Unit 3 Pre-Test Review – Exponential and Logarithmic Functions
MHF4U
Jensen

SOLUTIONS

Section 1: 6.1/6.2 – Log as Inverse

1) Sketch a graph of each function. Then, sketch a graph of the inverse of each function. Label each graph with its equation. Also, complete the table of information for each function

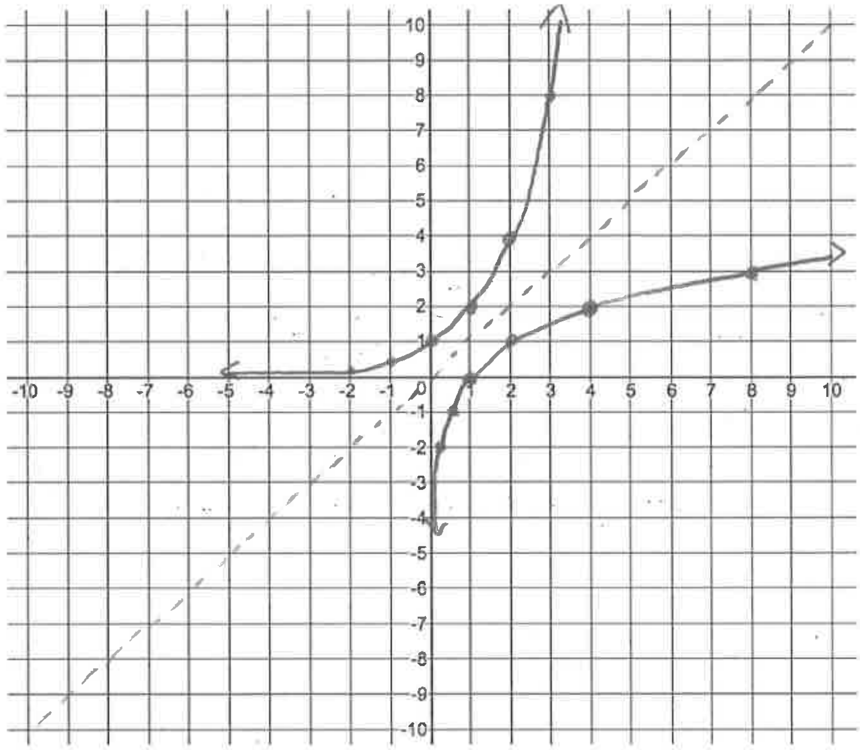
a) $f(x) = 2^x$

$f(x)=2^x$

x	y
-2	0.25
-1	0.5
0	1
1	2
2	4

$f^{-1}(x) = \log_2 x$

x	y
0.25	-2
0.5	-1
1	0
2	1
4	2



$f(x) = 2^x$	$f^{-1}(x) = \log_2 x$
x-int: NONE	x-int: $(1, 0)$
y-int: $(0, 1)$	y-int: NONE
Domain: $\{X \in \mathbb{R}\}$	Domain: $\{X \in \mathbb{R} \mid x > 0\}$
Range: $\{Y \in \mathbb{R} \mid y > 0\}$	Range: $\{Y \in \mathbb{R}\}$
Asymptote: $y = 0$	Asymptote: $x = 0$

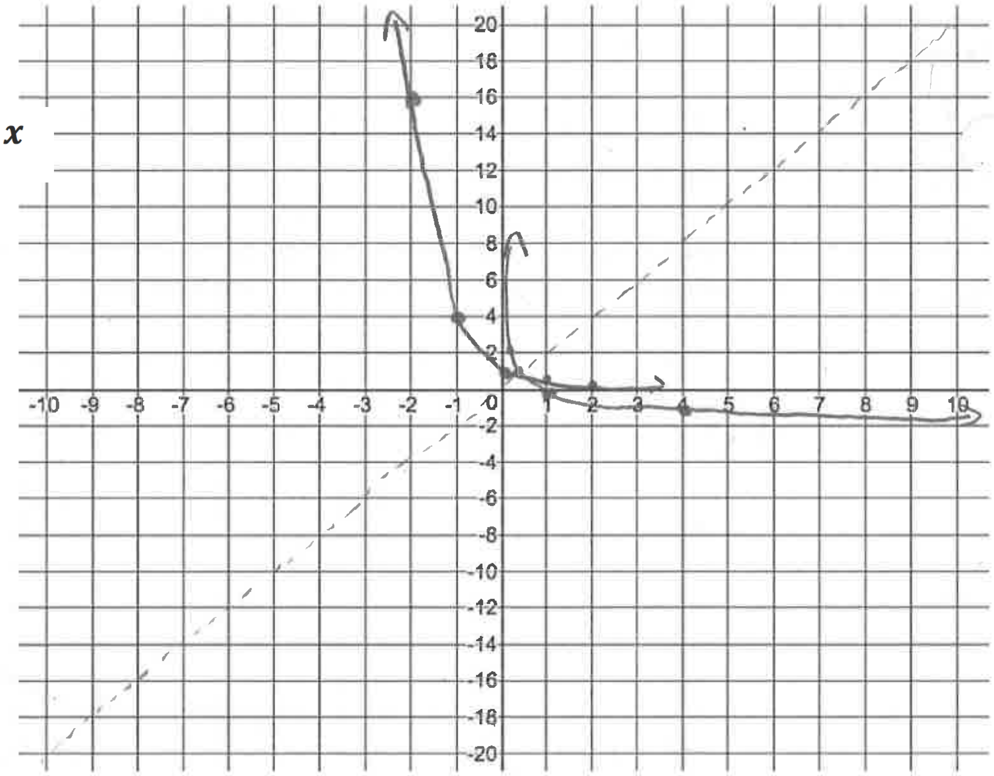
b) $g(x) = \left(\frac{1}{4}\right)^x$

$g(x) = \left(\frac{1}{4}\right)^x$

x	y
-2	16
-1	4
0	1
1	0.25
2	0.0625

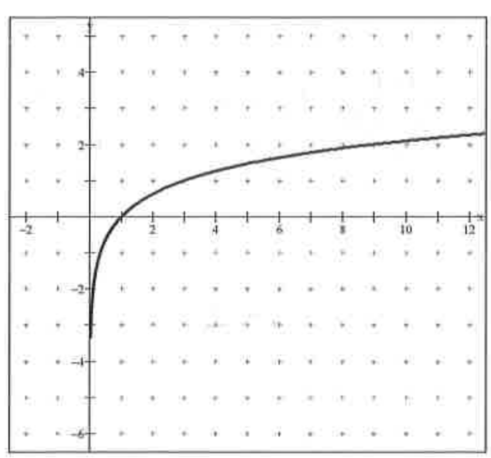
$g^{-1}(x) = \log_{\frac{1}{4}} x$

x	y
16	-2
4	-1
1	0
0.25	1
0.0625	2



$g(x) =$	$g^{-1}(x) =$
x-int: NONE	x-int: (1, 0)
y-int: (0, 1)	y-int: NONE
Domain: $\{x \in \mathbb{R}\}$	Domain: $\{x \in \mathbb{R} \mid x > 0\}$
Range: $\{y \in \mathbb{R} \mid y > 0\}$	Range: $\{y \in \mathbb{R}\}$
Asymptote: $y = 0$	Asymptote: $x = 0$

2) State the domain and range for the function, shown below.

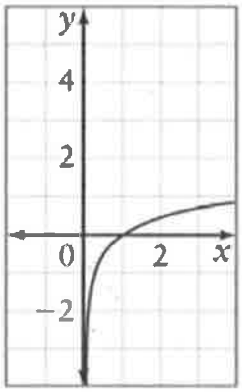


Domain: $\{x \in \mathbb{R} \mid x > 0\}$

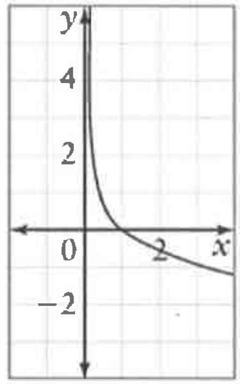
Range: $\{y \in \mathbb{R}\}$

3) Match each graph in the table with the graph of its inverse (A, B, or C). Then write an equation for each function

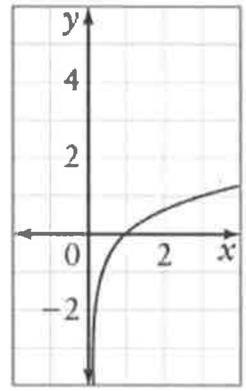
A)

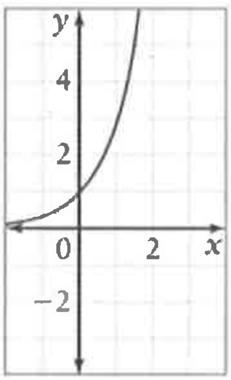
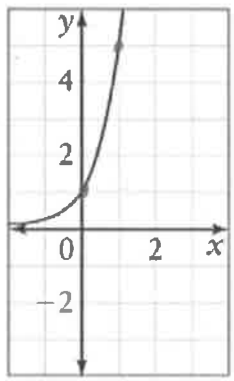
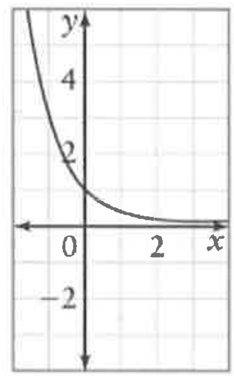


B)



C)



Graph: 		
Equation: $y=3^x$	$y=5^x$	$y=(\frac{1}{3})^x$
Letter of Graph of Inverse: C	A	B
Equation of inverse: $y=\log_3 x$	$y=\log_5 x$	$y=\log_{1/3} x$

4) Rewrite each equation in logarithmic form.

a) $4^3 = 64$

$3=\log_4(64)$

b) $28 = 3^x$

$x=\log_3(28)$

c) $6^3 = y$

$3=\log_6(y)$

d) $512 = 2^9$

$9=\log_2(512)$

5) Rewrite each equation in exponential form.

a) $7 = \log_2 128$

$2^7=128$

b) $x = \log_b n$

$b^x=n$

c) $5 = \log_3 243$

$3^5=243$

d) $19 = \log_b 4$

$b^{19}=4$

6) Evaluate without a calculator. Show your work.

Use either:
Rule: if $x^a = x^b$, then $a = b$
Rule: $\log_a(a^b) = b$

a) $\log_2 16$

$= \log_2(2^4)$
$= 4$

b) $\log_3 81$

$= \log_3(3^4)$
$= 4$

c) $\log_4\left(\frac{1}{16}\right)$

$= \log_4(4^{-2})$
$= -2$

d) $\log 0.000\,001$

$= \log(10^{-6})$
$= -6$

Section 2: 6.4 – Power Law of Logarithms

7) Evaluate each of the following without a calculator using the power law of logarithms.

a) $\log_2 32^3$

$= 3\log_2(2)^5$
$= 3(5)$
$= 15$

b) $\log 1000^{-2}$

$= -2\log 1000$
$= -2\log(10)^3$
$= -2(3)$
$= -6$

c) $\log 0.001^{-1}$

$= -1\log 0.001$
$= -1\log(10)^{-3}$
$= -1(-3)$
$= 3$

d) $\log_{\frac{1}{4}}\left(\frac{1}{16}\right)^4$

$= 4\log_{1/4}\left(\frac{1}{4}\right)^2$
$= 4(2)$
$= 8$

8) Solve for x, correct to 3 decimal places.

a) $x = \log_3 17$

$x = \frac{\log 17}{\log 3}$

$x \doteq 2.579$

b) $\log_2 0.35 = x$

$\frac{\log 0.35}{\log 2} = x$

$x \doteq -1.515$

c) $4^x = 10$

$x = \log_4 10$

$x = \frac{\log 10}{\log 4}$

$x \doteq 1.661$

d) $80 = 100\left(\frac{1}{2}\right)^x$

$0.8 = \left(\frac{1}{2}\right)^x$

$x = \log_{1/2}(0.8)$

$x = \frac{\log 0.8}{\log 0.5}$

$x \doteq 0.322$

9) Use the change of base formula to evaluate. Round to one decimal place.

a) $\log_9 12$

$= \frac{\log 12}{\log 9}$

≈ 1.1

b) $\log_{0.25} 52$

$= \frac{\log 52}{\log 0.25}$

≈ -2.9

10) Write as a single logarithm. Then evaluate without a calculator.

a) $\frac{\log 16}{\log 4}$

$= \log_4 (16)$

$= \log_4 (4)^2$

$= 2$

b) $\frac{\log\left(\frac{8}{27}\right)}{\log\left(\frac{2}{3}\right)}$

$= \log_{2/3} \left(\frac{8}{27}\right)$

$= \log_{2/3} \left(\frac{2}{3}\right)^3$

$= 3$

11) Solve, to two decimal places

a) $\log 4^x = 7$

$x \log 4 = 7$

$x = \frac{7}{\log 4}$

$x \approx 11.63$

b) $12 = \log_3 4^m$

$3^{12} = 4^m$

$531441 = 4^m$

$\log_4(531441) = m$

$\frac{\log(531441)}{\log(4)} = m$

$m \approx 9.51$

12) An investment earns 12% interest, compounded annually. The amount, A, that the investment is worth as a function of time, t, in years, is given by $A = 1500(1.12)^t$. Use the equation to determine...

a) the value of the investment after 4 years

$A = 1500\,(1.12)^4$

$A = \$\,2360.28$

b) how long it will take for the investment to double in value

$3000 = 1500\,(1.12)^t$

$2 = 1.12^t$

$\log 2 = \log(1.12^t)$

$\log 2 = t\log(1.12)$

$t = \frac{\log 2}{\log 1.12}$

$t = 6.12$ years

Section 3: 7.3 – Product and Quotient Laws of Logarithms

13) Write as a single logarithm

a) $\log_7 8 + \log_7 4 - \log_7 16$

$= \log_7\left[\frac{8(4)}{16}\right]$

$= \log_7 2$

b) $2\log a + \log(3b) - \frac{1}{2}\log c$

$= \log(a^2) + \log(3b) - \log c^{1/2}$

$= \log\left(\frac{3a^2b}{\sqrt{c}}\right)$

14) Write as a sum or difference of logarithms. Simplify if possible.

a) $\log(a^2bc)$

$= \log(a^2) + \log b + \log c$

$= 2\log a + \log b + \log c$

b) $\log\left(\frac{k}{\sqrt{m}}\right)$

$= \log k - \log(m^{1/2})$

$= \log k - \frac{1}{2}\log m$

15) Evaluate, using the laws of logarithms.

a) $\log_6 8 + \log_6 27$

$= \log_6(8 \times 27)$

$= \log_6(216)$

$= \log_6(6^3)$

$= 3$

b) $\log_4 128 - \log_4 8$

$= \log_4\left(\frac{128}{8}\right)$

$= \log_4(16)$

$= \log_4(4^2)$

$= 2$

c) $2\log 2 + 2\log 5$

$= \log(2^2) + \log(5^2)$

$= \log 4 + \log 25$

$= \log(4 \times 25)$

$= \log 100$

$= \log(10^2)$

$= 2$

d) $2\log 3 + \log\left(\frac{25}{2}\right)$

$= \log(3^2) + \log\left(\frac{25}{2}\right)$

$= \log(9) + \log\left(\frac{25}{2}\right)$

$= \log\left(9 \times \frac{25}{2}\right)$

$= \log\left(\frac{225}{2}\right)$

$= 2.05$

16) Simplify

a) $\log(2m + 6) - \log(m^2 - 9)$

$= \log\left[\frac{2(m+3)}{(m-3)(m+3)}\right]$

$= \log\left(\frac{2}{m-3}\right)$

b) $\log(x^2 + 2x - 15) - \log(x^2 - 7x + 12)$

$= \log\left(\frac{x^2+2x-15}{x^2-7x+12}\right)$

$= \log\left[\frac{(x+5)(x-3)}{(x-4)(x-3)}\right]$

$= \log\left(\frac{x+5}{x-4}\right)$

Section 4: 7.1/7.2 – Solving Exponential Equations

17) Write each as a power of 4

a) 64

$= 4^3$

b) $\frac{1}{16}$ $= \frac{1}{4^2}$

$= 4^{-2}$

c) $\left(\sqrt[3]{8}\right)^5$

$= 8^{\frac{5}{3}}$

$= \left(4^{\frac{3}{2}}\right)^{\frac{5}{3}}$

$= 4^{\frac{5}{2}}$

$4^x = 8$
$\log 4^x = \log 8$
$x \log 4 = \log 8$
$x = \frac{\log 8}{\log 4}$
$x = \frac{3}{2}$

18) Write 20 as a power of 5.

$5^x = 20$

$\log(5^x) = \log(20)$

$x\log(5) = \log(20)$

$x = \frac{\log(20)}{\log(5)}$

$20 = 5^{\frac{\log 20}{\log 5}}$

19) Solve each equation

a) $3^{5x} = 27^{x-1}$

$3^{5x} = (3^3)^{x-1}$

$3^{5x} = 3^{3x-3}$

$5x = 3x-3$

$2x = -3$

$\boxed{x = \frac{-3}{2}}$

b) $8^{2x+1} = 32^{x-1}$

$(2^3)^{2x+1} = (2^5)^{x-1}$

$2^{6x+3} = 2^{5x-5}$

$6x+3 = 5x-5$

$x = -8$

20) Solve exactly. Then use your calculator to evaluate correct to 3 decimal places.

a) $3^{x-2} = 5^x$

$\log(3^{x-2}) = \log(5^x)$

$(x-2)\log 3 = x\log 5$

$x\log 3 - 2\log 3 = x\log 5$

$x\log 3 - x\log 5 = 2\log 3$

$x(\log 3 - \log 5) = 2\log 3$

$x = \frac{2\log 3}{\log 3 - \log 5}$

$x \simeq -4.301$

b) $2^{k-2} = 3^{k+1}$

$\log(2^{k-2}) = \log(3^{k+1})$

$(k-2)\log(2) = (k+1)\log(3)$

$k\log 2 - 2\log 2 = k\log 3 + \log 3$

$k\log 2 - k\log 3 = \log 3 + 2\log 2$

$k(\log 2 - \log 3) = \log 3 + \log 4$

$k = \frac{\log 3 + \log 4}{\log 2 - \log 3}$

$k \simeq -6.129$

21) Solve the following equations; round to 2 decimal places where appropriate.

a) $3^x = 12$

$x = \log_3 12$

$x = \frac{\log 12}{\log 3}$

$x \simeq 2.26$

b) $10 = 2 \cdot 4^{x+2}$

$5 = 4^{x+2}$

$\log 5 = \log(4^{x+2})$

$\log 5 = (x+2)\log 4$

$\log 5 = x\log 4 + 2\log 4$

$\frac{\log 5 - \log 16}{\log 4} = x$

$x \simeq -0.84$

c) $3^x = 4^{1-x}$

$\log(3^x) = \log(4^{1-x})$

$x\log 3 = (1-x)\log 4$

$x\log 3 = \log 4 - x\log 4$

$x\log 3 + x\log 4 = \log 4$

$x(\log 3 + \log 4) = \log 4$

$x = \frac{\log 4}{\log 12}$

$x \simeq 0.56$

22) Solve each equation. Check for extraneous routes.

a) $4^{2x} - 4^x - 20 = 0$

$(4^x)^2 - 4^x - 20 = 0$

Let $k = 4^x$

$k^2 - k - 20 = 0$

$(k-5)(k+4) = 0$

$k = 5$ or $k = -4$

Case 1:

$4^x = 5$

$x = \log_4(5)$

$x = \frac{\log 5}{\log 4}$

$x \simeq 1.16$

Case 2:

$4^x = -4$

$x = \log_4(-4)$

Extraneous root.
(no solution)

$\boxed{x \simeq 1.16}$

b) $2^x + 12(2)^{-x} = 7$

$(2^x)(2^x) + 12(2^x)(2^{-x}) = 7(2^x)$

$2^{2x} + 12 = 7(2^x)$

$(2^x)^2 - 7(2^x) + 12 = 0$

Let $k = 2^x$

$k^2 - 7k + 12 = 0$

$(k-4)(k-3) = 0$

$k = 4$ or $k = 3$

Case 1:

$2^x = 4$

$x = \log_2(4)$

$x = \log_2(2^2)$

$\boxed{x = 2}$

Case 2:

$2^x = 3$

$x = \log_2(3)$

$x = \frac{\log 3}{\log 2}$

$\boxed{x \simeq 1.58}$

Section 5: 7.4 – Solving Logarithmic Equations

23) Solve each equation

a) $\log_4 x = 1.8$

$4^{1.8} = x$

$\boxed{x \simeq 12.13}$

b) $log_5 x - log_5(x-2) = 1$

$\log_5\left(\frac{x}{x-2}\right) = 1$

$5^1 = \frac{x}{x-2}$

$5(x-2) = x$

$5x - 10 = x$

$4x = 10$

$\boxed{x = \frac{5}{2}}$

c) $5^{2x} = 2(5)^x + 1$

$(5^x)^2 - 2(5^x) - 1 = 0$

Let $k = 5^x$

$k^2 - 2k - 1 = 0$

$k = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$k = \frac{2 \pm \sqrt{8}}{2}$

$k = \frac{2 \pm 2\sqrt{2}}{2}$

$k = \frac{2(1 \pm \sqrt{2})}{2}$

$k = 1 \pm \sqrt{2}$

Case 1:

$1 + \sqrt{2} = 5^x$

$x = \log_5(1 + \sqrt{2})$

$x = \frac{\log(1+\sqrt{2})}{\log(5)}$

$\boxed{x \simeq 0.548}$

Case 2:

$1 - \sqrt{2} = 5^x$

$\log_5(1 - \sqrt{2}) = x$

↑

extraneous root

24) Solve

a) $\log(2x + 10) = 2$

$10^2 = 2x + 10$

$100 = 2x + 10$

$90 = 2x$

$\boxed{x = 45}$

b) $1 - \log(2x) = 0$

$1 = \log(2x)$

$10^1 = 2x$

$\boxed{x = 5}$

25) Solve. Check for extraneous roots.

a) $\log_2 x + \log_2(x + 2) = 3$

$\log_2[x(x+2)] = 3$

$2^3 = x(x+2)$

$8 = x^2 + 2x$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x = -4$ or $\boxed{x = 2}$

↑

extraneous root

b) $\log_3(3x + 7) = 2$

$3^2 = 3x + 7$

$9 = 3x + 7$

$2 = 3x$

$\boxed{x = \frac{2}{3}}$

c) $\log_5(2x+1) = 1 - \log_5(x+2)$

$\log_5(2x+1) + \log_5(x+2) = 1$

$\log_5[(2x+1)(x+2)] = 1$

$5^1 = (2x+1)(x+2)$

$5 = 2x^2 + 5x + 2$

$0 = 2x^2 + 5x - 3$

P: -6, S: 5

$\left(\frac{3}{1}\right) = \frac{6}{2}$, $\frac{-1}{2}$

$0 = (x+3)(2x-1)$

$x = -3$ or $\boxed{x = \frac{1}{2}}$

↑ extraneous root

Section 6: 7.4 – Applications

Exponential Formulas

$A(t) = A_0(1+i)^t$	$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$	$A(t) = A_0(2)^{\frac{t}{D}}$
general, where *i* is percent growth(+) or decay(-)	half-life, *H* is the half-life period	doubling, *D* is the doubling period

Logarithmic Formulas

$pH = -\log[H^+]$	$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$	$M = \log\left(\frac{I}{I_0}\right)$
Where pH is acidity and [H+] is concentration of hydronium ions mol/L	Where β is loudness in dB and I is intensity of sound in W/m^2	Where M is magnitude measure by richters, I is intensity

26) When you drink a cup of coffee or a glass of cola, or when you eat a chocolate bar, the percent, P, of caffeine remaining in your bloodstream is related to the elapsed time, t, in hours by $t = 5\left(\frac{\log P}{\log 0.5}\right)$

a) How long will it take for the amount of caffeine to drop to 20% of the amount consumed?

$t = 5\left(\frac{\log 0.2}{\log 0.5}\right)$

$t \doteq 11.61$ hours

b) Suppose you drink a cup of coffee at 9:00 am, what percent of the caffeine will remain in your body at noon?

$3 = 5\left(\frac{\log P}{\log 0.5}\right)$

$\frac{3\log 0.5}{5} = \log P$

$\log 0.5^{0.6} = \log P$

$0.5^{0.6} = P$

$P \doteq 0.6598$

$\boxed{\text{About } 66\%}$

27) A 50-mg sample of cobalt-60 decays to 40 mg after 1.6 minutes. $A(t) = A_0\left(\frac{1}{2}\right)^{t/h}$

a) Determine the half-life of cobalt-60.

$40 = 50\left(\frac{1}{2}\right)^{1.6/h}$

$0.8 = 0.5^{1.6/h}$

$\log 0.8 = \log 0.5^{1.6/h}$

$\log 0.8 = \frac{1.6}{h}\log 0.5$

$\frac{\log 0.8}{\log 0.5} = \frac{1.6}{h}$

$h = \frac{1.6\log 0.5}{\log 0.8}$

$\simeq 5$ minutes.

b) How long will it take for the sample to decay to 5% of its initial amount?

$2.5 = 50\left(\frac{1}{2}\right)^{t/5}$

$0.05 = \left(\frac{1}{2}\right)^{t/5}$

$\log 0.05 = \log\left(\frac{1}{2}\right)^{t/5}$

$\log 0.05 = \frac{t}{5}\log\left(\frac{1}{2}\right)$

$\frac{\log 0.05}{\log 0.5} = \frac{t}{5}$

$t \simeq 21.61$ minutes

$pH = -\log[H^+]$

28) Determine the pH, correct to one decimal place, of a solution with each hydronium ion concentration.

a) 0.000 316 mol/L

$pH = -\log(0.000316)$

$pH \simeq 3.5$

b) 7.9×10^{-9} mol/L

$pH = -\log(7.9 \times 10^{-9})$

$pH \simeq 8.1$

29) Calculate the hydronium ion concentration, correct to two decimal places, if the pH of a solution is

a) 2.2

$2.2 = -\log[H^+]$

$-2.2 = \log[H^+]$

$10^{-2.2} = [H^+]$

$[H^+] \simeq 0.00631$ or 6.31×10^{-3} mol/L

b) 11.6

$11.6 = -\log[H^+]$

$-11.6 = \log[H^+]$

$[H^+] = 10^{-11.6}$

$[H^+] \simeq 2.51 \times 10^{-12}$ mol/L

30) Use the sound level scale in your notes to answer the following: $\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$

a) How many times as intense is a normal (β_2) conversation compared to a whisper (β_1)?

$60 - 30 = 10\log\left(\frac{I_2}{I_1}\right)$

$30 = 10\log\left(\frac{I_2}{I_1}\right)$

$3 = \log\left(\frac{I_2}{I_1}\right)$

$10^3 = \left(\frac{I_2}{I_1}\right)$

$\left(\frac{I_2}{I_1}\right) = 1000$

1000 times as intense

b) How many times as intense is normal (β_2) city traffic compared to a shout (β_1)?

$85 - 80 = 10\log\left(\frac{I_2}{I_1}\right)$

$0.5 = \log\left(\frac{I_2}{I_1}\right)$

$10^{0.5} = \left(\frac{I_2}{I_1}\right)$

$\left(\frac{I_2}{I_1}\right) \simeq 3.16$

About 3.16 times as intense

31) The intensity of sound in a library is estimated to be one thousandth that of normal conversation. What is the decibel rating for the library?

$$\beta_2 - 60 = 10\log\left(\frac{1}{1000}\right)$$
$$\beta_2 = 10\log\left(\frac{1}{1000}\right) + 60$$
$$\beta_2 = 10\log(10^{-3}) + 60$$
$$\beta_2 = 10(-3) + 60$$
$$\beta_2 = 30$$

$\boxed{\text{The library is 30 dB}}$

$M = \log\left(\frac{I}{I_0}\right)$

32) How many times as intense is an earthquake with a magnitude of 7.2 than an earthquake with a magnitude of 5.6? $7.2 - 5.6 = 1.6$

$$1.6 = \log\left(\frac{I}{I_0}\right)$$
$$10^{1.6} = \left(\frac{I}{I_0}\right)$$
$$\left(\frac{I}{I_0}\right) \simeq 39.8$$

$\boxed{\text{About 39.8 times as intense}}$

33) If an earthquake is 390 times as intense as an earthquake with a magnitude of 4.2 on the Richter scale, what is the magnitude of the more intense earthquake?

$$M - 4.2 = \log(390)$$
$$M = \log(390) + 4.2$$
$$M \simeq 6.79$$

$\boxed{\text{About 6.79}}$

34) The absolute magnitude of star A is $\overset{m_1}{-4.5}$ and that of star B is $\overset{m_2}{0.2}$. How many times as bright is star A than star B, to the nearest unit?

$m_2 - m_1 = \log\left(\frac{b_1}{b_2}\right)$

$$0.2 - (-4.5) = \log\left(\frac{b_1}{b_2}\right)$$
$$4.7 = \log\left(\frac{b_1}{b_2}\right)$$
$$10^{4.7} = \left(\frac{b_1}{b_2}\right)$$
$$\left(\frac{b_1}{b_2}\right) \simeq 50118.7$$

$\boxed{\text{About 50118.7 times brighter.}}$

35) An altimeter is a device that measures the height of a plane above the ground. It works based on air pressure according to the formula $h = 18400 log \frac{P_0}{P}$, where h is the height above the ground in metres, P is the air pressure at that height, and P_0 was the air pressure on the ground at takeoff. Air pressure is measure in kilopascals (kPa).

a) Air pressure on the ground was 102 kPa. If the airplane instruments measure a pressure of 32.5 kPa outside the plane, what is the height of the airplane to the nearest metre?

$$h = 18400 \log\left(\frac{102}{32.5}\right)$$

$$h \simeq 9140 \text{ m}$$

b) What is the outside air pressure for a plane flying at 11 000 metres? Assume a ground pressure 102.5 kPa. Round to one decimal place.

$$11\,000 = 18400 \log\left(\frac{102.5}{P}\right)$$

$$\frac{11\,000}{18400} = \log\left(\frac{102.5}{P}\right)$$

$$10^{\frac{11\,000}{18400}} = \frac{102.5}{P}$$

$$P = \frac{102.5}{10^{\frac{11000}{18400}}}$$

$$P \simeq 25.9 \text{ kPa.}$$

c) How high would a plane have to be flying when it encountered air pressure in the air that was half the air pressure on the ground? Round to the nearest meter.

$$h = 18400 \log(2)$$

$$h \simeq 5539 \text{ m}$$

Answer Key

See posted solutions for #1-3

4)a) $\log_4 64 = 3$ **b)** $\log_3 28 = x$ **c)** $\log_6 y = 3$ **d)** $\log_2 512 = 9$

5)a) $2^7 = 128$ **b)** $b^x = n$ **c)** $3^5 = 243$ **d)** $b^{19} = 4$

6)a) 4 **b)** 4 **c)** -2 **d)** -6

7)a) 15 **b)** -6 **c)** 3 **d)** 8

8)a) 2.579 **b)** -1.515 **c)** 1.661 **d)** 0.322

9)a) 1.1 **b)** -2.9

10)a) $\log_4 16 = 2$ **b)** $\log_{\frac{2}{3}}\left(\frac{8}{27}\right) = 3$

11)a) 11.63 **b)** 9.51

12)a) $2360.28 **b)** 6.12 years

13)a) $\log_7 2$ **b)** $\log\left(\frac{3a^2b}{\sqrt{c}}\right)$

14)a) $2\log a + \log b + \log c$ **b)** $\log k - \frac{1}{2}\log m$

15)a) 3 **b)** 2 **c)** 2 **d)** 2.05

16)a) $\log\left(\frac{2}{m-3}\right)$ **b)** $\log\left(\frac{x+5}{x-4}\right)$

17)a) 4^3 **b)** 4^{-2} **c)** $4^{\frac{5}{2}}$

18) $5^{\frac{\log 20}{\log 5}}$

19)a) $x = -\frac{3}{2}$ b) $x = -8$

20)a) $x = \frac{2\log 3}{\log 3 - \log 5} \cong -4.301$ **b)** $k = \frac{2\log 2 + \log 3}{\log 2 - \log 3} \cong -6.129$

21a) 2.26 **b)** -0.84 **c)** 0.56

22)a) $x = \frac{\log 5}{\log 4} \cong 1.16$ **b)** $x = 2$ or $x = \frac{\log 3}{\log 2} \cong 1.58$

23)a) 12.13 **b)** 2.5 **c)** $x = 0.548$

24)a) 45 **b)** 5

25)a) 2 **b)** $\frac{2}{3}$ **c)** $\frac{1}{2}$

26)a) 11.6 hours **b)** 66%

27)a) 5 min **b)** 21.6 min

28)a) 3.5 **b)** 8.1

29) a) 6.31×10^{-3} mol/L **b)** 2.51×10^{-12} mol/L

30) a) 1000 **b)** 3.2

31) 30 dB

32) 39.8

33) 6.8

34) a) 50119

35) a) 9140m **b)** 25.9 kPa **c)** 5539m