Unit 4 Pre-Test Review – Trig in Radians
MHF4U
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SOLUTIONS

ection 1: 4.1 – Radian Measure

1) Determine the approximate radian measure, to the nearest hundredth, for each angle.

a) $33° \times \frac{\pi}{180}$ ≈ 0.58

b) $138° \times \frac{\pi}{180}$ ≈ 2.41

c) $252° \times \frac{\pi}{180}$ ≈ 4.40

d) $347° \times \frac{\pi}{180}$ ≈ 6.06

2) Determine the approximate degree measure, to the nearest tenth, for each angle.

a) $1.24 \times \frac{180}{\pi}$ $\approx 71.0°$

b) $2.82 \times \frac{180}{\pi}$ $\approx 161.6°$

c) $4.78 \times \frac{180}{\pi}$ $\approx 273.9°$

d) $6.91 \times \frac{180}{\pi}$ $\approx 395.9°$

3) Determine the exact radian measure of each angle.

a) $75° \times \frac{\pi}{180}$ $= \frac{5\pi}{12}$

b) $20° \times \frac{\pi}{180}$ $= \frac{\pi}{9}$

c) $12° \times \frac{\pi}{180}$ $= \frac{\pi}{15}$

d) $9° \times \frac{\pi}{180}$ $= \frac{\pi}{20}$

4) Determine the exact degree measure of each angle.

a) $\frac{2\pi}{5} \times \frac{180}{\pi}$ $= 72°$

b) $\frac{4\pi}{9} \times \frac{180}{\pi}$ $= 80°$

c) $\frac{7\pi}{12} \times \frac{180}{\pi}$ $= 105°$

d) $\frac{11\pi}{18} \times \frac{180}{\pi}$ $= 110°$

5) An arc of a circle measuring 22.5 cm subtends a central angle of $\frac{4\pi}{3}$ radians. Find the approximate radius of the circle, to the nearest tenth of a cm.

$\theta = \frac{a}{r}$

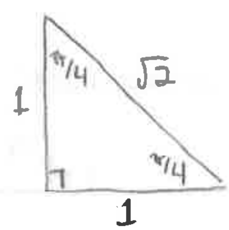
$\frac{4\pi}{3} = \frac{22.5}{r}$

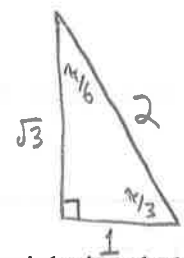
$r = \frac{22.5(3)}{4\pi}$

$r = 5.4\text{ cm}$

Section 2: 4.2 – Trig Ratios and Special Angles

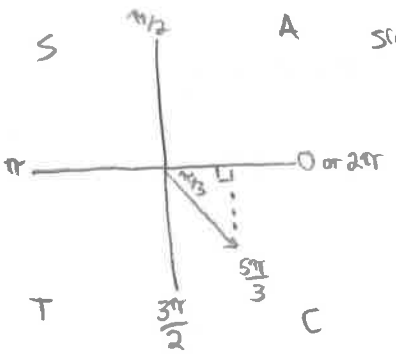
6) Draw both special triangles using radian measures





7) Find the exact value (using CAST and drawing the special triangles) for …

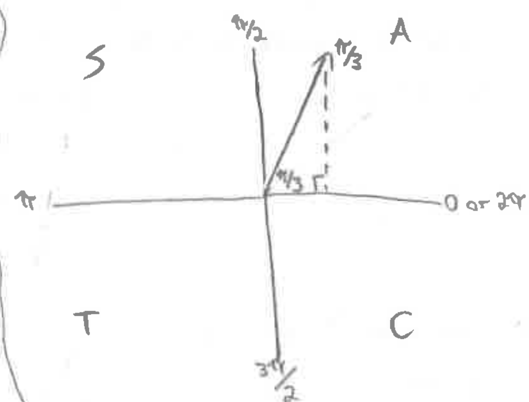
a) $\sin\frac{5\pi}{3}$



$\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$

$= -\frac{\sqrt{3}}{2}$

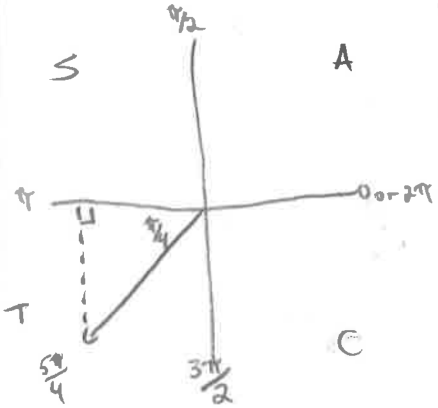
b) $\cot\frac{\pi}{3}$



$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)}$

$= \frac{1}{\sqrt{3}}$

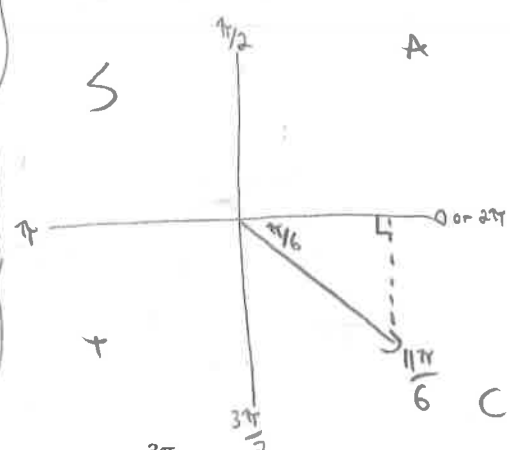
c) $\cos\frac{5\pi}{4}$



$\cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right)$

$= -\frac{1}{\sqrt{2}}$

d) $\sec\frac{11\pi}{6}$



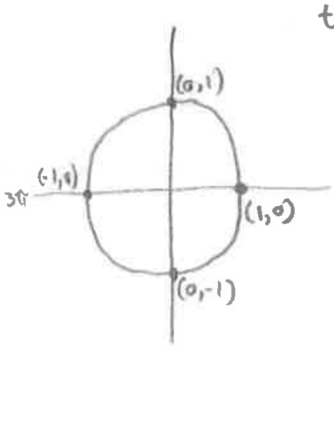
$\sec\left(\frac{11\pi}{6}\right) = \frac{1}{\cos\left(\frac{11\pi}{6}\right)}$

$= \frac{1}{\cos\left(\frac{\pi}{6}\right)}$

$= \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{2}{\sqrt{3}}$

e) $\tan 3\pi$



$\tan 3\pi = \tan(3\pi - 2\pi)$

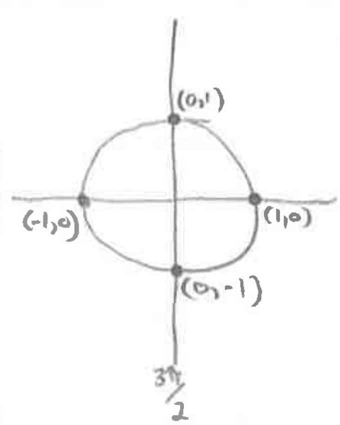
$= \tan\pi$

$= \frac{y}{x}$

$= \frac{0}{-1}$

$= 0$

f) $\sin\frac{3\pi}{2}$

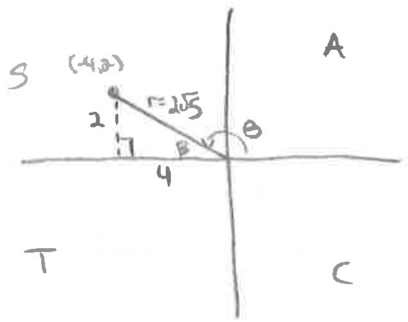


$\sin\left(\frac{3\pi}{2}\right) = \frac{y}{r}$

$= \frac{-1}{1}$

$= -1$

8) Suppose the terminal arm for the angle θ passes through the point (-4, 2). Find the exact values of $cot\theta$ and $sin\theta$.



$4^2+2^2=r^2$

$20=r^2$

$r=\sqrt{20}$

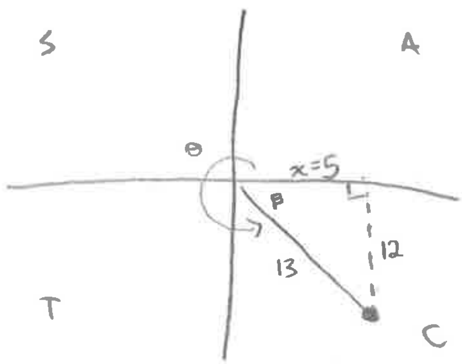
$r=\sqrt{4}(\sqrt{5})$

$r=2\sqrt{5}$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{\tan\beta} = -\frac{1}{\left(\frac{2}{4}\right)} = -2$$

$$\sin\theta = \sin\beta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$$

9) Suppose $\sin\theta = -\frac{12}{13}$, and $\frac{3\pi}{2} \le \theta \le 2\pi$. Find $\sec\theta$ and $\tan\theta$.



$x^2+12^2=13^2$

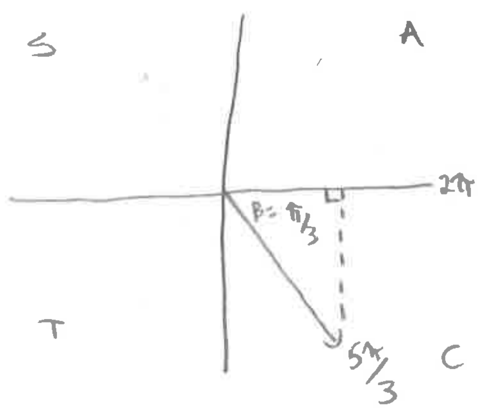
$x=\sqrt{25}$

$x=5$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\cos\beta} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

$$\tan\theta = -\tan\beta = -\frac{12}{5}$$

10) Determine exact values for all 6 trig ratios of $\frac{5\pi}{3}$ radians.



$$\sin\left(\tfrac{5\pi}{3}\right) = -\sin\left(\tfrac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \qquad \csc\left(\tfrac{5\pi}{3}\right) = -\frac{2}{\sqrt{3}}$$

$$\cos\left(\tfrac{5\pi}{3}\right) = \cos\left(\tfrac{\pi}{3}\right) = \frac{1}{2} \qquad \sec\left(\tfrac{5\pi}{3}\right) = 2$$

$$\tan\left(\tfrac{5\pi}{3}\right) = -\tan\left(\tfrac{\pi}{3}\right) = -\sqrt{3} \qquad \cot\left(\tfrac{5\pi}{3}\right) = -\frac{1}{\sqrt{3}}$$

11) Determine an exact value for each expression.

a) $\dfrac{\cot\frac{\pi}{4}}{\cos\frac{\pi}{3}\csc\frac{\pi}{2}}$

$$= \frac{1}{\left(\frac{1}{2}\right)(1)} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

b) $\cos\frac{\pi}{6}\csc\frac{\pi}{3} + \sin\frac{\pi}{4}$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right) + \frac{1}{\sqrt{2}} = 1 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}}$$

c) $\sec\left(\frac{5\pi}{4}\right) + \cot\left(\frac{2\pi}{3}\right)\sin\left(\frac{11\pi}{6}\right)$

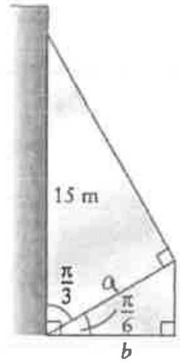
$$= \frac{1}{-\cos\left(\frac{\pi}{4}\right)} + \left(\frac{1}{-\tan\left(\frac{\pi}{3}\right)}\right)\left(-\sin\left(\frac{\pi}{6}\right)\right)$$

$$= \frac{1}{\left(-\frac{1}{\sqrt{2}}\right)} + \left(-\frac{1}{\sqrt{3}}\right)\left(-\frac{1}{2}\right)$$

$$= -\sqrt{2} + \frac{1}{2\sqrt{3}}$$

$$= \frac{-2\sqrt{6}}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} = \frac{-2\sqrt{6}+1}{2\sqrt{3}}$$

12) A ski lodge is constructed with one side along a vertical cliff such that it has a height of 15 m, as shown. Determine an exact measure for the base of the lodge, b.



$\cos\frac{\pi}{3} = \frac{a}{15}$

$\frac{1}{2} = \frac{a}{15}$

$a = \frac{15}{2}$

$\cos\frac{\pi}{6} = \frac{b}{\left(\frac{15}{2}\right)}$

$\frac{\sqrt{3}}{2} = \frac{2b}{15}$

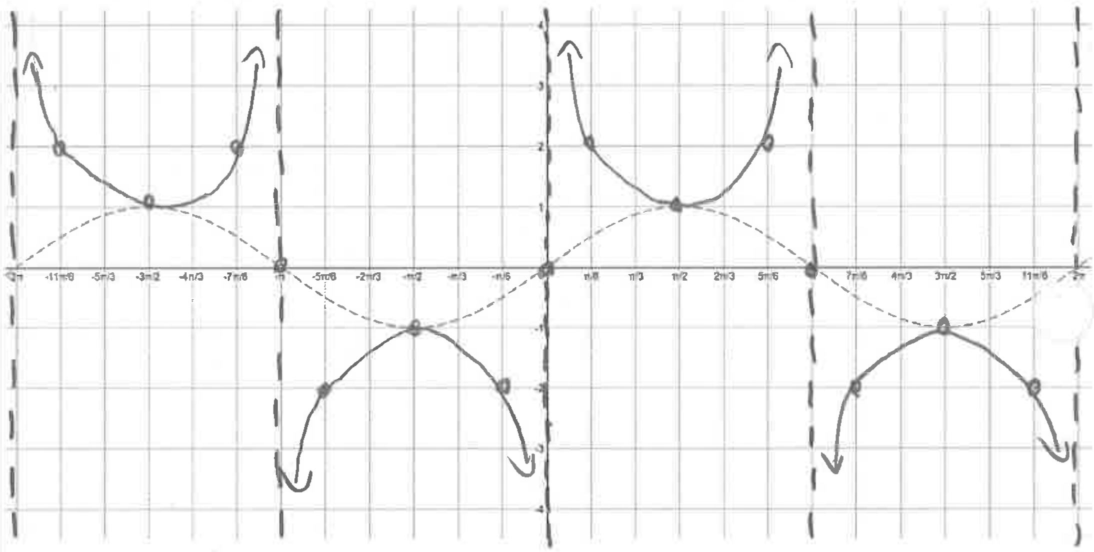
$15\sqrt{3} = 4b$

$b = \frac{15\sqrt{3}}{4}$ m

Section 3: 4.2 – Trig Ratios and Special Angles

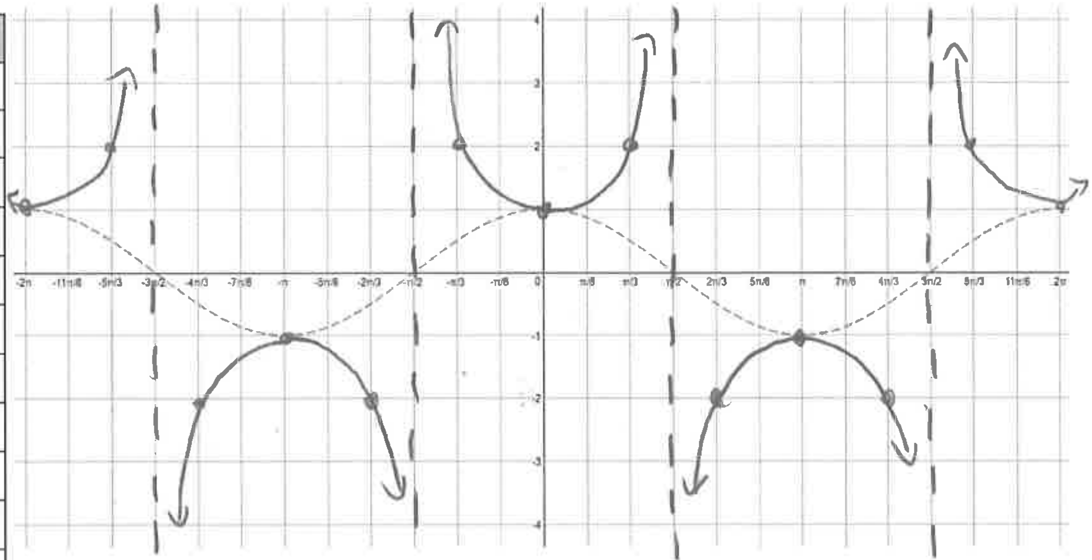
13) Use the given graph of $f(x) = \sin x$ to graph $g(x) = \csc x$. Complete the table of values as well.

x	$f(x)$	$g(x)$
0	0	und.
$\frac{\pi}{6}$	$\frac{1}{2}$	2
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \sim 0.87$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{3\pi}{6} = \frac{\pi}{2}$	1	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2} \sim 0.87$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{5\pi}{6}$	$\frac{1}{2}$	2
$\frac{6\pi}{6} = \pi$	0	und
$\frac{7\pi}{6}$	$-\frac{1}{2}$	-2
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2} \sim -0.87$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2} \sim -0.87$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	-2
$\frac{12\pi}{6} = 2\pi$	0	und



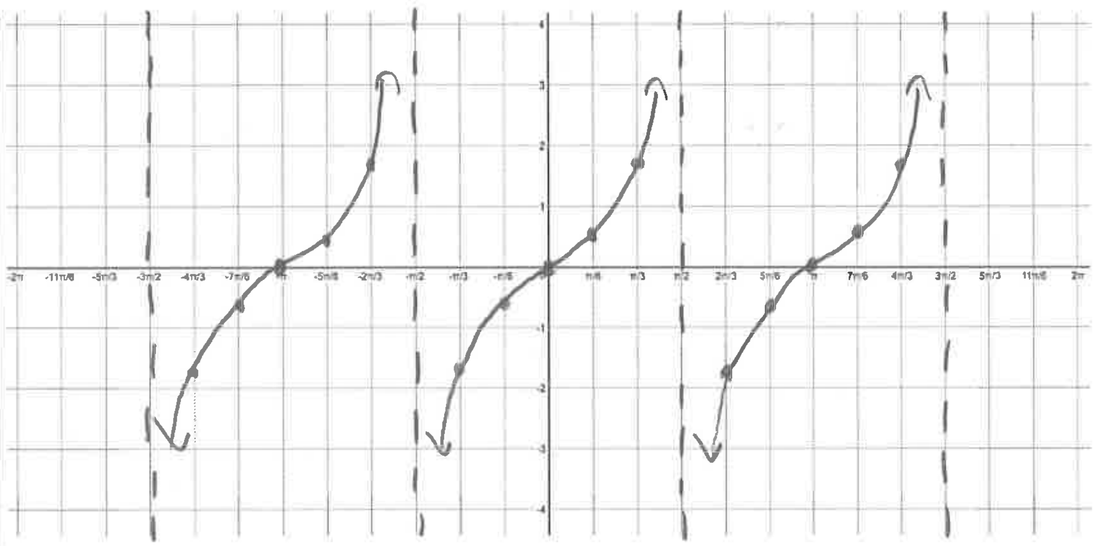
14) Use the given graph of $f(x) = \cos x$ to graph $g(x) = \sec x$. Complete the table of values as well.

x	$f(x)$	$g(x)$
0	1	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2} \sim 0.87$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{1}{2}$	2
$\frac{3\pi}{6} = \frac{\pi}{2}$	0	und
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\frac{1}{2}$	-2
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{6\pi}{6} = \pi$	-1	-1
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}} \sim -1.15$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{1}{2}$	-2
$\frac{9\pi}{6} = \frac{3\pi}{2}$	0	und
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$\frac{1}{2}$	2
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}} \sim 1.15$
$\frac{12\pi}{6} = 2\pi$	1	1



15) Graph the function $f(x) = \tan x$

x	$f(x)$
0	$\frac{0}{1} = 0$
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}} \sim 0.58$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\sqrt{3} \sim 1.73$
$\frac{3\pi}{6} = \frac{\pi}{2}$	und
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\sqrt{3} \sim -1.73$
$\frac{5\pi}{6}$	$-\frac{1}{\sqrt{3}} \sim -0.58$
$\frac{6\pi}{6} = \pi$	0
$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}} \sim 0.58$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$\sqrt{3} \sim 1.73$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	und
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\sqrt{3} \sim -1.73$
$\frac{11\pi}{6}$	$-\frac{1}{\sqrt{3}} \sim -0.58$
$\frac{12\pi}{6} = 2\pi$	0



Section 4: 5.3 – Transformations of Trig Functions

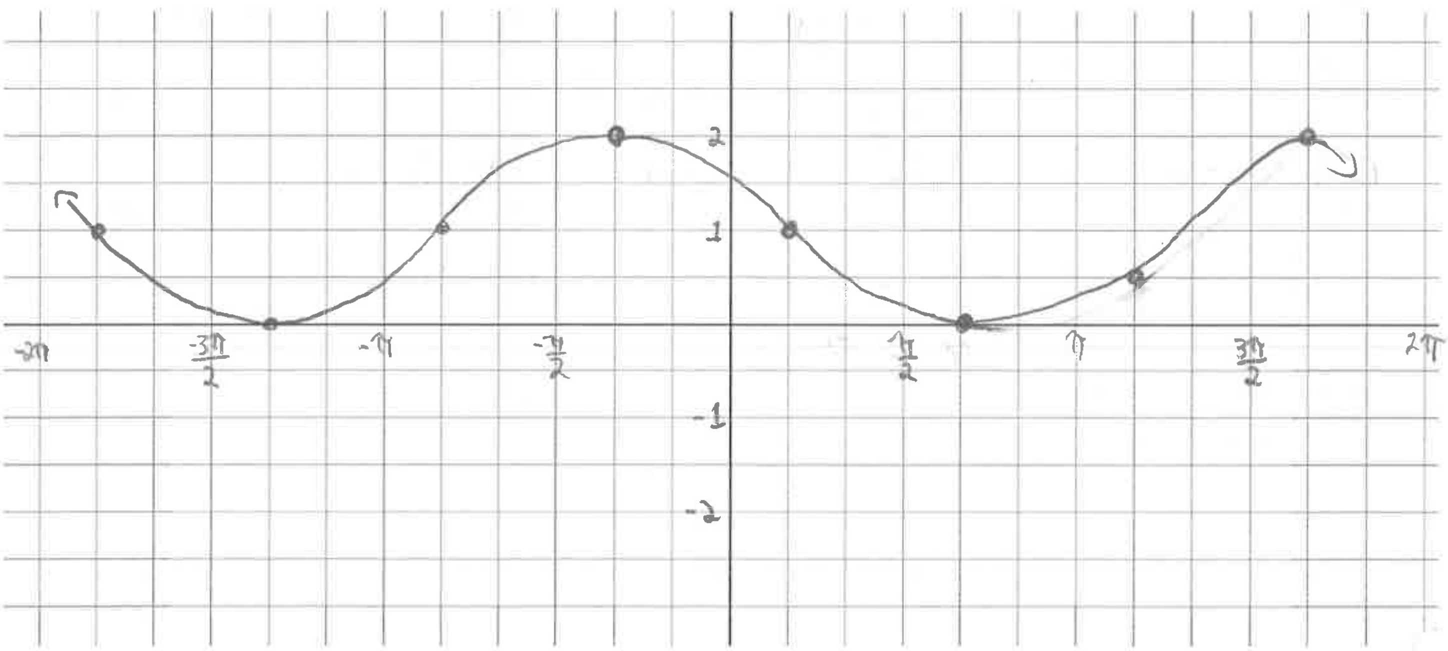
16) For each function, fill in the table of information and then graph two cycles of the transformed function using transformations of the parent function. Choose an appropriate scale.

a) $y = \cos\left(x + \frac{\pi}{3}\right) + 1$

Amplitude: $= \|a\| = 1$	Period: $\frac{2\pi}{\|k\|} = \frac{2\pi}{1} = 2\pi$
Phase shift: $\frac{\pi}{3}$ radians to the LEFT	Vertical shift: UP 1 unit
Max: $c + \|a\| = 1 + 1 = 2$	Min: $c - \|a\| = 1 - 1 = 0$

$y = \cos x$	
x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

$y = \cos\left(x + \frac{\pi}{3}\right) + 1$	
$x - \frac{\pi}{3}$	$y + 1$
$-\frac{\pi}{3} = -\frac{2\pi}{6}$	2
$\frac{\pi}{6}$	1
$\frac{4\pi}{6}$	0
$\frac{7\pi}{6}$	1
$\frac{10\pi}{6}$	2

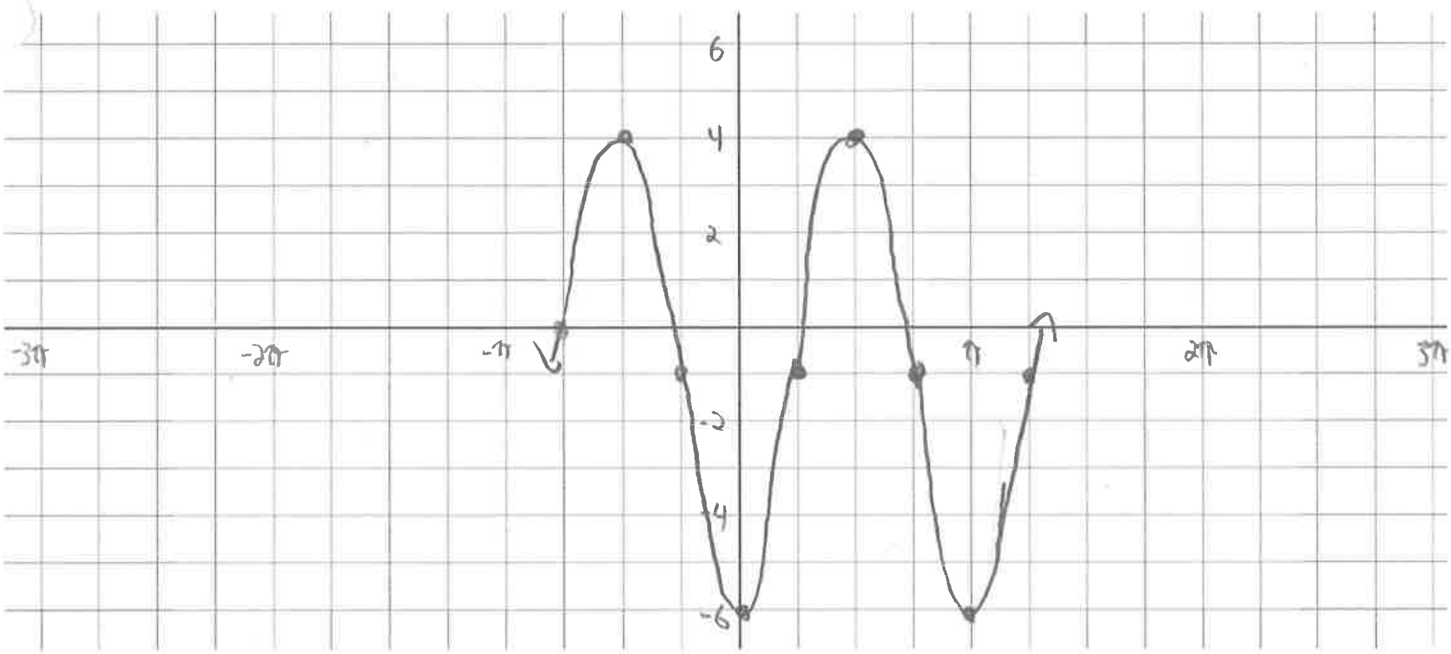


b) $y = 5\sin[2\left(x - \frac{\pi}{4}\right) - 1$

Amplitude: $= \lvert a \rvert = 5$	Period: $= \frac{2\pi}{\lvert k \rvert} = \frac{2\pi}{2} = \pi$
Phase shift: $\frac{\pi}{4}$ RIGHT	Vertical shift: 1 DOWN
Max: $c + \lvert a \rvert = -1 + 5 = 4$	Min: $c - \lvert a \rvert = -1 - 5 = -6$

$y = \sin x$	
x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$y = 5\sin[2(x - \frac{\pi}{4})] - 1$	
$\frac{x}{2} + \frac{\pi}{4}$	$5y - 1$
$\frac{\pi}{4}$	-1
$\frac{2\pi}{4}$	4
$\frac{3\pi}{4}$	-1
$\frac{4\pi}{4}$	-6
$\frac{5\pi}{4}$	-1

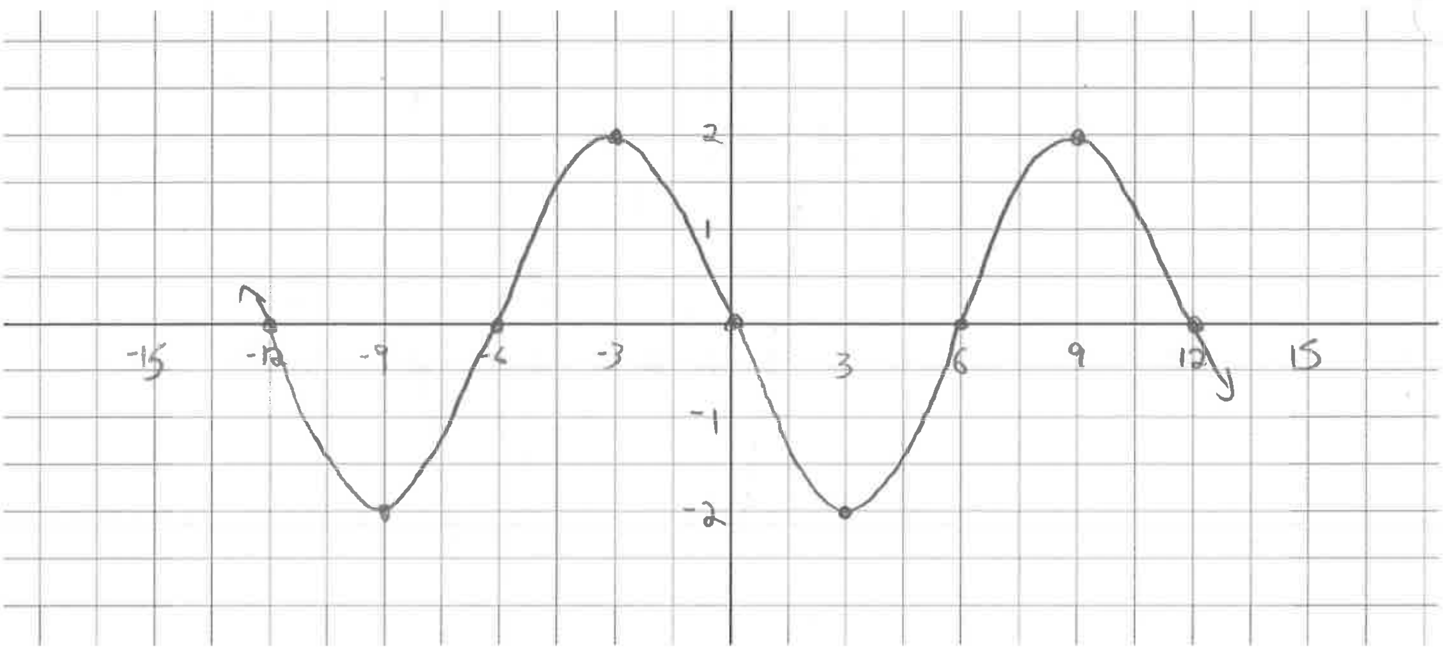


c) $y = -2\sin\left(\frac{\pi}{6}x\right)$

Amplitude: $= \lvert a \rvert = \lvert -2 \rvert = 2$	Period: $= \frac{2\pi}{\lvert k \rvert} = \frac{2\pi}{\left(\frac{\pi}{6}\right)} = 12$
Phase shift: NONE	Vertical shift: NONE
Max: $c + \lvert a \rvert = 0 + 2 = 2$	Min: $c - \lvert a \rvert = 0 - 2 = -2$

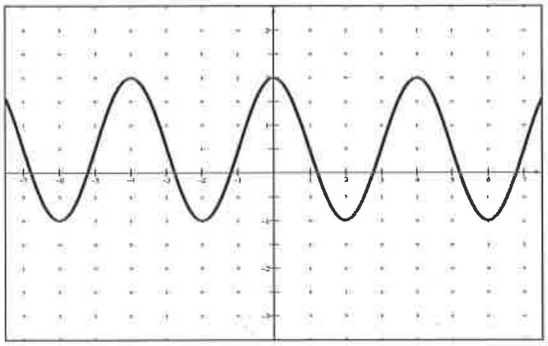
$y = \sin x$	
x	***y***
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$y = -2\sin\left(\frac{\pi}{6}x\right)$	
$\frac{6x}{\pi}$	$-2y$
0	0
3	-2
6	0
9	2
12	0



17) Model each graph shown as a sine and cosine function.

a)



$a = \frac{max - min}{2} = \frac{2-(-1)}{2} = \frac{3}{2} = 1.5$

$k = \frac{2\pi}{period} = \frac{2\pi}{4} = \frac{\pi}{2}$

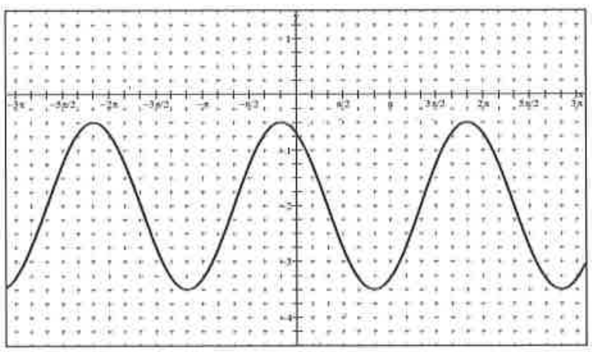
$c = max - |a| = 2 - 1.5 = 0.5$

$d_{cos} = 0$

$d_{sin} = d_{cos} - \frac{\pi}{2k} = 0 - \frac{\pi}{2(\frac{\pi}{2})} = 0 - \frac{\pi}{\pi} = 0 - 1 = -1$

$\boxed{y = 1.5\cos(\frac{\pi}{2}x) + 0.5 \quad y = 1.5\sin[\frac{\pi}{2}(x+1)] + 0.5}$

b)



$a = \frac{-0.5-(-3.5)}{2} = 1.5$

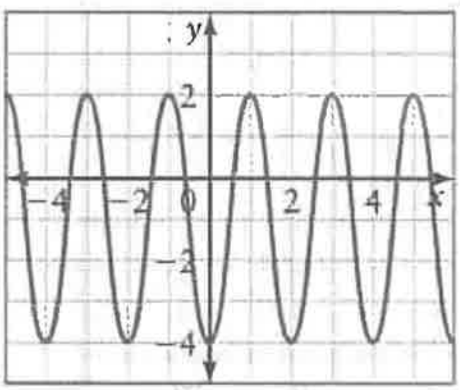
$k = \frac{2\pi}{2\pi} = 1$

$c = -0.5 - 1.5 = -2$

$d_{cos} = -\frac{\pi}{6}$

$d_{sin} = d_{cos} - \frac{\pi}{2k} = -\frac{\pi}{6} - \frac{\pi}{2} = \frac{-4\pi}{6} = -\frac{2\pi}{3}$

$\boxed{y = 1.5\cos(x + \frac{\pi}{6}) - 2 \quad y = 1.5\sin(x + \frac{2\pi}{3}) - 2}$



$a = \frac{2-(-4)}{2} = 3$

$k = \frac{2\pi}{2} = \pi$

$c = 2 - 3 = -1$

$d_{cos} = 1$

$d_{sin} = d_{cos} - \frac{\pi}{2k} = 1 - \frac{\pi}{2\pi} = 1 - \frac{1}{2} = \frac{1}{2}$

$\boxed{y = 3\cos[\pi(x-1)] - 1 \quad y = 3\sin[\pi(x - \frac{1}{2})] - 1}$

18) A cosine function has a maximum value of 7, a minimum value of 1, a period of $\frac{\pi}{2}$, and a phase shift of $\frac{3\pi}{4}$ radians to the right.

a) Write an equation for the function

$a = \frac{7-1}{2} = 3$

$d_{cos} = \frac{3\pi}{4}$

$k = \frac{2\pi}{(\frac{\pi}{2})} = \frac{4\pi}{\pi} = 4$

$c = 7 - 3 = 4$

$\boxed{y = 3\cos[4(x - \frac{3\pi}{4})] + 4}$

Write an equivalent sine equation for the function

$d_{sin} = d_{cos} - \frac{\pi}{2k} = \frac{3\pi}{4} - \frac{\pi}{2(4)} = \frac{6\pi}{8} - \frac{\pi}{8} = \frac{5\pi}{8}$

$\boxed{y = 3\sin[4(x - \frac{5\pi}{8})] + 4}$

Section 5: 5.3 – Trig Applications

19) Pitt Lake is a freshwater lake in southern British Columbia with the highest tidal change of any freshwater lake in the world. In a daily 24-hour period, the highest tide is traditionally at 11:00 am, reaching 5.8 m, and the lowest tide is traditionally at 11:00 pm, reaching only 0.4 m. (These are seasonal times and heights).

a) Determine a cosine function that gives the tidal height of the lake, h, in terms of the hours after midnight, t.

$a = \frac{5.8 - 0.4}{2} = 2.7$

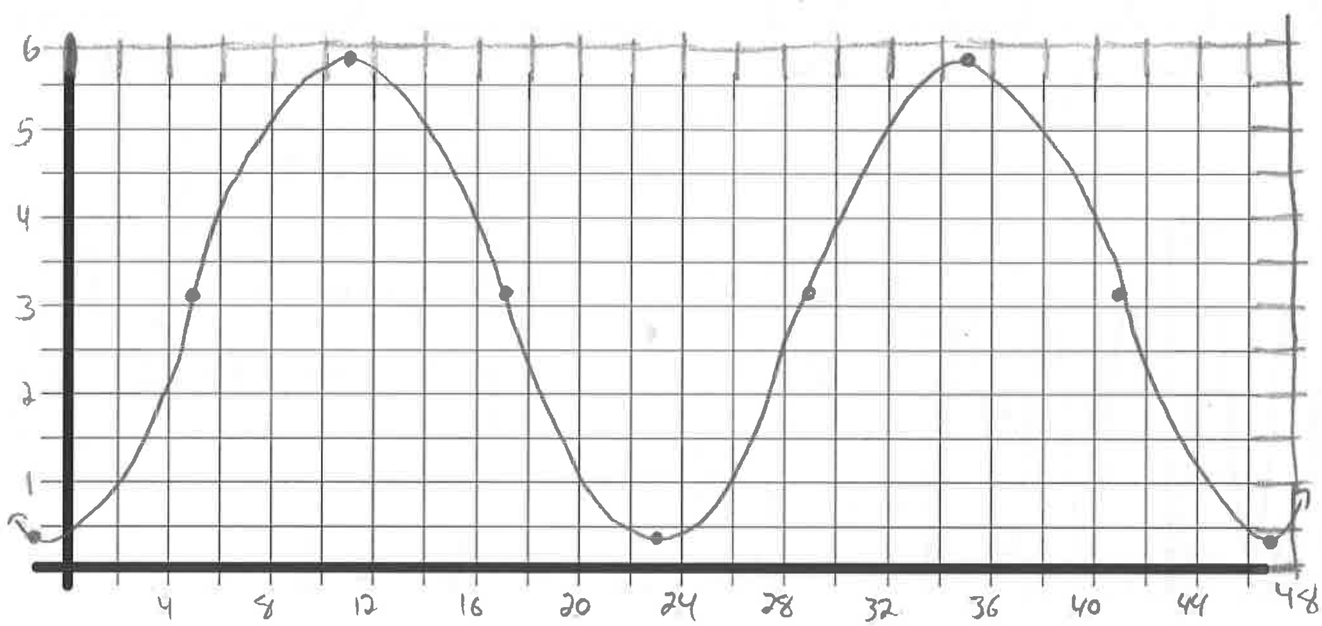
$d_{cos} = 11$

$k = \frac{2\pi}{24} = \frac{\pi}{12}$

$h(t) = 2.7\cos\left[\frac{\pi}{12}(t-11)\right] + 3.1$

$c = 5.8 - 2.7 = 3.1$

b) Create a sketch of this function (sketch first, label after)

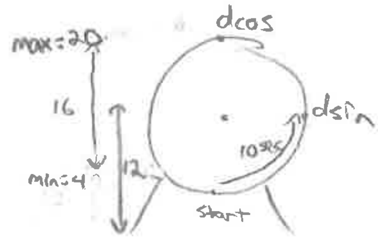


c) What is the tidal height at 1:30 pm?

$h(13.5) = 2.7\cos\left[\frac{\pi}{12}(13.5-11)\right] + 3.1$

$\approx 5.24\text{ m}$

20) A Ferris wheel at an amusement park completes one revolution every 40 seconds. The wheel has a diameter of 16 meters and its center is 12 meters above the ground.



Model the rider's height above the ground with a sine function

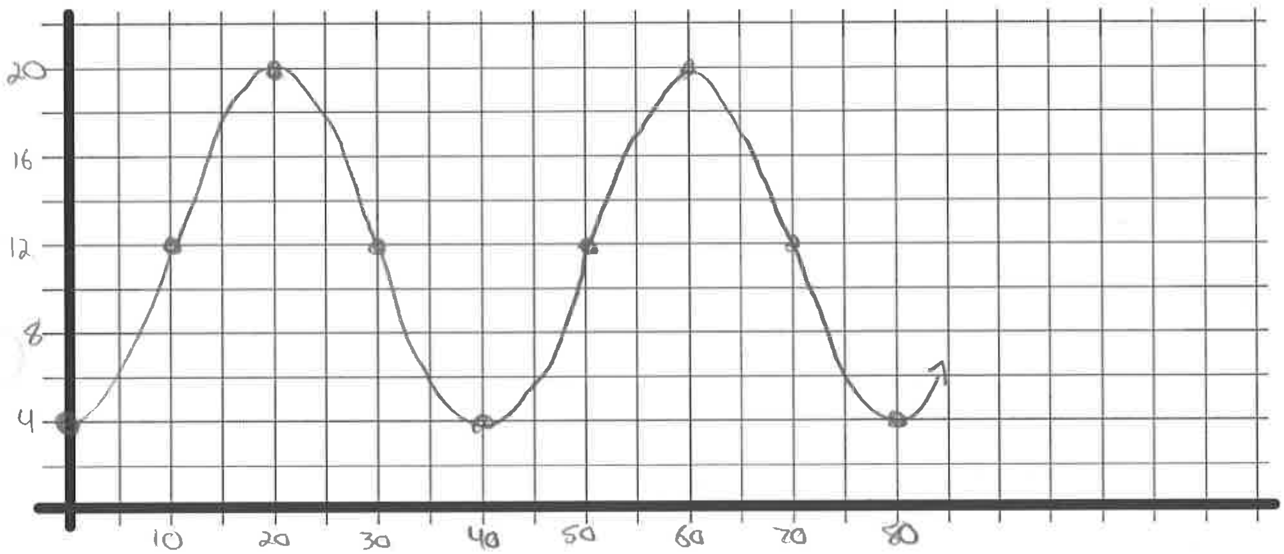
$a = \frac{20-4}{2} = 8$ $\qquad d_{sin} = 10$

$k = \frac{2\pi}{40} = \frac{\pi}{20}$

$\boxed{h(t) = 8\sin\left[\frac{\pi}{20}(t-10)\right] + 12}$

$c = 20 - 8 = 12$

b) Sketch a graph of the rider's height above the ground for 2 cycles.



21) A person who was listening to a siren reported that the frequency of the sound fluctuated with time, measured in seconds. The minimum frequency that the person heard was 500 Hz, and the maximum frequ3ency was 1000 Hz. The maximum frequency occurred at $t = 0$ and $t = 15$. The person also reported that, in 15 seconds, she heard the maximum frequency 6 times (including the times at $t = 0$ and $t = 15$). What is the equation of the cosine function that describes the frequency of this siren?

max at 0, 3, 6, 9, 12, 15

$a = \frac{1000-500}{2} = 250$

$k = \frac{2\pi}{3}$

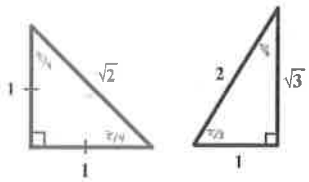
$\boxed{f(t) = 250\cos\left(\frac{2\pi}{3}t\right) + 750}$

$c = 1000 - 250 = 750$

$d_{cos} = 0$

Answer Key

1)a) 0.58 **b)** 2.41 **c)** 4.4 **d)** 6.06 **2)a)** 71° **b)** 161.6° **c)** 273.9 **d)** 395.9°
3)a) $\frac{5\pi}{12}$ **b)** $\frac{\pi}{9}$ **c)** $\frac{\pi}{15}$ **d)** $\frac{\pi}{20}$ **4)a)** 72° **b)** 80° **c)** 105° **d)** 110° **5)** 5.4 cm
6)



7)a) $-\frac{\sqrt{3}}{2}$ **b)** $\frac{1}{\sqrt{3}}$ **c)** $-\frac{1}{\sqrt{2}}$ **d)** $\frac{2}{\sqrt{3}}$ **e)** 0 **f)** -1
8) $cot\theta = -2 \quad and \quad sin\theta = \frac{1}{\sqrt{5}}$
9) $\sec\theta = \frac{13}{5}, \tan\theta = -\frac{12}{5}$
10) $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$; $\cos\frac{5\pi}{3} = \frac{1}{2}$; $\tan\frac{5\pi}{3} = -\sqrt{3}$; $\csc\frac{5\pi}{3} = \frac{-2}{\sqrt{3}}$; $\sec\frac{5\pi}{3} = 2$; $\cot\frac{5\pi}{3} = \frac{-1}{\sqrt{3}}$
11)a) 2 **b)** $\frac{\sqrt{2}+1}{\sqrt{2}}$ **c)** $\frac{-2\sqrt{6}+1}{2\sqrt{3}}$
12) $\frac{15\sqrt{3}}{4}$ m

See posted solutions for #13-16

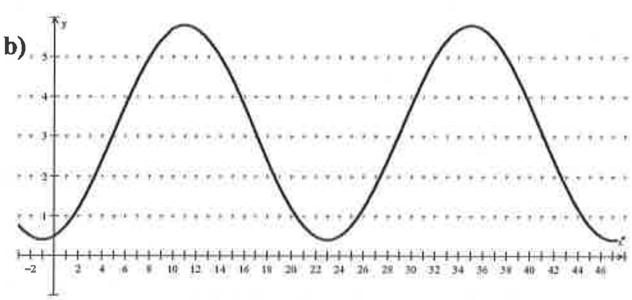
17)a) $y = \frac{3}{2}\cos\left(\frac{\pi}{2}x\right) + \frac{1}{2}; y = \frac{3}{2}\sin[\frac{\pi}{2}(x+1)] + \frac{1}{2}$

b) $y = \frac{3}{2}\cos\left(x + \frac{\pi}{6}\right) - 2; y = \frac{3}{2}\sin\left(x + \frac{2\pi}{3}\right) - 2$

c) $y = 3\cos[\pi(x-1)] - 1; y = 3\sin\left[\pi\left(x - \frac{1}{2}\right)\right] - 1$

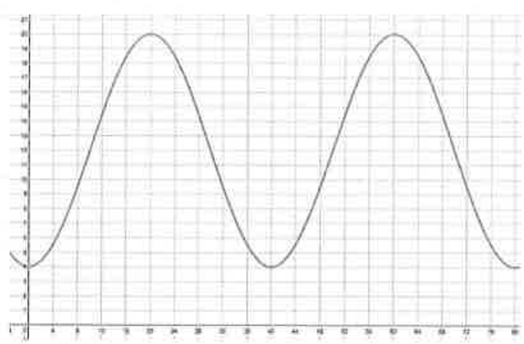
18)a) $y = 3\cos\left[4\left(x - \frac{3\pi}{4}\right)\right] + 4$ **b)** $y = 3\sin\left[4\left(x - \frac{5\pi}{8}\right)\right] + 4$

19) a) $h = 2.7\cos\left[\frac{\pi}{12}(t - 11)\right] + 3.1$ **b)**



c) 5.2 m

20)a) $y = 8\sin\left[\frac{\pi}{20}(t - 10)\right] + 12$ **b)**



21) $f(t) = 250\cos\left(\frac{2\pi}{3}t\right) + 750$