negative value means that her distance from the sensor is decreasing. Tracy's speed toward the sensor was 0.5 m/s.

y-intercept:

The y-intercept of 5 means that Tracy started walking at a distance of 5 meters from the sensor.

Equation of the relation:

$$y = -\frac{1}{2}x + 5$$

6.1b - Slope y-Intercept Form

Part 1: Graphing a Line Using the Slope and the y-Intercept:

Example 1: How can we graph $y = \frac{2}{3}x + 1$ without using a table of values?

a) The line $y = \frac{2}{3}x + 1$, has a slope: ______2/3______ and y-intercept: ______1______

b) Plot the y-intercept on the given grid

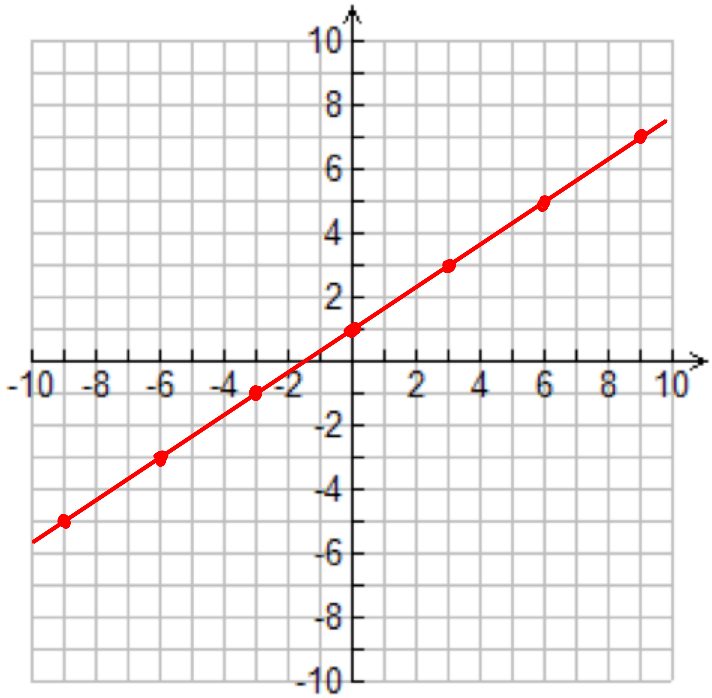
c) How can the slope be used to determine other points on this line?

Use the slope of $\frac{2}{3}$ which has a rise of __2__ and a run of __3__ to plot another point on the line.

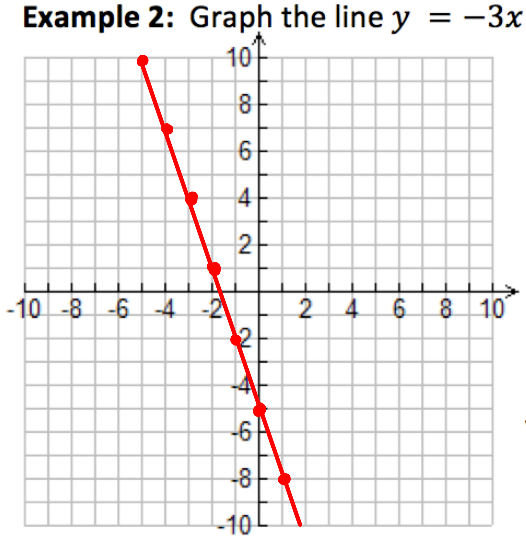
You could also use the opposite slope to plot points on the other side of the y-intercept.

The opposite slope, $\frac{-2}{-3}$, has a rise of __-2__ and a run of __-3__.

d) Use the slope to determine 2 other points on the line and draw in the line.



Example 2: Graph the line $y = -3x - 5$ using the slope and the y-intercept.



Slope: $m = -3 = \frac{-3}{1} = \frac{3}{-1}$

y-intercept: $b = -5$

Part 2: Find the Equation of a Line Graphically Given Two Points

Example 3:

a) Plot the points A($\overset{x_1}{-5}$, $\overset{y_1}{3}$) and B($\overset{x_2}{8}$, $\overset{y_2}{3}$)on the given grid.

b) What is the y-intercept for the line that passes through A and B?

$b = 3$

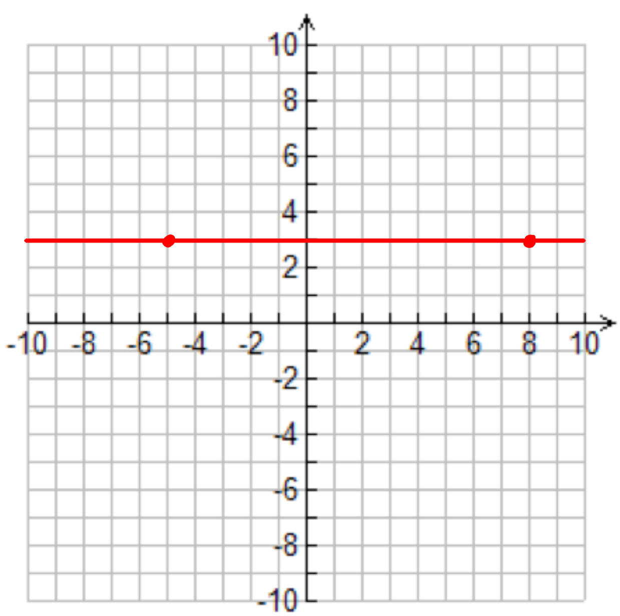
c) What is the slope for the line that passes through A and B?

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{8-(-5)} = \frac{0}{13} = 0$$

d) What is the equation for the line that passes through A and B?

$y = 3$

Note: the equation of a line is always in the form y=b. Every point on the line has a y-coordinate of 3.



Example 4: **a)** Plot the points A($\overset{x_1}{5}$, $\overset{y_1}{8}$) and B($\overset{x_2}{5}$, $\overset{y_2}{-3}$) on the given grid.

b) What is the y-intercept for the line that passes through A and B?

No y-intercept.

x-intercept = 5

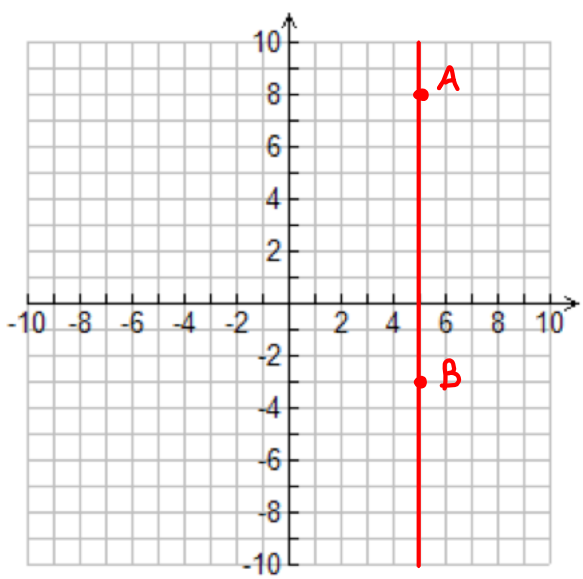
c) What is the slope for the line that passes through A and B?

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-8}{5-5} = \frac{-11}{0} = \text{undefined}$$

d) What is the equation for the line that passes through A and B?

$x = 5$

> **Note:** the equation of a vertical line is always in the form of x = the x-intercept. Notice that every point on this line has an x-coordinate of 5.



Part 3: Consolidation

a) In general, a horizontal line has a slope that is zero and an equation of the form $y = b$ where 'b' is the y-intercept.

b) In general, a vertical line has a slope that is undefined and an equation of the form $x = a$ where 'a' is the x-intercept.

c) State the steps required to graph a line using the slope and the y-intercept:

1. Plot the y-intercept
2. Use the slope to plot points on either side of the y-intercept
3. Draw a straight line through the points you plotted.

6.2 - Standard Form

The equation of a line can be written in two different forms:

1. Slope y-intercept form: $\boldsymbol{y = mx + b}$

where m is the slope, and b is the y-intercept

2. Standard form: $\boldsymbol{Ax + By + c = 0}$

where $A, B,$ and c are integers and A and B are both not zero.

You can change an equation from one form to the other by rearranging the equation.

Example 1:
Write the equation of the line 2x – 3y – 6 = 0 in slope y-intercept form by isolating the *y*.

$$2x-3y-6=0$$

$$\frac{\cancel{-3}y}{\cancel{-3}} = \frac{-2x+6}{-3}$$

$$y = \frac{-2}{-3}x + \frac{6}{-3}$$

$$y = \frac{2}{3}x - 2$$

Example 2: Write each equation in slope y-intercept form and state the slope and the y-intercept.

a) $3x + 5y - 15 = 0$

$$\frac{\cancel{5}y}{\cancel{5}} = \frac{-3x+15}{5}$$

$$y = \frac{-3}{5}x + \frac{15}{5}$$

$$y = \frac{-3}{5}x + 3$$

$$\boxed{\begin{array}{l} \text{slope} = m = \frac{-3}{5} \\ \text{y-intercept} = b = 3 \end{array}}$$

b) $7x - 3y + 21 = 0$

$$\frac{-\cancel{3}y}{-\cancel{3}} = \frac{-7x - 21}{-3}$$

$$y = \frac{-7}{-3}x - \frac{21}{-3}$$

$$y = \frac{7}{3}x + 7$$

$$\boxed{\begin{array}{l} \text{slope} = m = \frac{7}{3} \\ \text{y-intercept} = b = 7 \end{array}}$$

<u>**Example 3:**</u> Barney's Banquet Facility charges according to the equation 2x – y +200 = 0 where *x* is the number of people attending and *y* is the total cost.

a) Write the equation in slope y-intercept form.

$2x - y + 200 = 0$

$2x + 200 = y$

$y = 2x + 200$

b) What is the fixed cost?

fixed cost $= b = 200$ $\boxed{\$200}$

c) What is the rate of change of the cost?

rate of change $= m = 2$ $\boxed{\$2 \text{ per person}}$

d) What is the total cost if 125 people attend a banquet at Barney's?

y x

$y = 2(125) + 200$

$y = 250 + 200$

$y = 450$

$\boxed{\$450}$

e) If the total cost is $920, how many people attend the banquet?

y x

$920 = 2x + 200$

$920 - 200 = 2x$

$\frac{720}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$\frac{720}{2} = x$

$x = 360$

$\boxed{\text{360 people}}$

1. There are two forms in which the equation of a line can be written. What are they?

Slope y-intercept form: $y = mx + b$

standard Form: $Ax + By + c = 0$

2. It is possible to convert an equation from one form to the other by <u>Rearranging</u> the equation.

3. Write the slope-intercept form of the equation of each line:

a) $3x - 2y = -16$

$-2y = -3x - 16$

$y = \frac{-3}{-2}x - \frac{16}{-2}$

$y = \frac{3}{2}x + 8$

b) $13x - 11y = -12$

$-11y = -13x - 12$

$y = \frac{-13}{-11}x - \frac{12}{-11}$

$y = \frac{13}{11}x + \frac{12}{11}$

c) $9x - 7y = -7$

$-7y = -9x - 7$

$y = \frac{-9}{-7}x - \frac{7}{-7}$

$y = \frac{9}{7}x + 1$

d) $x - 3y = 6$

$-3y = -1x + 6$

$y = \frac{-1}{-3}x + \frac{6}{-3}$

$y = \frac{1}{3}x - 2$

e) $6x + 5y = -15$

$5y = -6x - 15$

$y = \frac{-6}{5}x - \frac{15}{5}$

$y = -\frac{6}{5}x - 3$

f) $4x - y = 1$

$4x - 1 = y$

$y = 4x - 1$

g) $11x - 4y = 32$

$-4y = -11x + 32$

$y = \frac{-11}{-4}x + \frac{32}{-4}$

$y = \frac{11}{4}x - 8$

h) $11x - 8y = -48$

$-8y = -11x - 48$

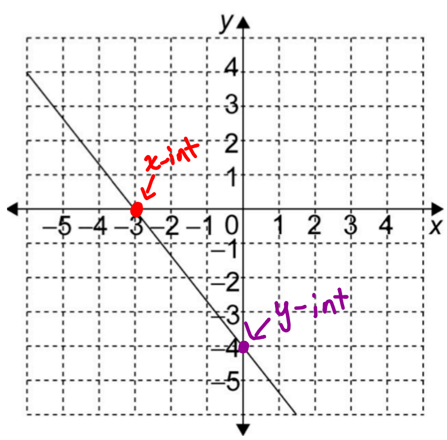
$y = \frac{-11}{-8}x - \frac{48}{-8}$

$y = \frac{11}{8}x + 6$

6.3 Graphing Using Intercepts

Part 1: Do It Now!

What are the x and y intercepts of the following line:



x-intercept: (-3,0)

y-intercept: (0,-4)

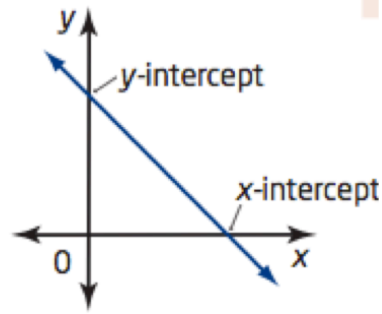
When a line is written in standard form, $Ax + By + C = 0$, or the form $Ax + By = -C$, it is easy to graph the line using x and y-intercepts.

The **x-intercept** is the x-coordinate of the point where the line crosses the x-axis.

At the x-intercept, $y = 0$.

The **y-intercept** is the y-coordinate of the point where the line crosses the y-axis.

At the y-intercept, $x = 0$.



Example 1:

Determine the intercepts for the line $2x - 3y - 6 = 0$ and use these points to graph the line.

To find the x-intercept, set $y = 0$ and solve:

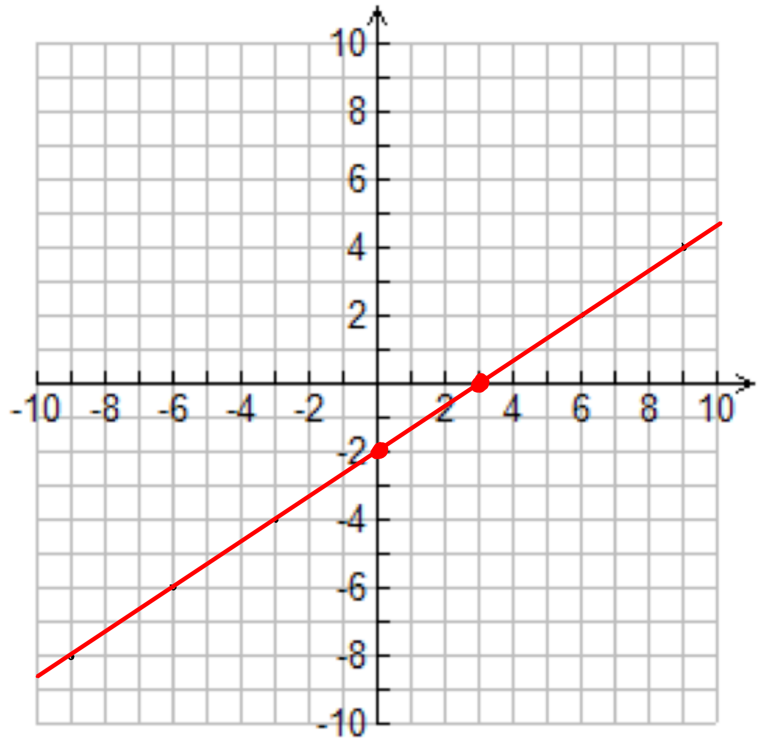
$$2x - 3(0) - 6 = 0$$
$$2x - 6 = 0$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$$
$$x = \frac{6}{2}$$
$$x = 3$$

x-int : $(3, 0)$

To find the y-intercept, set $x = 0$ and solve:

$$2(0) - 3y - 6 = 0$$
$$-3y - 6 = 0$$
$$\frac{-\cancel{3}y}{-\cancel{3}} = \frac{6}{-3}$$
$$y = \frac{6}{-3}$$
$$y = -2$$

y-int : $(0, -2)$



Example 2:
Determine the intercepts for the line $2x - y = 7$ and use these points to graph the line.

To find the x-intercept, set y=0 and solve:

$2x - 0 = 7$

$2x = 7$

$x = \frac{7}{2}$

$x = 3.5$

x-int: $(3.5, 0)$

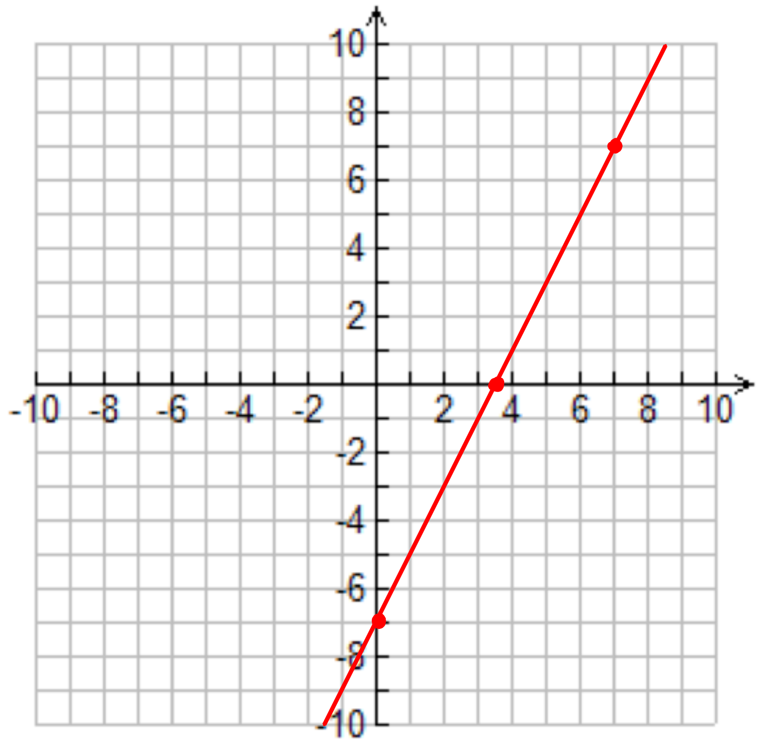
To find the y-intercept, set $x = 0$ and solve:

$2(0) - y = 7$

$-y = 7$

$y = -7$

y-int: $(0, -7)$



Example 3:

a) Determine the intercepts for the line $5x - 6y + 30 = 0$.

x-int

$5x - 6(0) + 30 = 0$

$5x + 30 = 0$

$5x = -30$

$x = \frac{-30}{5}$

$x = -6$

y-int

$5(0) - 6y + 30 = 0$

$-6y + 30 = 0$

$-6y = -30$

$y = \frac{-30}{-6}$

$y = 5$

b) Use the intercepts to determine the slope of the line.

Point 1: $(\overset{x_1}{-6}, \overset{y_1}{0})$

Point 2: $(\underset{x_2}{0}, \underset{y_2}{5})$

$m = \frac{5 - 0}{0 - (-6)} = \frac{5}{6}$

Remember:

Slope = $\frac{y_2 - y_1}{x_2 - x_1}$

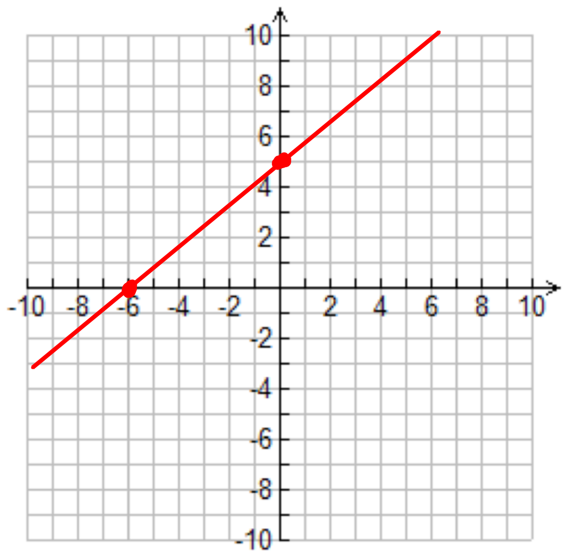
c) Write the equation of the line

$m = \frac{5}{6}$

$b = 5$

$$\boxed{y = \frac{5}{6}x + 5}$$

d) Graph the line



Example 4: Determine the slope of the line whose x-intercept is -4 and y-intercept is -6.

Point 1: $(\overset{x_1}{-4}, \overset{y_1}{0})$

Point 2: $(\underset{x_2}{0}, \underset{y_2}{-6})$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{0 - (-4)} = \frac{-6}{4} = \frac{-3}{2}$$

Consolidate:

State the steps needed to graph a line using the intercepts.

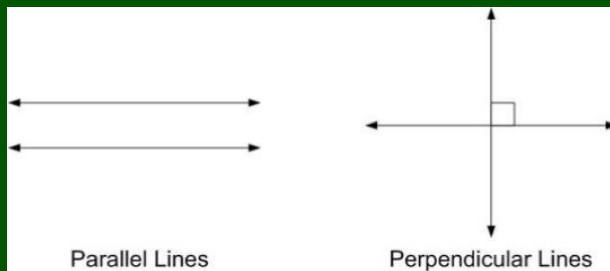
1) Solve for they y-intercept be setting $x = 0$

2) Solve for they x-intercept be setting $y = 0$

3) Plot the intercepts and draw a straight line through them

6.4 Parallel and Perpendicular Lines

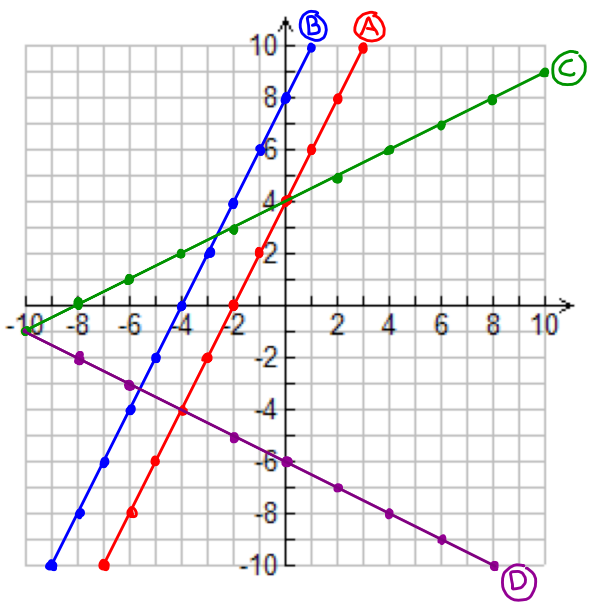
Parallel Lines – **lines in the same plane that never meet.**
Perpendicular Lines – **Two lines that cross at 90 degrees.**



DO IT NOW!

Instructions: Draw and label each of the following lines on the grid below:

A) y = 2x + 4 B) y = 2x + 8 C) y = $\frac{1}{2}x + 4$ D) y = $-\frac{1}{2}x - 6$



1) Which lines are parallel?

Lines A and B are parallel

2) What do you notice about the slopes of lines that are parallel?

They are equivalent (the same)

3) Which lines are perpendicular?

Line D is perpendicular to A and B

4) What do you notice about the slopes of lines that are perpendicular?

They are negative reciprocals of each other.

(the slopes have been "flipped" and the sign is changed)

5) What is the product of the perpendicular slopes?

$2\left(\frac{-1}{2}\right) = \frac{-2}{2} = -1$

Perpendicular slopes always have a product of -1.

6) Does the y-intercept matter when deciding if two lines are parallel or perpendicular?

NO, only the slopes matter.

Consolidation:

Parallel lines will have EQUIVALENT slopes.

Perpendicular lines will have slopes that are NEGATIVE RECIPROCALS. Their product is -1.

Example 1:

a) The equation of a line is $y = \overset{m}{\underline{3}}x - 4$. What is the slope of a line that is parallel to this line?

$m = 3$

b) The equation of a line is $y = -1x + 15$. What is the slope of a line that is parallel to this line?

$m = -1$

c) The equation of a line is $y = \underline{2}x + 1$. What is the slope of a line that is perpendicular to this line?

$m = 2 \; ; \; \perp m = \frac{-1}{2}$

d) The equation of a line is $y = \frac{3}{5}x + 2$. What is the slope of a line that is perpendicular to this line?

$m = \frac{3}{5} \; ; \; \perp m = -\frac{5}{3}$

e) The equation of a line is $y = -\frac{1}{7}x - \frac{3}{7}$. What is the slope of a line that is perpendicular to this line?

$m = -\frac{1}{7} \; ; \; \perp m = \frac{7}{1} = 7$

Example 2: The slopes of two lines are given. Determine whether the lines are parallel, perpendicular or neither.

a) $m = 3, m = -\frac{1}{3}$

$3\left(-\frac{1}{3}\right) = -\frac{3}{3} = -1$

$\therefore$ Perpendicular

b) $m = 5, m = -5$

Neither

c) $m = 6, m = \frac{1}{6}$

Neither

d) $m = -2, m = -2$

Parallel

Example 3:

a) Determine the slope of a line that is parallel to the line $2x - 3y - 6 = 0$.

$2x - 3y - 6 = 0$

$-3y = -2x + 6$

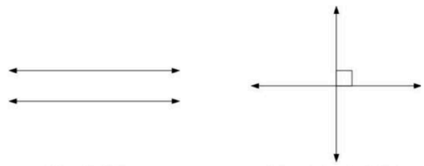
$y = \frac{2}{3}x - 2$

$m = \frac{2}{3}$

b) Determine the slope of a line that is perpendicular to the line $2x - 3y - 6 = 0$.

$m = \frac{2}{3}$; $\perp m = -\frac{3}{2}$

Consolidate:



a) Explain how you can determine if the two lines $3x - 4y - 12 = 0$ and $4x - 3y - 24 = 0$ are parallel, perpendicular or neither.

- Rearrange into slope intercept form ($y = mx + b$)
- Parallel if slopes are the same
- Perpendicular if slopes are negative reciprocals

b) Determine if the two lines $3x - 4y - 12 = 0$ and $4x - 3y - 24 = 0$ are parallel, perpendicular or neither.

$3x - 4y - 12 = 0$

$-4y = -3x + 12$

$y = \frac{3}{4}x - 3$

$m = \frac{3}{4}$

$4x - 3y - 24 = 0$

$-3y = -4x + 24$

$y = \frac{4}{3}x - 8$

$m = \frac{4}{3}$

The lines are neither parallel or perpendicular

6.5 - Equation of a Line Given Slope and a Point

DO IT NOW!

Instructions: Determine the equation of the line, in slope y-intercept form, that has a slope of 3 and goes through the point (2, -5)

Note: You can write the equation of a line once you know the slope and y-intercept.

$$y = \underbrace{m}_{\text{Slope}}x + \underbrace{b}_{y\text{-intercept}}$$

Step 1: State what you know about the line

Slope $= m = 3$

Point on line: $(\overset{x}{2}, \overset{y}{-5})$

Step 2: Determine the y-intercept of the line

To do this we can use the equation $y = mx + b$, substitute in values for m, x and y and then solve for the b value. Use the point on the line that is given for the x and y values.

$y = mx + b$

$-5 = 3(2) + b$

$-5 = 6 + b$

$-5 - 6 = b$

$b = -11$

STEP 3: Write the equation of the line in slope y-intercept form.

Note: When writing the final equation of the line, plug in values for m and b, not for x and y.

$y = 3x - 11$

Example 1: Find the equation of the line with a slope of $\frac{1}{2}$ that passes through (1, 5).

Step 1: State what you know about the line

Slope $= m = \frac{1}{2}$

Point: $(\overset{x}{1}, \overset{y}{5})$

Step 2: Determine the y-intercept of the line

$y = mx + b$

$5 = \frac{1}{2}(1) + b$

$5 = \frac{1}{2} + b$

$\frac{5}{1} - \frac{1}{2} = b$

$\frac{10}{2} - \frac{1}{2} = b$

$b = \frac{9}{2}$

STEP 3: Write the equation of the line in slope y-intercept form.

$y = \frac{1}{2}x + \frac{9}{2}$

Example 2: Find the equation of the line with a slope of 3 and that passes through (0, 2). Then graph the line.

Step 1: State what you know about the line

Slope: $m = 3$

Point: $(\overset{x}{0}, \overset{y}{2})$

This point is the y-intercept. If you notice this, you don't have to calculate it.

Step 2: Determine the y-intercept of the line

$y = mx + b$

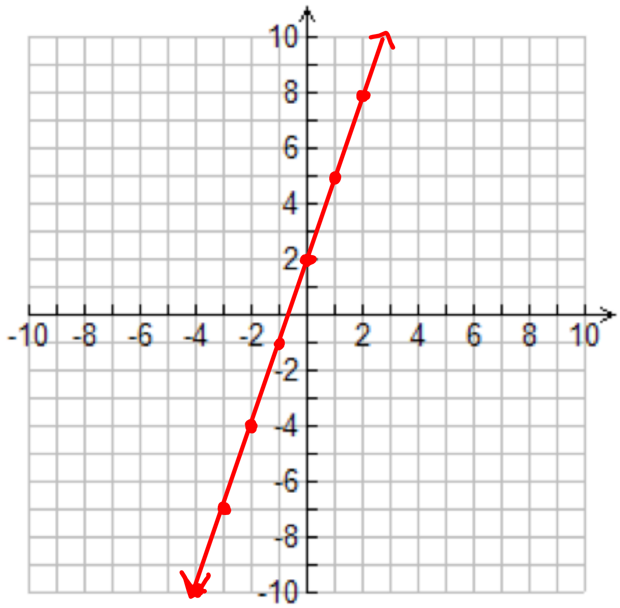
$2 = 3(0) + b$

$2 = b$

STEP 3: Write the equation of the line in slope y-intercept form.

$y = 3x + 2$

Step 4: Graph the line using the slope and y-intercept



Example 3: Determine the equation of a line that is parallel to the line y = -2x – 7 and passes through the point (1, -3).

Step 1: State what you know about the line

Remember: lines that are parallel have the same slope. They do not have the same y-intercept. You will still have to solve for that.

slope: $m = -2$

Point: $(\overset{x}{1}, \overset{y}{-3})$

Step 2: Determine the y-intercept of the line

$y = mx + b$

$-3 = -2(1) + b$

$-3 = -2 + b$

$-3 + 2 = b$

$-1 = b$

STEP 3: Write the equation of the line in slope y-intercept form.

$y = -2x - 1$

<u>Example 4:</u> Determine the equation of a line that is perpendicular to the line $2x - y + 4 = 0$ and passes through the point (-2, 5).

Hint: to determine the slope you will need to put the equation into $y = mx + b$ form so that you can see the slope and then take the negative reciprocal.

$$2x - y + 4 = 0$$
$$2x + 4 = y$$
$$y = 2x + 4$$

STEP 1: state what you know about the line

Slope of given line: $m = 2$

Slope of perpendicular line (find negative reciprocal):

$$\perp m = -\frac{1}{2}$$

Point on the perpendicular line: $(\overset{x}{-2}, \overset{y}{5})$

Step 2: Determine the y-intercept of the line (make sure to use the slope of the perpendicular line)

$$y = mx + b$$
$$5 = \frac{-1}{2}(-2) + b$$
$$5 = \frac{2}{2} + b$$
$$5 = 1 + b$$
$$5 - 1 = b$$
$$4 = b$$

STEP 3: Write the equation of the line in slope y-intercept form (make sure to use the slope of the perpendicular line).

$$y = -\frac{1}{2}x + 4$$

Consolidation:

To write the equation of a line you need to know the slope (m) and y-intercept (b).

You can use the slope of a line and a point on the line to calculate the y-intercept.

To find the slope of a perpendicular line, find the negative reciprocal.

6.6 Equation of a Line Given Two Points

Remember: You can write the equation of a line once you know the slope and y-intercept.

$$y = mx + b$$

Slope **y-intercept**

DO IT NOW!

Instructions: Write the equation of the following lines:

m   x y

a) Line with a slope of $\frac{3}{5}$ that passes through the point B(-5, 4).

$y = mx + b$

$4 = \left(\frac{3}{5}\right)(-5) + b$

$4 = \frac{-15}{5} + b$

$4 = -3 + b$

$4 + 3 = b$

$b = 7$

$$\boxed{y = \frac{3}{5}x + 7}$$

b) Line that is parallel to the line $y = 2x - 7$ and passes through the point (1, -3).

x y

$m = 2$

$y = mx + b$

$-3 = 2(1) + b$

$-3 = 2 + b$

$-3 - 2 = b$

$b = -5$

$$\boxed{y = 2x - 5}$$

c) Line that is perpendicular to the line 2x – 2y + 4 = 0 and passes through the point (-2, 5).

$-2y = -2x - 4$

$y = \frac{-2}{-2}x - \frac{4}{-2}$

$y = 1x + 2$

$m = 1$

$\perp m = -1$

Point: (-2,5)

$y = mx + b$

$5 = (-1)(-2) + b$

$5 = 2 + b$

$5 - 2 = b$

$b = 3$

$\boxed{y = -1x + 3}$

Today's Lesson: Find the equation of a line given two points on the line.

What do you need to write the equation of a line?

slope (m) and y-intercept (b)

If you are not given the slope of a line, how can you find it?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

How can you find the *y*-intercept?

use the slope (m) and the coordinates (x,y) of any point on the line to solve for the y-intercept (b).

Example 1: Determine the equation of a line that passes through the points M(4, -3) and N(2, 5).

x_1 y_1 x_2 y_2

Step 1: Calculate the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{2-4} = \frac{8}{-2} = -4$$

Step 2: Find the y-intercept

Note: to find the y-intercept you can use any point that is on the line for your x and y values.

$y = mx + b$

$5 = (-4)(2) + b$

$5 = -8 + b$

$5 + 8 = b$

$b = 13$

Step 3: Write the equation of the line

$$y = -4x + 13$$

Example 2: Determine the equation of a line that passes through the points P(0, 4) and Q(7, 0).

x_1 y_1 x_2 y_2

Step 1: Calculate the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-4}{7-0}$$

$$= \frac{-4}{7}$$

Step 2: Find the y-intercept

$y = mx + b$

$4 = (-\frac{4}{7})(0) + b$

$4 = b$

Step 3: Write the equation of the line

$$y = -\frac{4}{7}x + 4$$

Example 3: Determine the equation of a line that passes through the points A(-4, 2) and B(8, 11).
x_1 y_1 x_2 y_2

Step 1: Calculate the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{11-2}{8-(-4)}$$
$$= \frac{9}{12}$$
$$= \frac{3}{4}$$

Step 2: Find the y-intercept

$$y = mx + b$$
$$2 = \left(\frac{3}{4}\right)(-4) + b$$
$$2 = -\frac{12}{4} + b$$
$$2 = -3 + b$$
$$2 + 3 = b$$
$$b = 5$$

Step 3: Write the equation of the line

$$y = \frac{3}{4}x + 5$$

Example 4: On your own determine the equation of the line that passes through the points A(2,-4) and B(5,5)
x_1 y_1 x_2 y_2

SLOPE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5-(-4)}{5-2}$$
$$= \frac{9}{3}$$
$$= 3$$

Y-int

$$y = mx + b$$
$$5 = 3(5) + b$$
$$5 = 15 + b$$
$$5 - 15 = b$$
$$b = -10$$

$$\boxed{y = 3x - 10}$$

Example 5:

a) An appliance repair company charges \$205 for a repair that takes 3 hours. The same company charges \$505 for a repair that takes 8 hours. Determine an equation that represents the cost of a repair based on the number of hours that the repair takes. y

x

Hint: *you can write two coordinate points with the information given. (ind. variable, dep. variable)*

Point 1: $(\overset{x_1}{3}, \overset{y_1}{205})$

Point 2: $(\underset{x_2}{8}, \underset{y_2}{505})$

Slope

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{505 - 205}{8 - 3}$

$= \frac{300}{5}$

$= 60$

y-int

$y = mx + b$

$205 = 60(3) + b$

$205 = 180 + b$

$205 - 180 = b$

$b = 25$

$y = 60x + 25$ OR COST $= 60$(hours) $+ 25$

b) What is the cost of a repair that takes 7 hours?
y x

$y = 60x + 25$

$y = 60(7) + 25$

$y = 420 + 25$

$y = 445$

$\boxed{\$445}$

c) If a repair costs \$385, how many hours does it take?
y x

$y = 60x + 25$

$385 = 60x + 25$

$385 - 25 = 60x$

$\frac{360}{60} = \frac{\cancel{60}x}{\cancel{60}}$

$\frac{360}{60} = x$

$x = 6$

$\boxed{6 \text{ hours}}$

Consolidate:

To write the equation of a line you need the slope and y-intercept.

If you are not given the slope you can find it if you have 2 points on the line by using the

Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

6.7 - Linear Systems

Linear System: A set of two or more linear equations that are considered simultaneously

Point of Intersection: the point where two or more lines intersect

DO IT NOW!

Mike is considering joining a ski club for the winter season. He is considering the following two options:

Standard Rate: $50 per day and no registration fee
Frequent Skier Pass: $40 per day and $100 registration fee

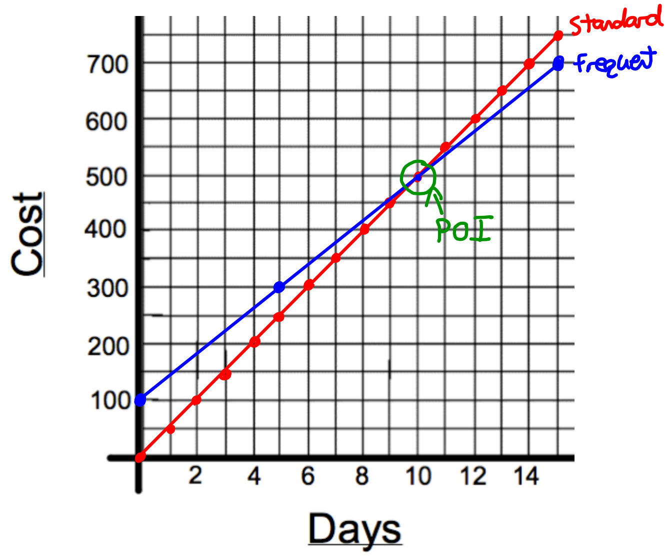
a) Write an equation that relates the total cost, C, in dollars, and the number of days, n, that Mike goes skiing if he chooses the Standard Rate:

$C = 50n$

b) Write an equation if he chooses the Frequent Skier Pass option:

$C = 40n + 100$

c) Graph both of the lines on the same graph



d) What is the point of intersection?

(10, 500)
# of days
cost

e) What is the cost of both plans at the point of intersection? What does this mean?

\$500 ; if you ski 10 times, it will cost the same for each plan.

f) Look to the right of the point of intersection, which plan is cheaper?

Frequent skier.

g) Look to the left of the point of intersection, which plan is cheaper?

Standard.

h) If Mike is going to go skiing 11 times this winter, which plan would you recommend to him?

Frequent skier.

Example 1: Graph $y = \frac{1}{2}x - 3$ and $x + y = -6$ on the same grid and identify the coordinates of the point of intersection.

Line 1: $y = \frac{1}{2}x - 3$

Line 2: $x + y = -6$

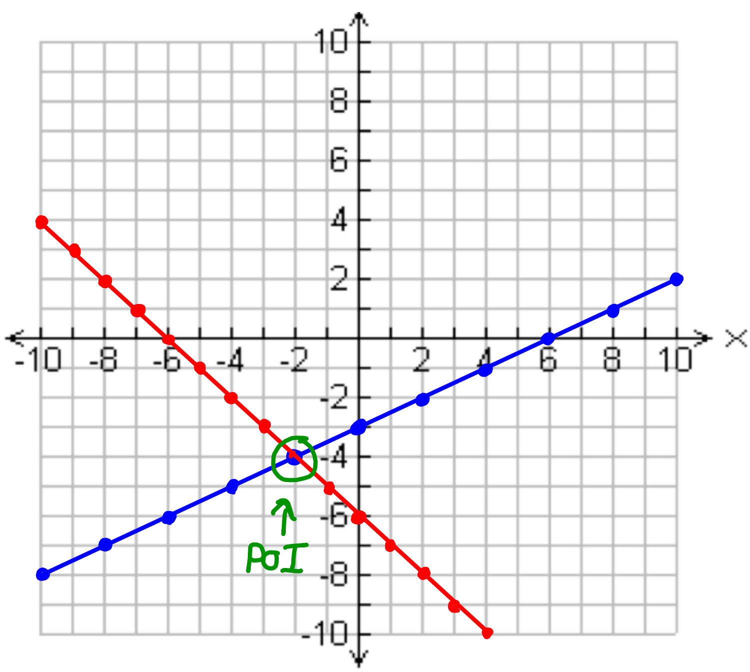
$y = -1x - 6$

Slope: $m = \frac{1}{2}$

Slope: $m = -1$

y-intercept: $b = -3$

y-intercept: $b = -6$



Point of Intersection: $(-2, -4)$

Check your answer: To verify the solution, (-2, -4), substitute the coordinates into both equations and check that they hold true. Use the left side/right side method.

Check: $y = \frac{1}{2}x - 3$

L.S.	**R.S.**
$= y$	$= \frac{1}{2}x - 3$
$= -4$	$= \frac{1}{2}(-2) - 3$
	$= -1 - 3$
	$= -4$

LS=RS

Check: $x + y = -6$

L.S.	**R.S.**
$= x + y$	$= -6$
$= -2 + (-4)$	
$= -2 - 4$	
$= -6$	

LS=RS

Example 2: Graph $y = 2x - 2$ and $y = x + 1$ on the same grid and identify the coordinates of the point of intersection.

Line 1: $y = 2x - 2$

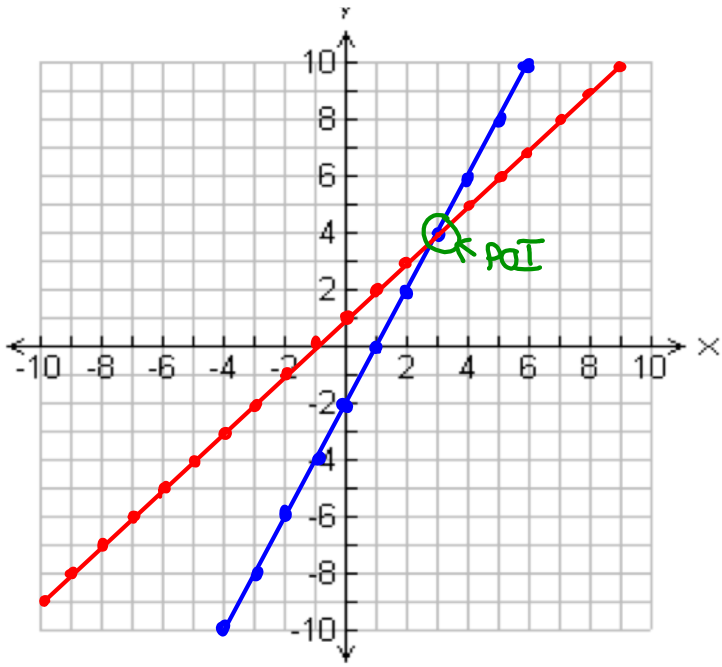
Slope: $m = 2$

y-intercept: $b = -2$

Line 2: $y = x + 1$

Slope: $m = 1$

y-intercept: $b = 1$



Point of Intersection: (3, 4)

Check your answer: To verify the solution, (3,4), substitute the coordinates into both equations and check that they hold true. Use the left side/right side method.

Check: $y = 2x - 2$

L.S.		**R.S.**
$= y$		$= 2x - 2$
$= 4$		$= 2(3) - 2$
		$= 6 - 2$
		$= 4$
	LS = RS	

Check: $y = x + 1$

L.S.		**R.S.**
$= y$		$= x + 1$
$= 4$		$= 3 + 1$
		$= 4$
	LS = RS	