Unit 3 Pre-test Solutions - Derivatives of Trig & Expo. Functions

1 a) $y = \cos(x)$

$y' = -\sin(x)$

b) $f(x) = -2\sin(x)$

$f'(x) = -2\cos(x)$

c) $y = \cos(x) - \sin(x)$

$y' = -\sin(x) - \cos(x)$

2) $y = 4\sin(x)$

$y' = 4\cos(x)$

$y'(\frac{\pi}{3}) = 4\cos(\frac{\pi}{3})$

$y'(\frac{\pi}{3}) = 4(\frac{1}{2})$

$y'(\frac{\pi}{3}) = 2$

3) Point:

$y(\frac{\pi}{4}) = 2\sin(\frac{\pi}{4}) + 4\cos(\frac{\pi}{4})$

$y(\frac{\pi}{4}) = 2(\frac{\sqrt{2}}{2}) + 4(\frac{\sqrt{2}}{2})$

$y(\frac{\pi}{4}) = \sqrt{2} + 2\sqrt{2}$

$y(\frac{\pi}{4}) = 3\sqrt{2}$

$(\frac{\pi}{4}, 3\sqrt{2})$

Slope:

$y' = 2\cos\theta - 4\sin\theta$

$y'(\frac{\pi}{4}) = 2\cos(\frac{\pi}{4}) - 4\sin(\frac{\pi}{4})$

$y'(\frac{\pi}{4}) = 2(\frac{\sqrt{2}}{2}) - 4(\frac{\sqrt{2}}{2})$

$y'(\frac{\pi}{4}) = \sqrt{2} - 2\sqrt{2}$

$y'(\frac{\pi}{4}) = -\sqrt{2}$

$m = -\sqrt{2}$

Eq^n:

$y = m\theta + b$

$3\sqrt{2} = -\sqrt{2}(\frac{\pi}{4}) + b$

$3\sqrt{2} + \frac{\pi}{4}\sqrt{2} = b$

$\boxed{y = -\sqrt{2}\,\theta + 3\sqrt{2} + \frac{\pi}{4}\sqrt{2}}$

4 a) $y = -\cos^2 x$

$y = -(\cos x)^2$

$y' = -2\cos x(-\sin x)$

$y' = 2\sin x\cos x$

$y' = \sin(2x)$

b) $y = \sin(2\theta) - 2\cos(2\theta)$

$y' = [\cos(2\theta)](2) - 2[-\sin(2\theta)](2)$

$y' = 2\cos(2\theta) + 4\sin(2\theta)$

c) $f(\theta) = -\frac{\pi}{2}\sin(2\theta - \pi)$

$f'(\theta) = -\frac{\pi}{2}[\cos(2\theta - \pi)](2)$

$f'(\theta) = -\pi\cos(2\theta - \pi)$

d) $f(x) = \sin(\sin x)$

$f'(x) = [\cos(\sin x)](\cos x)$

e) $f(x) = \cos(\cos x)$

$f'(x) = [-\sin(\cos x)](-\sin x)$

$f'(x) = [\sin(\cos x)](\sin x)$

f) $f(\theta) = \cos(7\theta) - \cos(5\theta)$

$f'(\theta) = [-\sin(7\theta)](7) - [-\sin(5\theta)](5)$

$f'(\theta) = -7\sin(7\theta) + 5\sin(5\theta)$

g) $y = 3x(\sin x)$

$y' = 3\sin x + \cos x(3x)$

$y' = 3\sin x + 3x\cos x$

h) $f(t) = 2t^2\cos(2t)$

$f'(t) = 4t\cos(2t) + [-\sin(2t)](2)(2t^2)$

$f'(t) = 4t\cos(2t) - 4t^2\sin(2t)$

i) $y = \pi t\sin(\pi t - 6)$

$y' = \pi\sin(\pi t - 6) + [\cos(\pi t - 6)](\pi)(\pi t)$

$y' = \pi\sin(\pi t - 6) + \pi^2 t\cos(\pi t - 6)$

j) $y = \cos(\sin\theta) + \sin(\cos\theta)$

$y' = [-\sin(\sin\theta)](\cos\theta) + [\cos(\cos\theta)](-\sin\theta)$

$y' = [-\sin(\sin\theta)]\cos\theta - [\cos(\cos\theta)]\sin\theta$

k) $f(x) = [\cos(\sin x)]^2$

$f'(x) = 2[\cos(\sin x)][-\sin(\sin x)](\cos x)$

$f'(x) = -2\cos x\,[\cos(\sin x)][\sin(\sin x)]$

l) $f(\theta) = \cos(7\theta) - \cos^2(5\theta)$

$f'(\theta) = [-\sin(7\theta)](7) - 2\cos(5\theta)[-\sin(5\theta)](5)$

$f'(\theta) = -7\sin(7\theta) + 10\cos(5\theta)\sin(5\theta)$

m) $y = 3\sin(2x) - 4\cos(2x)$

$y' = 3[\cos(2x)](2) - 4[-\sin(2x)](2)$

$y' = 6\cos(2x) + 8\sin(2x)$

n) $y = \tan(3x)$

$y' = [\sec^2(3x)](3)$

$y' = 3\sec^2(3x)$

o) $y = (2-\cos x)^{-1}$

$y' = -1(2-\cos x)^{-2}(\sin x)$

$y' = \dfrac{-\sin x}{(2-\cos x)^2}$

p) $y = x\tan(2x)$

$y' = 1\tan(2x) + [\sec^2(2x)](2)(x)$

$y' = \tan(2x) + 2x\sec^2(2x)$

q) $y = [\sin(2x)]e^{3x}$

$y' = 2\cos(2x)e^{3x} + 3e^{3x}[\sin(2x)]$

$y' = e^{3x}[2\cos(2x) + 3\sin(2x)]$

r) $y = \cos^2(2x) = [\cos(2x)]^2$

$y' = 2[\cos(2x)](-\sin(2x))(2)$

$y' = -4\cos(2x)\sin(2x)$

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$F(x) = 2\cos(3x)$

$F'(x) = 2[-\sin(3x)](3)$

$F'(x) = -6\sin(3x)$

max slope is 6. when?

$6 = -6\sin(3x)$

$-1 = \sin(3x)$

$3x = \frac{3\pi}{2}$

$x = \frac{\pi}{2}$

Point:

$F(\frac{\pi}{2}) = 2\cos(\frac{3\pi}{2})$

$F(\frac{\pi}{2}) = 0$

$(\frac{\pi}{2}, 0)$

Eq^n:

$y = mx + b$

$0 = (6)(\frac{\pi}{2}) + b$

$b = -3\pi$

$$\boxed{y = 6x - 3\pi}$$

6) $V(t) = 130\sin(5t) + 18$

$Max = 18 + 130 = 148$

$148 = 130\sin(5t) + 18$

$1 = \sin(5t)$

$5t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

$t = \frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}$

Max of 148 V when $t = \frac{\pi + 4\pi k}{10}, k \in \mathbb{Z}, k \geq 0$

$Min = 18 - 130 = -112\,V$

$-112 = 130\sin(5t) + 18$

$-1 = \sin(5t)$

$5t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}, \ldots$

$t = \frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}, \frac{15\pi}{10}, \ldots$

Min of -112 V when $t = \frac{3\pi + 4\pi k}{10}, k \in \mathbb{Z}, k \geq 0$

7) $h(t) = 4\sin(t)$

$v(t) = h'(t) = 4\cos(t)$

$v(5) = 4\cos(5)$

$v(5) \simeq 1.13\ cm/s$

$a(t) = v'(t) = -4\sin(t)$

$a(5) = -4\sin(5)$

$a(5) \simeq 3.84\ cm/s^2$

8) a) $T = 2\pi\sqrt{\frac{0.2}{9.8}}$

$h(t) = A\cos\left(\frac{2\pi t}{T}\right)$

$h(t) = 8\cos\left[\frac{\cancel{2\pi} t}{\cancel{2\pi}\sqrt{\frac{0.2}{9.8}}}\right]$

$h(t) = 8\cos(7t)$

b)

$h'(t) = 8[-\sin(7t)](7)$

$h'(t) = -56\sin(7t)$

c) $max = 0 + 56 = 56\ cm/s$

$56 = -56\sin(7t)$

$-1 = \sin(7t)$

$7t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$

$t = \frac{3\pi}{14}, \frac{7\pi}{14}, \frac{11\pi}{14}, \ldots$

$\boxed{t = \frac{3\pi + 4\pi k}{14}\ sec;\ k \in \mathbb{Z}, k \geq 0}$

(9) a) $f(x) = \left(\frac{1}{2}\right)^x$

$f'(x) = \left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right)$

b) $g(x) = -2e^x$

$g'(x) = -2e^x$

c) $y = 5^x$

$y' = 5^x \ln(5)$

d) $y = 5(2)^x$

$y' = 5(2)^x \ln(2)$

e) $y = (52)^{2x}$

$y' = (52)^{2x}[\ln(52)](2)$

$y' = 2(52)^{2x} \ln(52)$

f) $y = -2(10)^{3x}$

$y' = -2(10)^{3x}[\ln(10)](3)$

$y' = -6(10)^{3x} \ln(10)$

g) $y = e^{3x^2-2x+1}$

$y' = e^{3x^2-2x+1}(6x-2)$

h) $y = (x-1)e^{2x}$

$y' = 1e^{2x} + 2e^{2x}(x-1)$

$y' = e^{2x}[1 + 2x - 2]$

$y' = e^{2x}(2x-1)$

i) $y = 3x + e^{-x}$

$y' = 3 + e^{-x}(-1)$

$y' = 3 - e^{-x}$

j) $y = e^x \cos(2x)$

$y' = e^x \cos(2x) + [-\sin(2x)](2)e^x$

$y' = e^x[\cos(2x) - 2\sin(2x)]$

k) $g(x) = \left(\frac{1}{3}\right)^{4x} - 2e^{\sin x}$

$g'(x) = \left(\frac{1}{3}\right)^{4x}\left[\ln\left(\frac{1}{3}\right)\right](4) - 2e^{\sin x}(\cos x)$

$g'(x) = 4\ln\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{4x} - 2e^{\sin x}(\cos x)$

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Point:

$y(1) = 2(3)^1$

$y(1) = 6$

Slope:

$y' = 2(3)^x \ln(3)$

$y'(1) = 2(3)^1 \ln(3)$

$y'(1) = 6\ln(3)$

$m = 6\ln(3)$

Eq^n:

$y = mx + b$

$6 = 6\ln(3)(1) + b$

$b = 6 - 6\ln(3)$

$\boxed{y = 6\ln(3)x + 6 - 6\ln(3)}$

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Point

$y(\ln 2) = -3e^{\ln(2)}$

$= -3(2)$

$= -6$

Slope

$y' = -3e^x$

$y'(\ln 2) = -6$

$m = -6$

Eq^n:

$y = mx + b$

$-6 = -6[\ln(2)] + b$

$b = -6 + 6\ln(2)$

$\boxed{y = -6x - 6 + 6\ln(2)}$

(12) Slope:

$x+3y-9=0$

$3y=-x+9$

$y=\frac{-1}{3}x+3$

$\perp m=3$

when is tangent slope 3?

$y=x\ln(x)$

$y'=1\ln(x)+\frac{1}{x}(x)$

$y'=\ln(x)+1$

$3=\ln(x)+1$

$2=\ln(x)$

$x=e^2$

Point:

$y(e^2)=e^2\ln(e^2)$

$=e^2(2)$

$=2e^2$

$(e^2,2e^2)$

Eq^n:

$y=mx+b$

$2e^2=3e^2+b$

$b=-e^2$

$\boxed{y=3x-e^2}$

(13) Slope:

$y'=2[\cos(\pi x)](\pi)$

$y'=2\pi\cos(\pi x)$

$y'(\frac{1}{2})=2\pi\cos(\frac{\pi}{2})$

$y'(\frac{1}{2})=0$

$m=0$

Point:

$y(\frac{1}{2})=2\sin(\frac{\pi}{2})$

$y(\frac{1}{2})=2$

Eq^n:

$y=mx+b$

$2=0(\frac{1}{2})+b$

$b=2$

$\boxed{y=2}$

(14) $f(x) = x^2 e^{2x}$

$f'(x) = 2x(e)^{2x} + 2e^{2x}(x^2)$

$f'(x) = x(e)^{2x}(2+2x)$

$x_1 = 0 \qquad x_2 = -1$

$f(0) = 0 \qquad f(-1) = e^{-2}$

	$-\infty$ to -1	-1 to 0	0 to ∞
	-2	-0.5	1
$f'(x)$	+	−	+
$f(x)$	↗	↘	↗

MAX (at -1) MIN (at 0)

$$\boxed{\begin{array}{l}\text{Max at } (-1, e^{-2}) \\ \text{Min at } (0,0)\end{array}}$$

(15) $y = \frac{1}{2}x(2)^{3x+1}$

$y' = \frac{1}{2}(2)^{3x+1} + (2)^{3x+1}[\ln(2)](3)(\frac{1}{2}x)$

$0 = \frac{1}{2}(2)^{3x+1}[1 + 3x\ln(2)]$

$0 = 1 + 3x\ln(2)$

$x = \frac{-1}{3\ln(2)}$

$x \simeq -0.48$

$y(-0.48) \simeq -0.18$

	$-\infty$ to -0.48	-0.48 to ∞
	-1	0
y'	−	+
y	↘	↗

Min (at -0.48)

$$\boxed{\text{Min at } (-0.48, -0.18)}$$

16) a) $100 = 1000e^{100k}$

$0.1 = e^{100k}$

$\ln(0.1) = 100k$

$k = \frac{\ln(0.1)}{100}$

$k \approx -0.023$

b) $500 = 1000e^{-0.023t}$

$0.5 = e^{-0.023t}$

$\ln(0.5) = -0.023t$

$t \approx 30.1$ days

c) $A(300) = 1000e^{-0.023(300)}$

$A(300) \approx 1.01$ grams

d) $A(t) = 1000e^{-0.023t}$

$A'(t) = 1000e^{-0.023t}(-0.023)$

$A'(t) = -23e^{-0.023t}$

$A'(50) = -23e^{-0.023(50)}$

$A'(50) \approx -7.3$ g/day.

(17) a) $x^2+y^2=36$

$2x+2y\left(\frac{dy}{dx}\right)=0$

$\frac{dy}{dx}=\frac{-2x}{2y}$

$\frac{dy}{dx}=\frac{-x}{y}$

b) $x^3-xy+y^2=4$

$3x^2-\left[1y+\frac{dy}{dx}(x)\right]+2y\left(\frac{dy}{dx}\right)=0$

$3x^2-y-x\left(\frac{dy}{dx}\right)+2y\left(\frac{dy}{dx}\right)=0$

$-x\left(\frac{dy}{dx}\right)+2y\left(\frac{dy}{dx}\right)=y-3x^2$

$\frac{dy}{dx}(-x+2y)=y-3x^2$

$\frac{dy}{dx}=\frac{y-3x^2}{-x+2y}$

c) $2\sin x\cos y=1$

$2\cos x\cos y+(-\sin y)\frac{dy}{dx}(2)\sin x=0$

$2\cos x\cos y=2\sin x\sin y\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx}=\frac{\cos x\cos y}{\sin x\sin y}$

d) $(4-x)y^2=x^3$

$-1y^2+2y\left(\frac{dy}{dx}\right)(4-x)=3x^2$

$\frac{dy}{dx}(2y)(4-x)=3x^2+y^2$

$\frac{dy}{dx}=\frac{3x^2+y^2}{2y(4-x)}$

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Slope:

$$\frac{1}{2}x^2 + \frac{1}{8}y^2 = 1$$

$$x + \frac{1}{4}y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{y}{4}\left(\frac{dy}{dx}\right) = -x$$

$$\frac{dy}{dx} = \frac{-4x}{y}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=2}} = \frac{-4(1)}{2}$$

$$= -2$$

$$m = -2$$

Eq^n

$$y = mx + b$$

$$2 = -2(1) + b$$

$$b = 4$$

$$\boxed{y = -2x + 4}$$

(19) $y = 2^{x-2y}$

$$y' = 2^{x-2y}(\ln 2)(1 - 2y')$$

$$y' = 2^{x-2y}(\ln 2) - 2^{x-2y}(\ln 2)(2y')$$

$$y' + 2y'(2)^{x-2y}(\ln 2) = 2^{x-2y}(\ln 2)$$

$$y'\left[1 + 2(2)^{x-2y}\ln 2\right] = 2^{x-2y}(\ln 2)$$

$$y' = \frac{2^{x-2y}(\ln 2)}{1 + 2(2)^{x-2y}\ln 2}$$

$$y'(2,1) = \frac{2^0(\ln 2)}{1 + 2\ln 2}$$

$$m = \frac{\ln(2)}{1 + 2\ln(2)}$$