Unit 3 Pretest – Derivatives of Trig and Exponential Functions

MCV4U

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1) Find the derivatives of each of the following functions

a) $y = \cos x$ **b)** $f(x) = -2\sin x$ **c)** $y = \cos x - \sin x$

2) Determine the slope of the function $y = 4\sin x$ at $x = \frac{\pi}{3}$.

3) Find the equation of the line that is tangent to the curve $y = 2\sin\theta + 4\cos\theta$ at $\theta = \frac{\pi}{4}$.

4) Differentiate each of the following:

a) $y = -\cos^2 x$

b) $y = \sin(2\theta) - 2\cos(2\theta)$

c) $f(\theta) = -\frac{\pi}{2}\sin(2\theta - \pi)$

d) $f(x) = \sin(\sin x)$

e) $f(x) = \cos(\cos x)$

f) $f(\theta) = \cos(7\theta) - \cos(5\theta)$

g) $y = 3x \sin x$

h) $f(t) = 2t^2 \cos(2t)$

i) $y = \pi t \sin(\pi t - 6)$

j) $y = \cos(\sin\theta) + \sin(\cos\theta)$

k) $f(x) = \cos^2(\sin x)$

l) $f(\theta) = \cos(7\theta) - \cos^2(5\theta)$

m) $y = 3\sin(2x) - 4\cos(2x)$

n) $y = \tan(3x)$

o) $y = \frac{1}{2-\cos x}$

p) $y = x\tan(2x)$

q) $y = [\sin(2x)]e^{3x}$

r) $y = \cos^2(2x)$

5) Find an equation of a line that is tangent to the curve $f(x) = 2\cos(3x)$ and whose slope is a maximum.

6) A voltage of the power supply in an AC-DC coupled circuit is given by the function $V(t) = 130\sin(5t) + 18$, where t is time, in seconds, and V is the voltage, in volts, at time t.

a) Find the maximum and minimum voltages and the times at which they occur.
b) Determine the period, T, in seconds, the frequency, f, in hertz, and the amplitude, A, in volts, for this signal.

7) The movement of an engine piston can be modelled by $h = 4\sin t$ where h is the height of the piston, in cm, above the neutral position and t is time, in seconds.

a) Determine the velocity of the piston when $t = 5$.
b) Determine the acceleration of the piston when $t = 5$

8) A simple pendulum has a length of 20 cm and a max horizontal displacement of 8 cm.

a) Determine a function that gives the horizontal position of the bob as a function of time.
b) Determine a function that gives the velocity of the bob as a function of time.
c) Find the max velocity of the bob. When does it occur?

9) Differentiate each function with respect to x.

a) $f(x) = \left(\frac{1}{2}\right)^x$

b) $g(x) = -2e^x$

c) $y = 5^x$

d) $y = 5(2)^x$

e) $y = (52)^{2x}$

f) $y = -2(10)^{3x}$

g) $y = e^{3x^2-2x+1}$

h) $y = (x-1)e^{2x}$

i) $y = 3x + e^{-x}$

j) $y = e^x \cos(2x)$

k) $g(x) = \left(\frac{1}{3}\right)^{4x} - 2e^{\sin x}$

10) Find the equation of the line that is tangent to the curve $y = 2(3)^x$ at $x = 1$.

11) Find the equation of the line that is tangent to the curve $y = -3e^x$ at $x = \ln 2$.

12) Find the equation of the tangent to $y = x \ln x$ that is perpendicular to $x + 3y - 9 = 0$.

13) Find the equation of the tangent to $y = 2\sin(\pi x)$ when $x = \frac{1}{2}$.

14) Find all local extrema for $f(x) = x^2e^{2x}$

15) Find all local extrema for $y = \frac{1}{2}x(2)^{3x+1}$

16) Continuous growth or decay follows the formula $A = ce^{kt}$, where c is the initial amount, and k is a rate factor. The mass of a radioactive substance is 1000 g on day 1, and only 100 g after 100 days. Find…

a) k
b) the half-life
c) the amount that remains after 300 days
d) the rate of decay after 50 days

17) Find $\frac{dy}{dx}$ by implicit differentiation

a) $x^2 + y^2 = 36$

b) $x^3 - xy + y^2 = 4$

c) $2 \sin x \cos y = 1$

d) $(4 - x)y^2 = x^3$

18) Use implicit differentiation to find an equation of the tangent line to the ellipse $\frac{x^2}{2} + \frac{y^2}{8} = 1$ at $(1,2)$

19) Find the slope of the tangent at $(2,1)$ for the curve $y = 2^{x-2y}$

20) Differentiate with respect to x

a) $y = \ln x$
b) $y = \log_3 x$
c) $y = 4\log_5(2x - 1)$

Answers:

1)a) $\frac{dy}{dx} = -\sin x$ **b)** $f'(x) = -2\cos x$ **c)** $y' = -\sin x - \cos x$

2) 2

3) $y = -\sqrt{2}\theta + \frac{\sqrt{2}}{4}\pi + 3\sqrt{2}$

4)a) $\frac{dy}{dx} = \sin(2x)$ **b)** $y' = 2\cos(2\theta) + 4\sin(2\theta)$ **c)** $f'(\theta) = -\pi\cos(2\theta - \pi)$ **d)** $f'(x) = [\cos(\sin x)](\cos x)$ **e)** $f'(x) = [\sin(\cos x)](\sin x)$
f) $f'(\theta) = -7\sin(7\theta) + 5\sin(5\theta)$ **g)** $y' = 3x\cos x + 3\sin x$ **h)** $f'(t) = -4t^2\sin(2t) + 4t\cos(2t)$
i) $y' = \pi^2 t[\cos(\pi t - 6)] + \pi\sin(\pi t - 6)$ **j)** $y' = -\sin(\sin\theta)(\cos\theta) - \cos(\cos\theta)(\sin\theta)$ **k)** $f'(x) = -2\cos(\sin x)[\sin(\sin x)](\cos x)$
l) $f'(\theta) = -7\sin(7\theta) + 10\cos(5\theta)[\sin(5\theta)]$ **m)** $y' = 6\cos(2x) + 8\sin(2x)$ **n)** $3\sec^2(3x)$ **o)** $y' = -\frac{\sin x}{(2-\cos x)^2}$
p) $y' = 2x\sec^2(2x) + \tan(2x)$ **q)** $y' = e^{3x}[3\sin(2x) + 2\cos(2x)]$ **r)** $y' = -4\cos(2x)\sin(2x)$

5) $y = 6x - 3\pi$ (answers may vary)

6)a) max voltage: 148 V at time $t = \frac{(4k+1)\pi}{10}, k \in \mathbb{Z}, k \geq 0$
min voltage: -112 V at time $t = \frac{(4k+3)\pi}{10}, k \in \mathbb{Z}, k \geq 0$

b) period: $T = \frac{2\pi}{5}$ seconds; frequency: $f = \frac{5}{2\pi}$ Hz; amplitude: $A = 130$ V

7)a) 1.13 cm/s **b)** 3.8 cm/s^2

8)a) $h(t) = 8\cos(2.2\pi t)$ **b)** $v(t) = -17.6\pi\sin(2.2\pi t)$ **c)** max velocity of 55.3 cm/s when $t = \frac{3+4k}{4}, k \in \mathbb{Z}, k \geq 0$

9)a) $f'(x) = \left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right)$ **b)** $g'(x) = -2e^x$ **c)** $y' = 5^x\ln(5)$ **d)** $y' = 5(2)^x\ln(2)$ **e)** $y' = 2(52)^{2x}\ln(52)$
f) $y = -6(10)^{3x}\ln(10)$ **g)** $y' = (e^{3x^2-2x+1})(6x-2)$ **h)** $f'(x) = e^{2x}(2x-1)$ **i)** $y' = 3 - e^{-x}$ **j)** $y' = e^x[-2\sin(2x) + \cos(2x)]$
k) $g'(x) = [4\ln\left(\frac{1}{3}\right)]\left(\frac{1}{3}\right)^{4x} - (2e^{\sin x})(\cos x)$

10) $y = 6x\ln(3) - 6\ln(3) + 6$

11) $y = -6x + 6\ln(2) - 6$

12) $y = 3x - e^2$

13) $y = 2$

14) $y' = 2xe^{2x}(1+x)$; min at (0, 0), max at $\left(-1, \frac{1}{e^2}\right)$

15) $(-0.48, -0.18)$ is a local minimum

16)a) $k \sim -0.023$ **b)** $t \sim 30$ days **c)** $A(300) \cong 1g$ **d)** $A'(50) \cong -7.3$ g/day

17)a) $y' = -\frac{x}{y}$ **b)** $y' = \frac{y-3x^2}{2y-x}$ **c)** $y' = \frac{\cos(x)\cos(y)}{\sin(x)\sin(y)}$ **d)** $y' = \frac{-y^2-3x^2}{(2x-8)y}$

18) $y = -2x + 4$

19) $\frac{\ln(2)}{2\ln(2)+1}$

20)a) $y' = \frac{1}{x}$ **b)** $y' = \frac{1}{x\ln 3}$ **c)** $y' = \frac{8}{(2x-1)\ln 5}$