Unit 2 Pre-Test Review – Factor Theorem and Inequalities
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SOLUTIONS

Section 1: 2.1 - Long and Synthetic Division / Remainder Theorem

1) What is the remainder when $x^4 - 4x^2 - 2x + 3$ is divided by $x + 1$? Do not divide. Support your answer with an explanation.

$$P(-1) = (-1)^4 - 4(-1)^2 - 2(-1) + 3$$
$$= 1 - 4 + 2 + 3$$
$$= 2$$

Based on the remainder theorem, the remainder will be 2.

2) Is $x - 3$ a factor of the polynomial $3x^2 - 8x - 3$? Do not divide. Support your answer with an explanation.

$$P(3) = 3(3)^2 - 8(3) - 3$$
$$= 27 - 24 - 3$$
$$= 0$$

Yes because the remainder is 0.

3) Divide $\frac{f(x)}{g(x)}$ and state the answer in quotient form. Use synthetic division where possible.

a) $f(x) = x^4 - 4x^2 - 2x + 3, g(x) = x - 2$

2	1	0	-4	-2	3
	↓	2	4	0	-4
	1	2	0	-2	-1
x	x^3	x^2	x	#	R

$$\frac{x^4 - 4x^2 - 2x + 3}{x - 2} = x^3 + 2x^2 - 2 - \frac{1}{x - 2}$$

b) $f(x) = x^5 - x^4 + 2x^3 + 3x - 2,\ g(x) = x^2 + 2$

$$x^2 + 0x + 2 \overline{)\ x^5 - x^4 + 2x^3 + 0x^2 + 3x - 2}$$

$x^3 - 1x^2 + 0x + 2$

$x^5 + 0x^4 + 2x^3$ ↓

$-1x^4 + 0x^3 + 0x^2$

$-1x^4 + 0x^3 - 2x^2$

$0x^3 + 2x^2 + 3x$

$0x^3 + 0x^2 + 0x$

$2x^2 + 3x - 2$

$2x^2 + 0x + 4$

$R = 3x - 6$

$$\frac{x^5 - x^4 + 2x^3 + 3x - 2}{x^2 + 2} = x^3 - x^2 + 2 + \frac{3x - 6}{x^2 + 2}$$

4) Perform each division. Express the answer in quotient form and write the statement that could be used to check the division.

a) $x^3 + 9x^2 - 5x + 3$ divided by $x - 2$

$$\begin{array}{c|cccc} 2 & 1 & 9 & -5 & 3 \\ & \downarrow & 2 & 22 & 34 \\ \hline & 1 & 11 & 17 & 37 \\ & x^2 & x & \# & R \end{array}$$

Q.F.: $\dfrac{x^3+9x^2-5x+3}{x-2} = x^2+11x+17+\dfrac{37}{x-2}$

As product:

$x^3+9x^2-5x+3 = (x-2)(x^2+11x+17)+37$

b) $12x^3 - 2x^2 + x - 11$ divided by $3x + 1$

$$\begin{array}{r} 4x^2-2x+1 \\ 3x+1 \overline{)\,12x^3-2x^2+x-11} \\ 12x^3+4x^2 \\ \hline -6x^2+x \\ -6x^2-2x \\ \hline 3x-11 \\ 3x+1 \\ \hline R=-12 \end{array}$$

Q.F.: $\dfrac{12x^3-2x^2+x-11}{3x+1} = 4x^2-2x+1-\dfrac{12}{3x+1}$

Product:

$12x^3-2x^2+x-11 = (3x+1)(4x^2-2x+1)-12$

c) $-8x^4 - 4x + 10x^3 - x^2 + 15$ divided by $2x - 1$

$$\begin{array}{r} -4x^3+3x^2+1x-\frac{3}{2} \\ 2x-1 \overline{)\,-8x^4+10x^3-x^2-4x+15} \\ -8x^4+4x^3 \\ \hline 6x^3-1x^2 \\ 6x^3-3x^2 \\ \hline 2x^2-4x \\ 2x^2-1x \\ \hline -3x+15 \\ -3x+\frac{3}{2} \\ \hline R=\frac{27}{2} \end{array}$$

Q.F.: $\dfrac{-8x^4+10x^3-x^2-4x+15}{2x-1} = -4x^3+3x^2+x-\dfrac{3}{2}+\dfrac{27}{2(2x-1)}$

Product:

$-8x^4+10x^3-x^2-4x+15 = (2x-1)(-4x^3+3x^2+x-\dfrac{3}{2})+\dfrac{27}{2}$

d) $x^3 + 4x^2 - 3$ divided by $x - 2$

$$\begin{array}{c|cccc} 2 & 1 & 4 & 0 & -3 \\ & \downarrow & 2 & 12 & 24 \\ \hline & 1 & 6 & 12 & 21 \\ & x^2 & x & \# & R \end{array}$$

Q.F.: $\dfrac{x^3+4x^2-3}{x-2} = x^2+6x+12+\dfrac{21}{x-2}$

Product:

$x^3+4x^2-3 = (x-2)(x^2+6x+12)+21$

5) Determine the value of k such that when $f(x) = x^4 + kx^3 - 3x - 5$ is divided by $x - 3$, the remainder is -10.

$f(3) = (3)^4 + k(3)^3 - 3(3) - 5$

$-10 = 81 + 27k - 9 - 5$

$-10 = 27k + 67$

$-77 = 27k$

$k = \frac{-77}{27}$

Section 2: 2.2 – Factor Theorem

6) Suppose the cubic polynomial $8x^3 + mx^2 + nx - 6$ has both $2x + 3$ and $x - 1$ as factors. Find m and n. Do not divide.

$0 = 8\left(-\frac{3}{2}\right)^3 + m\left(-\frac{3}{2}\right)^2 + n\left(-\frac{3}{2}\right) - 6$

$0 = -27 + \frac{9}{4}m - \frac{3}{2}n - 6$

$33 = \frac{9}{4}m - \frac{3}{2}n$

① $132 = 9m - 6n$

$0 = 8(1)^3 + m(1)^2 + n(1) - 6$

$0 = 8 + m + n - 6$

② $-2 = m + n$

① $132 = 9m - 6n$

6x② $-12 = 6m + 6n$ +

$120 = 15m$

$m = 8$

sub $m=8$ into ②

$-2 = 8 + n$

$-10 = n$

7) Factor each of the following

a) $x^3 - 4x^2 + x + 6$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 6$

$f(2) = 0$ ∴ $x - 2$ is a factor

2	1	-4	1	6
	↓	2	-4	-6 +
x	1	-2	-3	0
	x^2	x	#	R

$x^3 - 4x^2 + x + 6 = (x-2)(x^2 - 2x - 3)$

$= (x-2)(x-3)(x+1)$

b) $3x^3 - 5x^2 - 26x - 8$

Possible factors: $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm 8, \pm \frac{4}{3}, \pm \frac{8}{3}$

$f(-2) = 0$ ∴ $x + 2$ is a factor

-2	3	-5	-26	-8
	↓	-6	22	8 +
x	3	-11	-4	0
	x^2	x	#	R

P: $\frac{-9}{1} = \frac{-12}{3}$, -12, $\frac{1}{3}$, -11, S

$3x^3 - 5x^2 - 26x - 8 = (x+2)(3x^2 - 11x - 4)$

$= (x+2)(x-4)(3x+1)$

c) $-4x^3 - 4x^2 + 16x + 16$

$= -4(x^3 + x^2 - 4x - 4)$

$= -4\left[x^2(x+1) - 4(x+1)\right]$

$= -4(x+1)(x^2-4)$

$= -4(x+1)(x-2)(x+2)$

DOC: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

d) $x^3 - 64 = x^3 - 4^3$

$= (x-4)(x^2+4x+4^2)$

$= (x-4)(x^2+4x+16)$

Section 3: 2.3&2.6 – Factoring to Solve Equations and Inequalities

8) Determine the real roots of each equation.

a) $(5x^2 + 20)(3x^2 - 48) = 0$

$5x^2 + 20 = 0$

$5x^2 = -20$

$x^2 = -4$

$x = \pm\sqrt{-4}$

∴ No solutions

$3x^2 - 48 = 0$

$3x^2 = 48$

$x^2 = 16$

$x = \pm\sqrt{16}$

$x_1 = 4$ $x_2 = -4$

b) $(2x^2 - x - 13)(x^2 + 1) = 0$

$x^2 + 1 = 0$

$x = \pm\sqrt{-1}$

∴ No solutions

NOT Factorable so use QF

$x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(2)(-13)}}{2(2)}$

$x = \dfrac{1 \pm \sqrt{105}}{4}$

$x_1 = \dfrac{1 + \sqrt{105}}{4}$ $x_2 = \dfrac{1 - \sqrt{105}}{4}$

9) Solve the following polynomial equations.

a) $2x^3 + 1 = x^2 + 2x$

$2x^3 - x^2 - 2x + 1 = 0$

$x^2(2x-1) - 1(2x-1) = 0$

$(2x-1)(x^2-1) = 0$

$(2x-1)(x-1)(x+1) = 0$

$x_1 = \frac{1}{2}$ $x_2 = 1$ $x_3 = -1$

b) $x^3 + 6x^2 + 11x + 6 = 0$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 6$

$f(-1) = 0$, so $x+1$ is a factor

-1	1	6	11	6
	↓	-1	-5	-6
	1	5	6	0
	x^2	x	#	R

$(x+1)(x^2+5x+6) = 0$

$(x+1)(x+2)(x+3) = 0$

$x_1 = -1$ $x_2 = -2$ $x_3 = -3$

c) $x^5 - 4x^3 - x^2 + 4 = 0$

$x^3(x^2-4) - 1(x^2-4) = 0$

$(x^2-4)(x^3-1) = 0$

$(x-2)(x+2)(x-1)(x^2+1x+1) = 0$

$x_1 = 2$ $x_2 = -2$ $x_3 = 1$

No solutions

$b^2 - 4ac = -3$

d) $3x^3 + 2x^2 - 11x - 10 = 0$

Possible factors: $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 5, \pm \frac{5}{3}, \pm 10, \pm \frac{10}{3}$

$f(-1) = 0$, so $x+1$ is a factor

-1	3	2	-11	-10
	↓	-3	1	10
	3	-1	-10	0
	x^2	x	#	R

P: -30

S: -1

$\frac{-2}{1}$ $\frac{-6}{3}$ $\frac{5}{3}$

$(x+1)(3x^2 - x - 10) = 0$

$(x+1)(x-2)(3x+5) = 0$

$x_1 = -1$ $x_2 = 2$ $x_3 = \frac{-5}{3}$

10) Solve the following polynomial inequalities. (Refer to #9 where you factored the polynomials)

a) $2x^3 + 1 < x^2 + 2x$

$2x^3 - x^2 - 2x + 1 < 0$

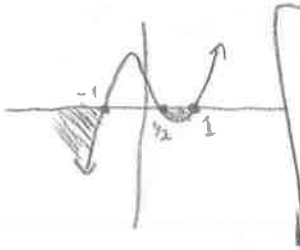
$(2x-1)(x-1)(x+1) < 0$

x-int at $x = -1, \frac{1}{2}, 1$

+ L.C., odd degree

Q3 → Q1

	$-\infty$	-1		as	1	∞
	-2	0	0.75		2	
$2x-1$	$-$	$-$	$+$		$+$	
$x-1$	$-$	$-$	$-$		$+$	
$x+1$	$-$	$+$	$+$		$+$	
overall	$\ominus$	$+$	$\ominus$		$+$	



Solution:

when $x < -1$ or $0.5 < x < 1$

OR

when $x \in (-\infty, -1) \cup (0.5, 1)$

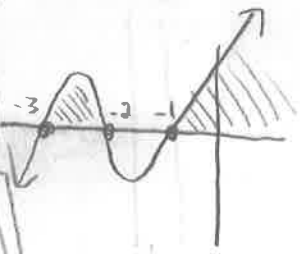
b) $x^3 + 6x^2 + 11x + 6 > 0$

$(x+1)(x+2)(x+3) > 0$

x-int at $x = -3, -2, -1$

+ L.C., odd degree

Q3 → Q1



	$-\infty$	-3		-2		-1	∞
	-4	-2.5		-1.5		0	
$x+1$	$-$	$-$		$-$		$+$	
$x+2$	$-$	$-$		$+$		$+$	
$x+3$	$-$	$+$		$+$		$+$	
overall	$-$	$\oplus$		$-$		$\oplus$	

SOLUTION:

when $-3 < x < -2$ OR $x > -1$

when $x \in (-3, -2) \cup (-1, \infty)$

11) Where is the polynomial $y = 8x^3 + 1$ positive? Justify your solution.

SOC

$8x^3 + 1 > 0$

$(2x+1)\left[(2x)^2 - (2x)(1) + (1)^2\right] > 0$

$(2x+1)(4x^2 - 2x + 1) > 0$

$x = -\frac{1}{2}$

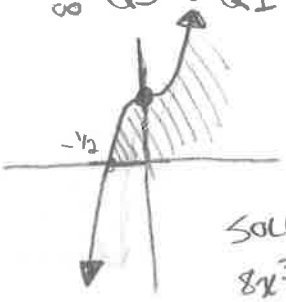
check $b^2 - 4ac = (-2)^2 - 4(4)(1)$

$= -12$

∴ NO ROOTS

Positive L.C. and odd degree.

∴ Q3 → Q1.



SOLUTION:

$8x^3 + 1 > 0$ when $x > -\frac{1}{2}$

when $x \in (-\frac{1}{2}, \infty)$

12) Solve $6x^3 + 13x^2 - 41x + 12 \leq 0$ using a sign chart.

Possible zeros: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 2, \pm\frac{2}{3}, \pm 3, \pm\frac{3}{2}, \pm 4, \pm\frac{4}{3}, \pm 6, \pm 12$

$f(-4) = 0$; ∴ $x+4$ is a factor

-4	6	13	-41	12
	↓	-24	44	-12 +
x	6	-11	3	0
	x^2	x	#	R

P

18

$\frac{-3}{2} = \frac{-9}{6}$ $\quad$ $\frac{-2}{6} = \frac{-1}{3}$

-11

S

$(x+4)(6x^2 - 11x + 3) \leq 0$

$(x+4)(2x-3)(3x-1) \leq 0$

x-int at $x = -4, \frac{1}{3}, \frac{3}{2}$

	$-\infty$	-4		0.33 $1/3$		1.5 $3/2$	∞
	-5		0		1		2
$x+4$	$-$		$+$		$+$		$+$
$2x-3$	$-$		$-$		$-$		$+$
$3x-1$	$-$		$-$		$+$		$+$
overall	$\ominus$		$+$		$\ominus$		$+$

SOLUTION:

when $x \leq -4$ or $\frac{1}{3} \leq x \leq \frac{3}{2}$

when $x \in (-\infty, -4] \cup [\frac{1}{3}, \frac{3}{2}]$

Section 4: 2.4 – Families of Polynomials

13) Find the equation for the family of quartic polynomials that have real roots of 3 (order 2) and $2 \pm \sqrt{2}$.

factors:

$x = 3$

$x - 3 = 0$

$(x-3)$ order 2

$x = 2 \pm \sqrt{2}$

$x - 2 = \pm\sqrt{2}$

$(x-2)^2 = 2$

$x^2 - 4x + 4 = 2$

$x^2 - 4x + 2 = 0$

$(x^2 - 4x + 2)$

Equation for family:

$y = k(x-3)^2(x^2 - 4x + 2)$

14) A family of cubic polynomials has roots of -2, -3 and -5. Find the member of this family that passes through the point (2,-35). What is this polynomials *y*-intercept?

$f(x) = k(x+2)(x+3)(x+5)$

$-35 = k(2+2)(2+3)(2+5)$

$-35 = k(4)(5)(7)$

$-35 = 140k$

$k = -\frac{1}{4}$

$f(0) = -\frac{1}{4}(0+2)(0+3)(0+5)$

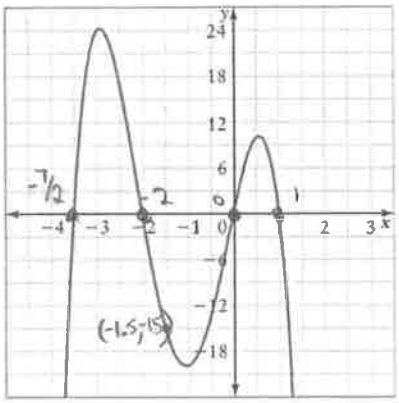
$f(0) = -\frac{1}{4}(30)$

$f(0) = -\frac{15}{2}$

The equation is $f(x) = -\frac{1}{4}(x+2)(x+3)(x+5)$.
It has a y-intercept of $(0, -\frac{15}{2})$

15) Find an equation for each of the following functions

a)



$f(x) = k(2x+7)(x+2)(x)(x-1)$

$-15 = k[2(-1.5)+7](-1.5+2)(-1.5)(-1.5-1)$

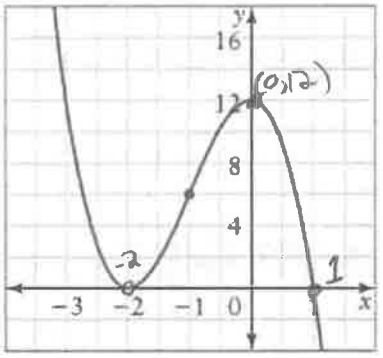
$-15 = k(4)(0.5)(-1.5)(-2.5)$

$-15 = 7.5k$

$k = -2$

$f(x) = -2(x)(2x+7)(x+2)(x-1)$

b)



$f(x) = k(x+2)^2(x-1)$

$12 = k(0+2)^2(0-1)$

$12 = k(4)(-1)$

$12 = -4k$

$k = -3$

$f(x) = -3(x+2)^2(x-1)$

ANSWER KEY

1) $P(-1) = 2$ =remainder . This was found using remainder theorem.

2) $P(3) = 0$, so $x-3$ is a factor because remainder is 0 (Factor Theorem)

3)a) $\frac{x^4-4x^2-2x+3}{x-2} = x^3+2x^2-2-\frac{1}{x-2}$ **b)** $\frac{x^5-x^4+2x^3+3x-2}{x^2+2} = x^3-x^2+2+\frac{3x-6}{x^2+2}$

4)a) $\frac{x^3+9x^2-5x+3}{x-2} = x^2+11x+17+\frac{37}{x-2}$; $x^3+9x^2-5x+3 = (x-2)(x^2+11x+17)+37$

b) $\frac{12x^3-2x^2+x-11}{3x+1} = 4x^2-2x+1-\frac{12}{3x+1}$; $12x^3-2x^2+x-11 = (3x+1)(4x^2-2x+1)-12$

c) $\frac{-8x^4-4x+10x^3-x^2+15}{2x-1} = -4x^3+3x^2+x-\frac{3}{2}+\frac{27}{2(2x-1)}$; $-8x^4-4x+10x^3-x^2+15 = (2x-1)\left(-4x^3+3x^2+x-\frac{3}{2}\right)+\frac{27}{2}$

d) $\frac{x^3+4x^2-3}{x-2} = x^2+6x+12+\frac{21}{x-2}$; $x^3+4x^2-3 = (x-2)(x^2+6x+12)+21$

5) $k = -\frac{77}{27}$

6) $m = 8$, $n = -10$

7)a) $(x+1)(x-3)(x-2)$ **b)** $(x+2)(3x+1)(x-4)$ **c)** $-4(x+1)(x+2)(x-2)$ **d)** $(x-4)(x^2+4x+16)$

8)a) (-4, 0) and (4, 0) **b)** $\left(\frac{1-\sqrt{105}}{4},0\right)$ and $\left(\frac{1+\sqrt{105}}{4},0\right)$

9) a) $x = -1, 1, \frac{1}{2}$ **b)** $x = -1, -2, -3$ **c)** $x = 1, -2, 2$ **d)** $x = -1, -\frac{5}{3}, 2$

10)a) $x \in (-\infty,-1) \cup (0.5, 1)$ **b)** $x \in (-3,-2) \cup (-1,\infty)$

11) $x \in \left(-\frac{1}{2},\infty\right)$

12) $x \in (-\infty,-4] \cup \left[\frac{1}{3},\frac{3}{2}\right]$

13) $P(x) = k(x-3)^2(x^2-4x+2)$

14) $f(x) = -\frac{1}{4}(x+2)(x+3)(x+5)$, y-int is $\left(0,-\frac{15}{2}\right)$

15)a) $P(x) = -2x(x-1)(x+2)(2x+7)$ **b)** $P(x) = -3(x+2)^2(x-1)$