Unit 2 Pretest – Curve Sketching
MCV4U
Jensen

1) Find the increasing and decreasing intervals for each function.

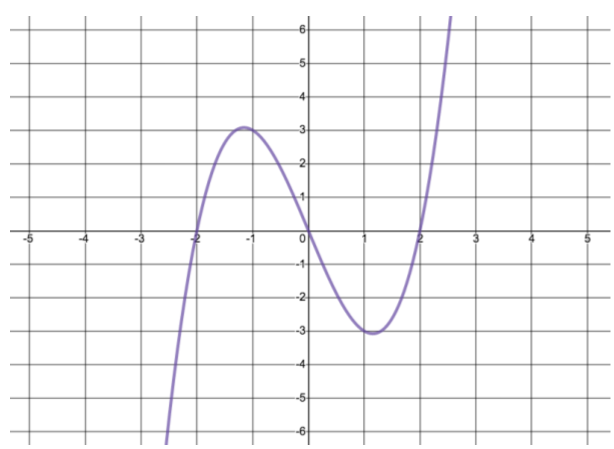
a) $f(x) = 7 + 6x - x^2$

b) $y = x^3 - 48x + 5$

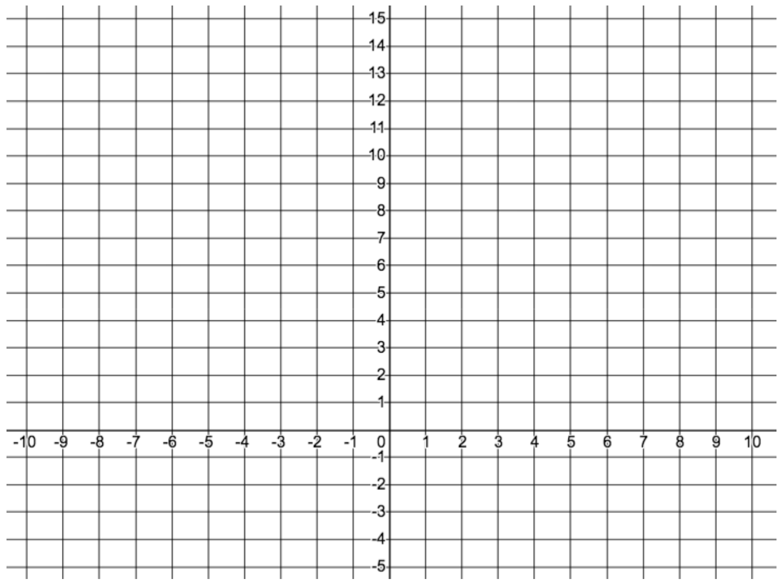
c) $g(x) = x^4 - 18x^2$

d) $f(x) = x^3 + 10x - 9$

2) Given the graph of $f'(x)$, state the intervals of increase and decrease for the function $f(x)$. Then sketch a possible graph of $f(x)$.

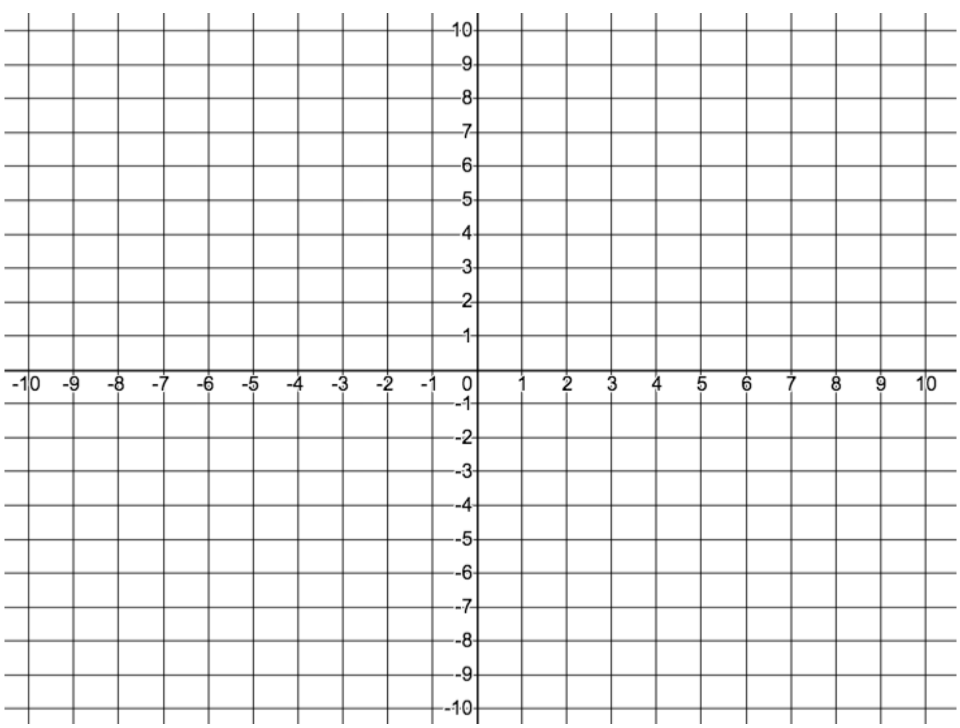


3) Sketch a continuous graph that satisfies the following set of conditions: $f'(x) > 0$ when $x < -4$ and $x > 3$, $f'(x) < 0$ when $-4 < x < 3$ and $f(1) = 5$.



4) Given the following information about $y = f(x)$, sketch a graph for the function on the axes provided below. Label an appropriate scale for the sketch.

Local minimum $(6, -3)$. Local maximum $(0,3)$. Points of inflection at $(-2,1)$ and $(8, -1)$. Increasing when $x < 0$ and $x > 6$; decreasing when $0 < x < 3$ and $3 < x < 6$. Concave up when $x < -2$ and $3 < x < 8$; concave down when $-2 < x < 3$ and $x > 8$. HA at $y = 0$. VA at $x = 3$. y-intercept at $(0,3)$. x-intercepts at $(2,0)$ and $(4,0)$.



5) Find the local extrema for each function and classify them as local max or local min.

a) $f(x) = 16 - x^4$

b) $g(x) = x^3 + 9x^2 - 21x - 12$

6) The speed, in km/h, of a certain car t seconds after passing a police radar location is given by the function $v(t) = 3t^2 - 24t + 88$.

a) Find the min speed of the car.

b) The radar tracks the car on the interval $2 < t < 5$. Find the max speed of the car on this interval.

7) Determine the absolute extreme values of each function on the given interval.

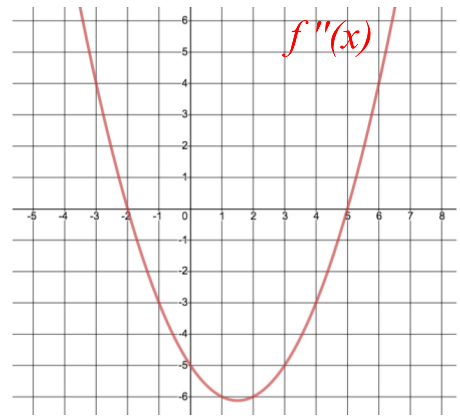
a) $y = x^2 - 3x + 2; -4 \leq x \leq 4$

b) $g(x) = 2x^3 - 24x + 3; -4 \leq x \leq 2$

8) For the function $f(x) = x^4 - 2x^3 - 12x^2 + 3$, determine the points of inflection and the intervals of concavity.

9) For the function $f(x) = 2x^3 - x^4$, determine the critical points and classify them using the second derivative test.

10) Given the graph of $f''(x)$, state the intervals of concavity for $f(x)$.



11) For each function, state equations for any asymptotes.

a) $f(x) = \frac{x^2-4}{x}$

b) $g(x) = \frac{2x-3}{2x-4}$

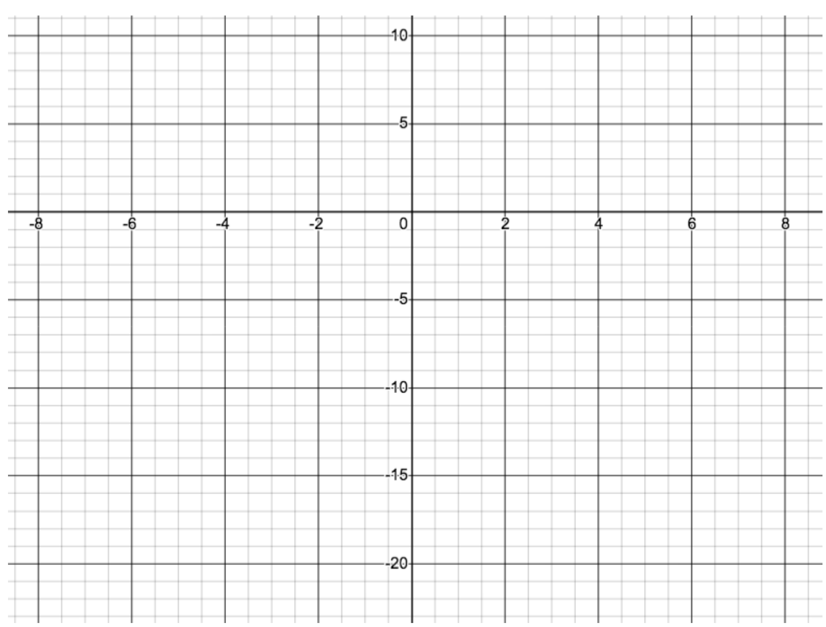
c) $y = \frac{x^2+1}{x^2-3x-10}$

d) $\frac{x-1}{x^2+2x+1}$

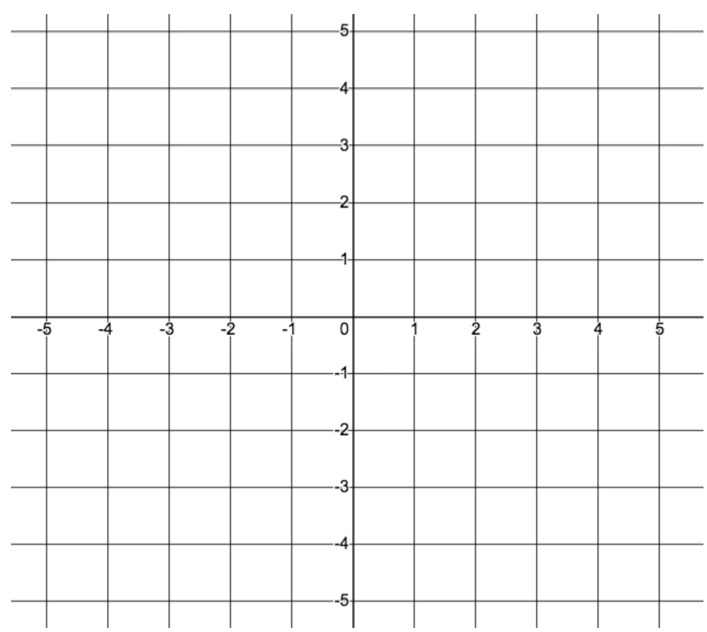
12) State the equation of the tangent to the graph of $f(x) = \frac{x+1}{x^2+1}$ at the point where $x = -1$.

13) Analyze and sketch each function using the algorithm for curve sketching

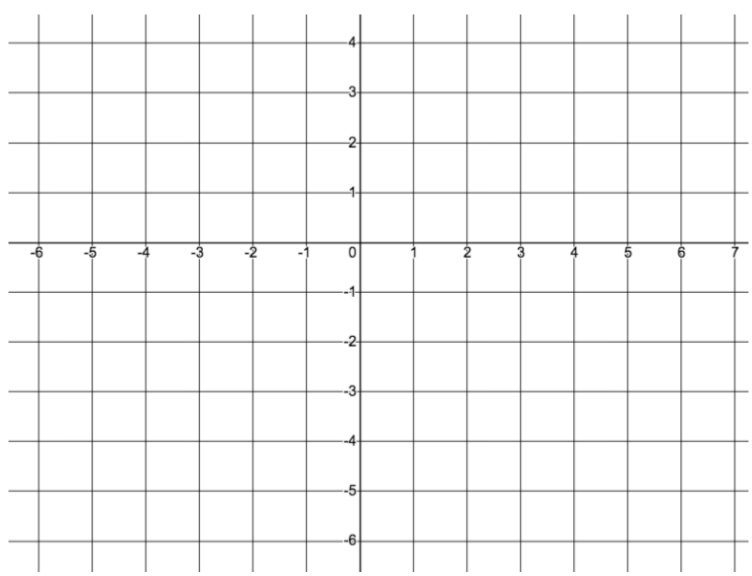
a) $k(x) = \frac{1}{4}x^4 - \frac{9}{2}x^2$



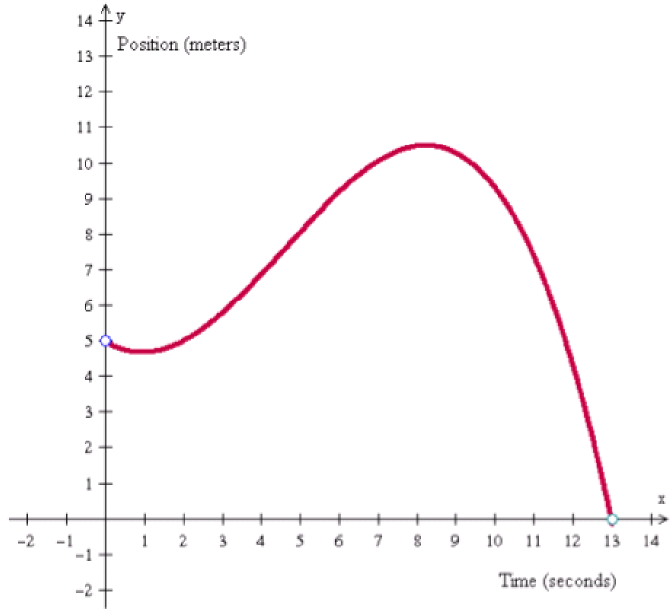
b) $h(x) = 2x^3 - 3x^2 - 3x + 2$



c) $f(x) = \frac{x^2+2x-4}{x^2}$



14) Consider the graph to the right.



a) How many times does the function have a derivative = 0? How do you know?

b) State the sign of the first and second derivatives at the following times.

i) t = 3 s **ii)** t = 7 s **iii)** t = 10 s

15) A garbage can is in the shape of a cylinder with no lid. It needs to have a volume of 5000 cm^3. What will be the radius and height of the can that uses the least amount of material to construct it? (Hint: Surface Area)

16) A carpenter builds an open box with a square base. She has 8 m^2 of wood available. Find the volume of the largest box she can build.

17) Sandy will make a closed rectangular jewellery box with a square base from two different woods. The wood from the top and botton costs \$0.0020/cm^2. The wood for the sides costs \$0.0030/cm^2. Find the dimensions that minimize the cost for a box with volume 4000 cm^3.

18) A music store sells an average of 120 CDs per week at \$24 each. A market survey indicates that for each \$0.75 decrease in price, five additional CDs will be sold per week. The cost of producing x CDs is $C(x) = -0.003x^2 + 3x + 2000$. What price and quantity of CDs maximizes profit?

19) A boat leaves a dock at 2:00 p.m., heading west at 15 km/h. Another boat heads south at 12 km/h and reaches the same dock at 3:00 p.m. When were the boats closest to each other?

Answers:

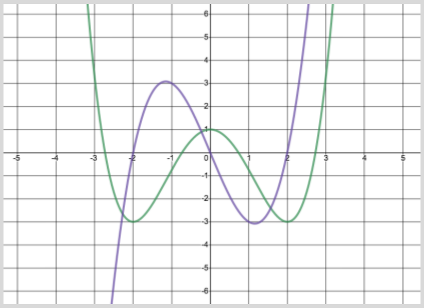
1)a) increasing: $(-\infty, 3)$
decreasing: $(3, \infty)$

b) increasing: $(-\infty, -4) \cup (4, \infty)$
decreasing: $(-4, 4)$

c) increasing: $(-3, 0) \cup (3, \infty)$
decreasing: $(-\infty, -3) \cup (0, 3)$

d) increasing: $(-\infty, \infty)$
decreasing: never

2) increasing: $(-2, 0) \cup (2, \infty)$
decreasing: $(-\infty, -2) \cup (0, 2)$



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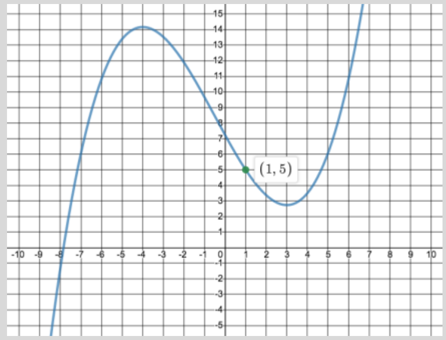

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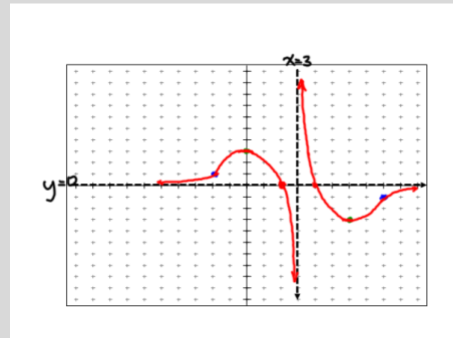


5)a) $(0, 16)$ is a local max **b)** local max at $(-2, 233)$; local min at $(1, -23)$

6)a) 40 km/h **b)** 52 km/h

7)a) absolute min: $(1.5, -0.25)$
absolute max: $(-4, 30)$

b) absolute min: $(-4, -29)$ and $(2, -29)$
absolute max: $(-2, 35)$

8) concave up: $x < -1$ and $x > 2$
concave down: $-1 < x < 2$

points of inflection: $(-1, -6)$ and $(2, -45)$

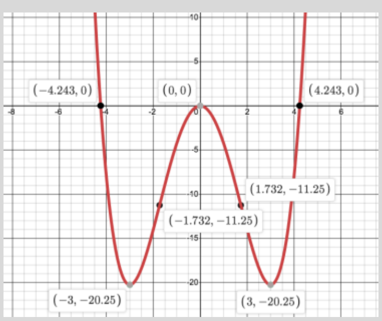
9) $(0, 0)$ is a point of inflection NOT a local min or max; $(1.5, 1.7)$ is a local max

10) concave up: $x < -2$, $x > 5$
concave down: $-2 < x < 5$

11)a) VA: $x = 0$; SA: $y = x$ **b)** VA: $x = 2$; HA: $y = 1$ **c)** VA: $x = 5$ and $x = -2$; HA: $y = 1$
d) VA: $x = -1$; HA: $y = 0$

12) $y = \frac{1}{2}x + \frac{1}{2}$

13)a)



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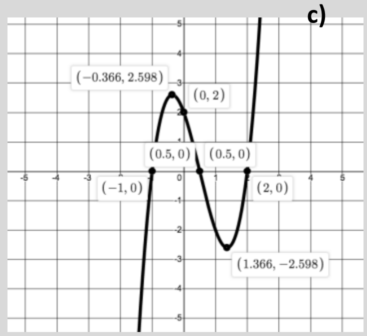
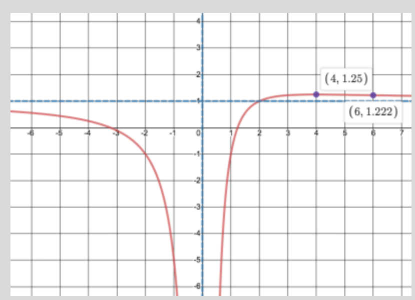


c)



14) a) twice – there are two turning points for the graph, so two points with $y' = 0$

b) i) $y' > 0,\ y'' > 0$ **ii)** $y' > 0,\ y'' < 0$ **iii)** $y' < 0,\ y'' < 0$

15) $r = h = 11.7$ cm

16) $V \cong 2.17$ m^3

17) 12.1 cm X 18.2 cm X 18.2 cm

18) approximately 133 CDs at a price of \$22.05

19) 2:23 pm