Unit 1 Pre-Test Review – Derivative Rules Unit 1

MCV4U

Jensen

SOLUTIONS

1) Differentiate each function.

a) $h(t) = t^3 - 2t^2 + \frac{1}{t^2}$

$h(t) = t^3 - 2t^2 + t^{-2}$

$h'(t) = 3t^2 - 4t - 2t^{-3}$

$h'(t) = 3t^2 - 4t - \frac{2}{t^3}$

b) $p(n) = -n^5 + 5n^3 + \sqrt[3]{n^2}$

$p(n) = -n^5 + 5n^3 + n^{2/3}$

$p'(n) = -5n^4 + 15n^2 + \frac{2}{3}n^{-1/3}$

$p'(n) = -5n^4 + 15n^2 + \frac{2}{3n^{1/3}}$

c) $p(r) = r^6 - \frac{2}{5\sqrt{r}} + r - 1$

$p(r) = r^6 - \frac{2}{5}r^{-1/2} + r - 1$

$p'(r) = 6r^5 + \frac{1}{5}r^{-3/2} + 1$

$p'(r) = 6r^5 + \frac{1}{5r^{3/2}} + 1$

2) Differentiate using the product rule.

a) $f(x) = (5x + 3)(2x - 11)$

$f'(x) = 5(2x - 11) + 2(5x + 3)$

$f'(x) = 10x - 55 + 10x + 6$

$f'(x) = 20x - 49$

b) $h(t) = \left(2t^2 + \sqrt[3]{t}\right)(4t - 5)$

$h'(t) = \left(4t + \frac{1}{3}t^{-2/3}\right)(4t - 5) + 4\left(2t^2 + t^{1/3}\right)$

$h'(t) = 16t^2 - 20t + \frac{4t}{3t^{2/3}} - \frac{5}{3t^{2/3}} + 8t^2 + 4t^{1/3}$

$h'(t) = 24t^2 - 20t + \frac{4t^{1/3}}{3} + 4t^{1/3} - \frac{5}{3t^{2/3}}$

$h'(t) = 24t^2 - 20t + \frac{16t^{1/3}}{3} - \frac{5}{3t^{2/3}}$

c) $g(x) = (-1.5x^6 + 1)(3 - 8x)$

$g'(x) = -9x^5(3 - 8x) + (-8)(-1.5x^6 + 1)$

$g'(x) = -27x^5 + 72x^6 + 12x^6 - 8$

$g'(x) = 84x^6 - 27x^5 - 8$

d) $p(n) = (11n + 2)(-5 + 3n^2)$

$p'(n) = 11(-5 + 3n^2) + 6n(11n + 2)$

$p'(n) = -55 + 33n^2 + 66n^2 + 12n$

$p'(n) = 99n^2 + 12n - 55$

3) Determine an equation for the tangent to the graph of $y = (-3x+8)(x^3-7)$ at $x = 2$.

$\frac{dy}{dx} = -3(x^3-7) + 3x^2(-3x+8)$

$\left.\frac{dy}{dx}\right|_{x=2} = -3[(2)^3-7] + 3(2)^2[-3(2)+8]$

$= -3(1) + 12(2)$

$= 21$

$m = 21$

$y(2) = [-3(2)+8][(2)^3-7]$

$y(2) = (2)(1)$

$(2,2)$

$\underline{Eq^n}$

$y = mx+b$

$2 = 21(2)+b$

$b = -40$

$\boxed{y = 21x - 40}$

4) Determine $f''(-2)$ for $f(x) = (4-x^2)(3x+1)$

$f'(x) = (-2x)(3x+1) + 3(4-x^2)$

$f'(x) = -6x^2 - 2x + 12 - 3x^2$

$f'(x) = -9x^2 - 2x + 12$

$f''(x) = -18x - 2$

$f''(-2) = -18(-2) - 2$

$\boxed{f''(-2) = 34}$

5) Determine the first and second derivative of each function.

a) $g(x) = \frac{2}{3}x^3 + \frac{1}{2}x^4 - 3$

$g'(x) = 2x^2 + 2x^3$

$g''(x) = 4x + 6x^2$

b) $h(x) = (2x-3)(3x+1)$

$h'(x) = 2(3x+1) + 3(2x-3)$

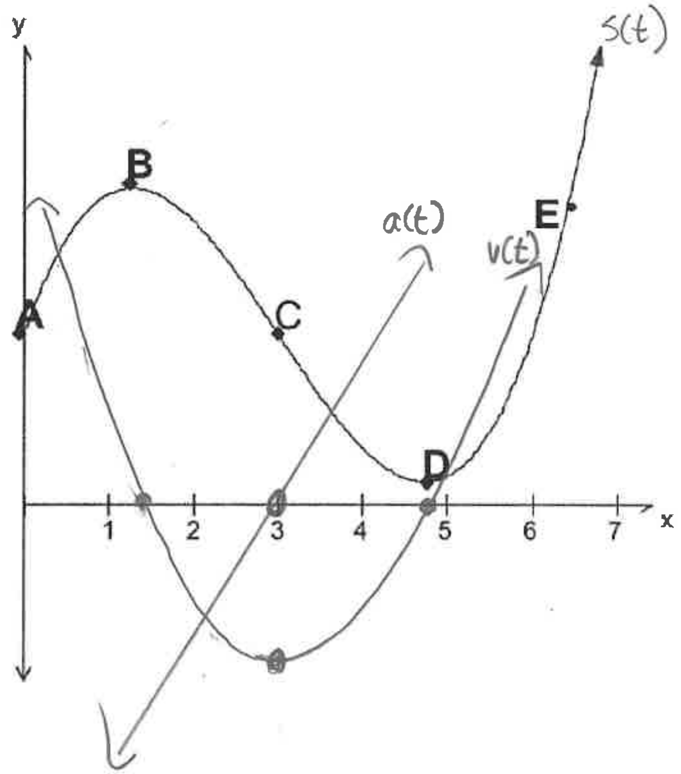
$h'(x) = 6x + 2 + 6x - 9$

$h'(x) = 12x - 7$

$h''(x) = 12$

6) For the distance time graph given,

a) sketch the velocity and acceleration function.



b) Complete the table to determine the motion of the object.

Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	Slope of $s(t)$	Motion of particle
(A, B)	+	−	−	Positive and decreasing	Moving Forward slowing down
(B, C)	−	−	+	Negative and decreasing	Moving in reverse speeding up
(C, D)	−	+	−	Negative and increasing	Moving in reverse slowing down
(D, E)	+	+	+	Positive and increasing	Moving Forward speeding up

7) A toy missile is shot into the air. Its height, h, in meters, after t seconds can be modelled by the function $h(t) = -4.9t^2 + 15t + 0.4$, $t \geq 0$.

a) Determine the height of the toy missile at 2 seconds.

$h(2) = -4.9(2)^2 + 15(2) + 0.4$

$h(2) = 10.8$ m

b) Determine the rate of change of the height of the toy missile at 1 s and 4 s.

$h'(t) = -9.8t + 15$

$h'(1) = -9.8(1) + 15$

$h'(1) = 5.2$ m/s

$h'(4) = -9.8(4) + 15$

$h'(4) = -24.2$ m/s

c) How long does it take the toy missile to return to the ground?

$0 = -4.9t^2 + 15t + 0.4$

$$t = \frac{-15 \pm \sqrt{(15)^2 - 4(-4.9)(0.4)}}{2(-4.9)}$$

$t \simeq 3.09$ s

d) How fast was the toy missile travelling when it hit the ground?

$h'(3.09) = -9.8(3.09) + 15$

$\simeq -15.28$ m/s

8) Differentiate using the quotient rule.

a) $y = \frac{x-2}{2x+5}$

$$y' = \frac{1(2x+5)-2(x-2)}{(2x+5)^2}$$

$$y' = \frac{2x+5-2x+4}{(2x+5)^2}$$

$$y' = \frac{9}{(2x+5)^2}$$

b) $y = \frac{x^2-4}{2x+5}$

$$y' = \frac{2x(2x+5)-2(x^2-4)}{(2x+5)^2}$$

$$y' = \frac{4x^2+10x-2x^2+8}{(2x+5)^2}$$

$$y' = \frac{2x^2+10x+8}{(2x+5)^2}$$

9) Determine the slope of the tangent to $y = \frac{3x}{x^2-4x+3}$ at $x = 4$.

$$y' = \frac{3(x^2-4x+3) - (2x-4)(3x)}{(x^2-4x+3)^2}$$

$$y' = \frac{3x^2-12x+9-6x^2+12x}{(x^2-4x+3)^2}$$

$$y' = \frac{-3x^2+9}{(x^2-4x+3)^2}$$

$$y'(4) = \frac{-3(4)^2+9}{[(4)^2-4(4)+3]^2}$$

$$y'(4) = \frac{-39}{9}$$

$$y'(4) = \frac{-13}{3}$$

10) Differentiate each of the following.

a) $f(x) = (3x-2)^2$

$$f'(x) = 2(3x-2)(3)$$

$$f'(x) = 6(3x-2)$$

$$f'(x) = 18x - 12$$

b) $y = (3x^2 - x)^3$

$$y' = 3(3x^2-x)^2(6x-1)$$

$$y' = 3(9x^4-6x^3+x^2)(6x-1)$$

$$y' = (27x^4-18x^3+3x^2)(6x-1)$$

$$y' = 162x^5-27x^4-108x^4+18x^3+18x^3-3x^2$$

$$y' = 162x^5-135x^4+36x^3-3x^2$$

c) $h(x) = \sqrt[3]{3x+5x^4}$

$$h'(x) = \frac{1}{3}(3x+5x^4)^{-2/3}(3+20x^3)$$

$$h'(x) = \frac{20x^3+3}{3(3x+5x^4)^{2/3}}$$

d) $f(x) = (2x-3)^3(3x-1)^2$

$$f'(x) = 3(2x-3)^2(2)(3x-1)^2 + 2(3x-1)(3)(2x-3)^3$$

$$f'(x) = 6(2x-3)^2(3x-1)[3x-1+(2x-3)]$$

$$f'(x) = 6(2x-3)^2(3x-1)(5x-4)$$

e) $y = \frac{(2x-5)^4}{(x+1)^3}$

$$y' = \frac{4(2x-5)^3(2)(x+1)^3 - 3(x+1)^2(1)(2x-5)^4}{[(x+1)^3]^2}$$

$$y' = \frac{(2x-5)^3(x+1)^2[8(x+1) - 3(2x-5)]}{(x+1)^6}$$

$$y' = \frac{(2x-5)^3(8x+8-6x+15)}{(x+1)^4}$$

$$y' = \frac{(2x-5)^3(2x+23)}{(x+1)^4}$$

f) $y = \frac{8x^3}{\sqrt{3x-2}} = \frac{8x^3}{(3x-2)^{1/2}}$

$$y' = \frac{24x^2(3x-2)^{1/2} - \frac{1}{2}(3x-2)^{-1/2}(3)(8x^3)}{[(3x-2)^{1/2}]^2}$$

$$y' = \frac{(3x-2)^{-1/2}[24x^2(3x-2) - 12x^3]}{(3x-2)^1}$$

$$y' = \frac{72x^3 - 48x^2 - 12x^3}{(3x-2)^{1/2}(3x-2)^1}$$

$$y' = \frac{60x^3 - 48x^2}{(3x-2)^{3/2}} = \frac{12x^2(5x-4)}{(3x-2)^{3/2}}$$

11) Find an equation for the tangent at $x = 1$ to the curve $y = \left(\frac{2x}{x+1}\right)^6$.

Point:

$y(1) = \left[\frac{2(1)}{1+1}\right]^6$

$y(1) = (1)^6$

$y(1) = 1$

$(1, 1)$

Slope:

$$y' = 6\left(\frac{2x}{x+1}\right)^5\left[\frac{2(x+1) - 1(2x)}{(x+1)^2}\right]$$

$$y' = 6\left[\frac{32x^5}{(x+1)^5}\right]\left[\frac{2}{(x+1)^2}\right]$$

$$y' = \frac{384x^5}{(x+1)^7}$$

$y'(1) = 3$

$m = 3$

Eq^n

$y = mx + b$

$1 = 3(1) + b$

$b = -2$

$\boxed{y = 3x - 2}$

12) Find all tangents to the curve $y = 4x^3$ that have slope of 3.

$y' = 12x^2$

$3 = 12x^2$

$x = \pm\sqrt{\frac{1}{4}}$

$x = \pm\frac{1}{2}$

Point 1:

$y(\frac{1}{2}) = 4(\frac{1}{2})^3$

$y(\frac{1}{2}) = \frac{1}{2}$

$(\frac{1}{2}, \frac{1}{2})$

Point 2:

$y(-\frac{1}{2}) = 4(-\frac{1}{2})^3$

$y(-\frac{1}{2}) = -\frac{1}{2}$

$(-\frac{1}{2}, -\frac{1}{2})$

Eq^n 1:

$y = mx + b$

$\frac{1}{2} = 3(\frac{1}{2}) + b$

$b = -1$

$\boxed{y = 3x - 1}$

Eq^n 2:

$-\frac{1}{2} = 3(-\frac{1}{2}) + b$

$b = 1$

$\boxed{y = 3x + 1}$

13) Suppose a particle travels according to the position function in meters $s(t) = \frac{t^3}{3} - 2t^2 + 3t - 4$.

a) At what two times is the particle stationary (stopped)? That is, when is the velocity zero.

$v(t) = s'(t) = t^2 - 4t + 3$

$0 = (t-3)(t-1)$

$t_1 = 3\,s \quad t_2 = 1\,s$

b) How far does the particle travel between the two stationary times?

Distance $= s(3) - s(1)$

$= \left[\frac{3^3}{3} - 2(3)^2 + 3(3) - 4\right] - \left[\frac{1^3}{3} - 2(1)^2 + 3(1) - 4\right]$

$= -4 - (-2.67)$

$= -1.33$

1.33 m in reverse.

14) When the price is \$1.75 each, 3000 fruit bars will be sold. If the price of a fruit bar is raised to \$2.00, sales will drop to 2500.

x = # sold $\quad p$ = price $\quad n$ = # of increases

a) Determine the demand, or price, function

$x = 3000 - 500n \rightarrow n = \frac{3000 - x}{500} = 6 - 0.002x$

$p = 1.75 + 0.25n$

$p(x) = 1.75 + 0.25(6 - 0.002x)$

$p(x) = 1.75 + 1.5 - 0.0005x$

$\boxed{p(x) = 3.25 - 0.0005x}$

b) Determine the marginal revenue from the sale of 2700 bars

$R(x) = x(3.25 - 0.0005x)$

$R(x) = 3.25x - 0.0005x^2$

$R'(x) = 3.25 - 0.001x$

$R'(2700) = 3.25 - 0.001(2700)$

$R'(2700) = 0.55$

\$0.55 per bar sold.

c) The cost for the bars is given by the function $C(x) = 30 + 0.25x$. Determine the marginal cost of purchasing 3000 bars.

$C'(x) = 0.25$

$C'(3000) = \$0.25$ per bar

d) Determine the marginal profit function for the sale of the fruit bars.

$P(x) = R(x) - C(x)$

$P(x) = 3.25x - 0.0005x^2 - (30+0.25x)$

$P(x) = -0.0005x^2 + 3x - 30$

$\boxed{P'(x) = -0.001x+3}$

e) Determine the marginal profit from the sale of 3000 bars.

$P'(3000) = -0.001(3000) + 3$

$P'(3000) = \$0$ / bar

15) The mass, in grams, of the first x meters of a wire is represented by the function $f(x) = \sqrt{4x - 1}$.

a) Determine the average linear density of a segment of the wire from $x = 3$ to $x = 7$.

Average LD $= \dfrac{f(7) - f(3)}{7-3} = \dfrac{\sqrt{27} - \sqrt{11}}{4} \simeq 0.47$ g/m

b) Determine the linear density at $x = 4$ and $x = 10$. What does these values confirm about the wire?

$f'(x) = \frac{1}{2}(4x-1)^{-1/2}(4)$

$f'(x) = \dfrac{2}{\sqrt{4x-1}}$

$f'(4) = \frac{2}{\sqrt{15}} \simeq 0.516$ g/m

$f'(10) = \frac{2}{\sqrt{39}} \simeq 0.32$ g/m

$\therefore$ the material is not homogenous.

Answers:

a) $h'(t) = 3t^2 - 4t - \frac{2}{t^3}$ **b)** $p'(n) = -5n^4 + 15n^2 + \frac{2}{3\sqrt[3]{n}}$ **c)** $p'(r) = 6r^5 + \frac{1}{5\sqrt{r^3}} + 1$

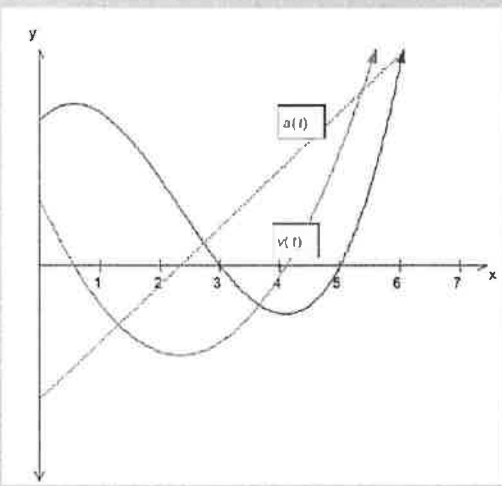
2)a) $f'(x) = 20x - 49$ **b)** $h'(t) = 24t^2 - 20t + \frac{16}{3}t^{\frac{1}{3}} - \frac{5}{3t^{\frac{2}{3}}}$ **c)** $g'(x) = 84x^6 - 27x^5 - 8$ **d)** $p'(n) = 99n^2 + 12n - 55$

3) $y = 21x - 40$

4) 34

5)a) $g'(x) = 2x^2 + 2x^3$ $\quad g''(x) = 4x + 6x^2$ **b)** $h'(x) = 12x - 7$ $\quad h''(x) = 12$

6)a)



b)

Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	Slope of $s(t)$	Motion of particle
(A, B)	+	-	-	positive slope that is decreasing	Slowing down and moving forward
(B, C)	-	-	+	Negative slope that is decreasing	Speeding up and moving in reverse
(C, D)	-	+	-	Negative slope that is increasing	Slowing down and moving in reverse
(D, E)	+	+	+	Positive slope that is increasing	Speeding up and moving forward

7)a) 10.8 m **b)** 5.2 m/s at 1 second; -24.2 m/s at 4 seconds **c)** 3.088 seconds **d)** -15.26 m/s

8)a) $y' = \frac{9}{(2x+5)^2}$ **b)** $y' = \frac{2x^2+10x+8}{(2x+5)^2}$

9) $-\frac{13}{3}$

10)a) $f'(x) = 18x - 12$ **b)** $y' = 54x^5 - 45x^4 + 12x^3 - x^2$ **c)** $h'(x) = \frac{20x^3+3}{3(\sqrt[3]{3x+5x^4})^2}$ **d)** $f'(x) = 6(2x-3)^2(3x-1)(5x-4)$

e) $y' = \frac{(2x-5)^3(2x+23)}{(x+1)^4}$ **f)** $\frac{dy}{dx} = \frac{12x^2(5x-4)}{(3x-2)^{\frac{3}{2}}}$

11) $y = 3x - 2$

12) $y = 3x - 1$ and $y = 3x + 1$

13)a) $t = 1$ and $t = 3$ **b)** $\frac{4}{3}$ meters backwards

14)a) $p(x) = 3.25 - 0.0005x$ **b)** \$0.55/bar **c)** \$0.25/bar **d)** $p'(x) = 3 - 0.001x$ **e)** \$0/bar

15)a) -0.470 g/m **b)** $f'(4) = 0.516$; $f'(10) = 0.320$; the confirm that the material of which the wire is composed of is not homogenous.